Lecture 2: Mechanics

Content:

- basic terms and quantities
- velocity and acceleration
- force, moment of force, momentum
- work, power
- mechanical energy
- Kepler’s laws
- Newton’s gravitational law
- free fall, motion in gravitational field

basic terms and quantities

The general study of the relationships between motion, forces, and energy is called **mechanics**.

Motion is the action of changing location or position.

Motion may be divided into three basic types - translational, rotational, and oscillatory.

The study of motion without regard to the forces or energies that may be involved is called **kinematics**. It is the simplest branch of mechanics.

The branch of mechanics that deals with both motion and forces together is called **dynamics** and the study of forces in the absence of changes in motion or energy is called **statics**.

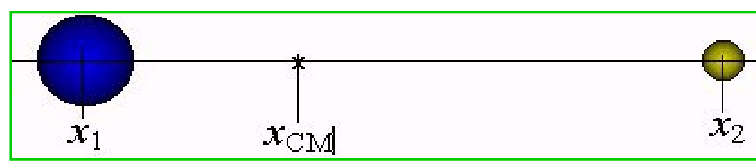
basic terms and quantities

Point (point mass) is a mass which doesn‘t have volume (mass focused into one point),

body (physical object) is an identifiable collection of matter, which may be more or less constrained to move together by translation or rotation,

rigid body is an idealization of a solid body in which deformation is neglected,

barycentre is is the center of mass of two or more bodies that are orbiting each other,



trajectory - path that a moving object follows through space as a function of time.

basic terms and quantities

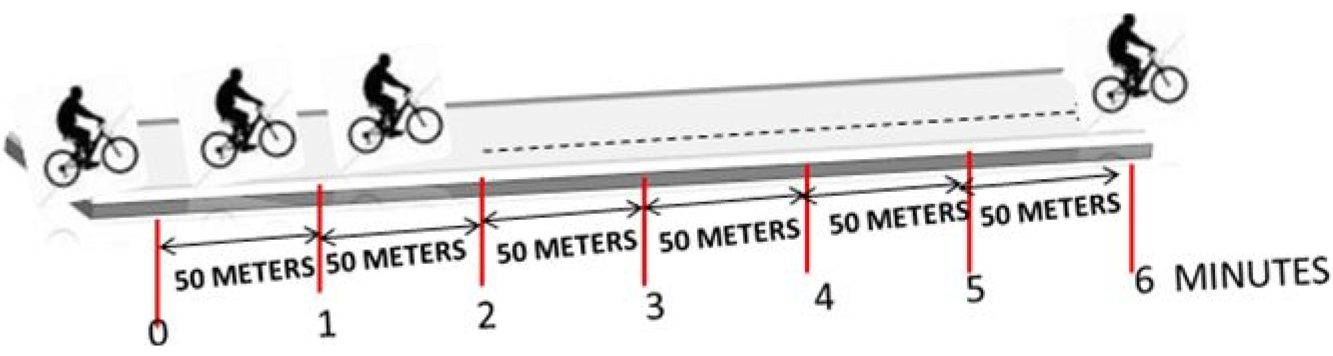
Velocity (speed): The velocity *v* of an object is the <u>rate of change of its position</u> with respect to a frame of reference, and is a function of time.

It is in general a vector quantity.

Its size can be evaluated:

$$v = \frac{ds}{dt} = s' \quad \left[m \cdot s^{-1}\right]$$

where s is path (distance) [m] and t is time [s].



basic terms and quantities

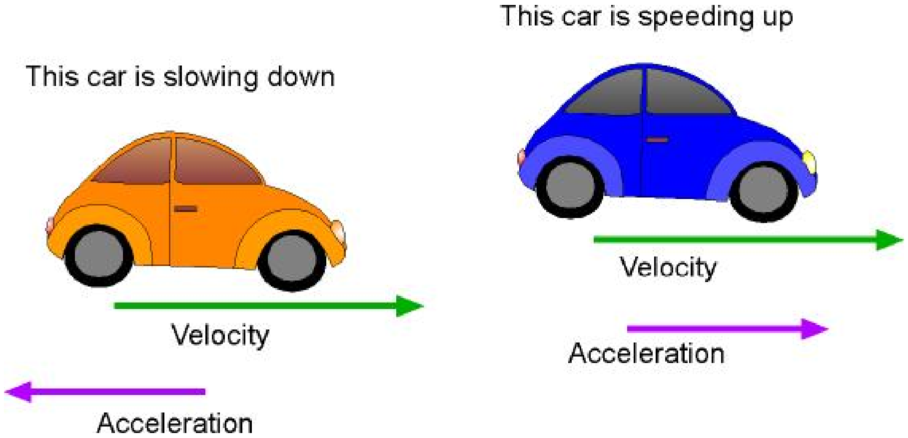
Acceleration: is the <u>rate of change</u> <u>of velocity</u> of an object.

It is in general a vector quantity.

Its size can be evaluated:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v' = s'' \left[m \cdot s^{-2}\right]$$

where v is velocity, s is path (distance) [m] and t is time [s].



basic terms and quantities

Angular velocity (speed): is the <u>rate of change of angular displacement (angle)</u> as a function of time.

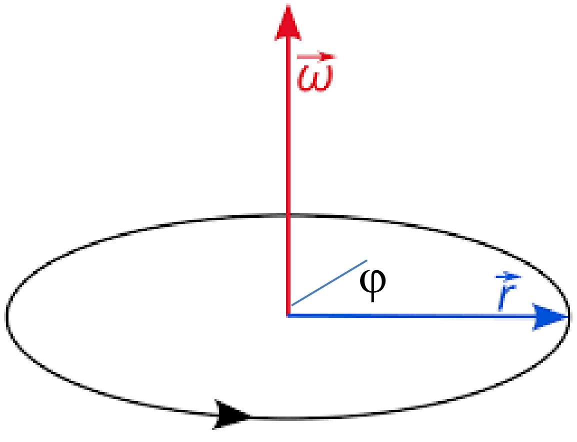
It is defined for rotating bodies (because tangential velocity is a function of radius).

It is a vector quantity (pointing in the direction of rotation axis).

$$\left|\vec{\omega}\right| = \frac{d\varphi}{dt}$$

Symbol: $\vec{\omega}$

Unit: radians per second [rad·s^{-1}]



Relationship between angular and tangential velocity: $\left|\vec{v}_t\right| = \left|\vec{\omega}\right| r$

And normal acceleration: $\left|\vec{a}_n\right| = \left|\vec{\omega}\right|^2 r$

Tangent vs normal component

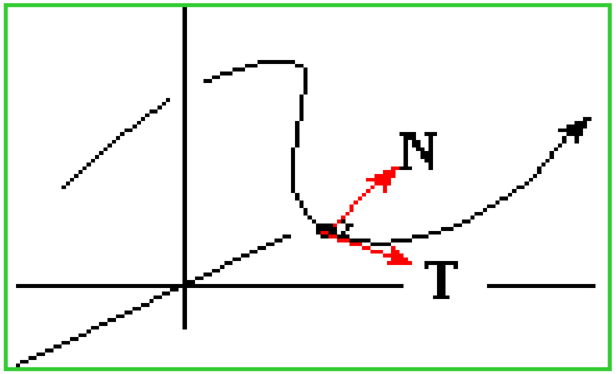
- each vector of velocity and/or acceleration can be split into 2 perpendicular components along a curvilinear path:

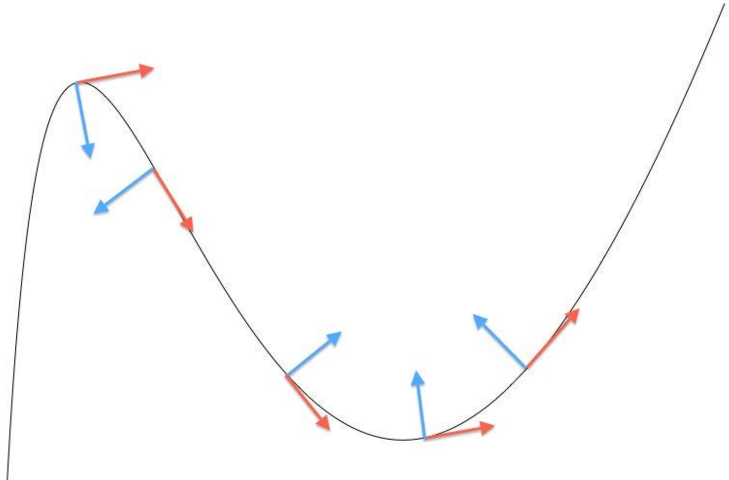
- tangent component (e.g. a_t)

(connected with the change of the vector size)

- normal component (e.g. a_n)

(connected with the change of vector direction)





classification of motions

a_t	a_n	a	motion
$a_t = 0$	$a_n = 0$	$a = 0$	uniform straight-line motion; velocity has constant direction and size
$a_t = 0$	$a_n \neq 0$	$a \neq 0$	uniform curvilinear motion; velocity has constant size, but the direction is changed (e.g. motion along a circular line)
$a_t =$ const.	$a_n = 0$	$a \neq 0$	uniformly accelerated, straight-line motion, velocity has the same direction, velocity size is changed
$a_t =$ const.	$a_n \neq 0$	$a \neq 0$	uniformly accelerated, curvilinear motion, velocity direction is changed, velocity size is changed
$a_t \neq$ const.	$a_n = 0$	$a \neq 0$	non-uniformly accelerated, straight-line motion, velocity has the same direction, velocity size is changed
$a_t \neq$ const.	$a_n \neq 0$	$a \neq 0$	non-uniformly accelerated, curvilinear motion, velocity direction is changed, velocity size is changed
		$a =$ const.	motion with constant acceleration (e.g. free fall in Earth gravity field)
a - total acceleration, $a = a_t + a_n$, a_t - tangent component of acceleration, a_n - normal component of acceleration,			

basic terms and quantities

Force: is any interaction that, when unopposed, will change the motion of an object. It is a vector quantity.

If the mass of the object is constant, this law implies that the acceleration of an object is directly proportional to the force acting on the object (in the direction of the force).

Its size can be evaluated (2. Newton‘s law):

$$\vec{F} = m\vec{a} \quad \left[kg \cdot m \cdot s^{-2}\right] = [N]$$

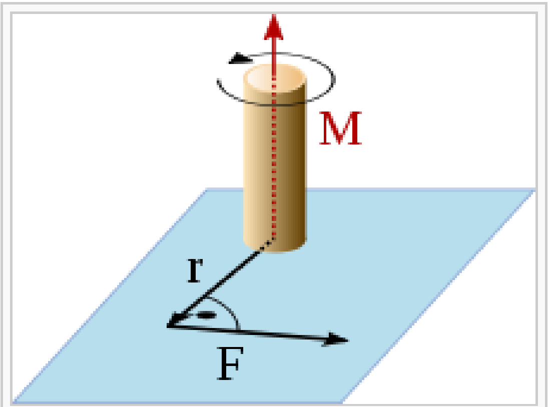
where m is mass [kg] and **a** is acceleration [m·s^{-2}].

It is a vector quantity.

basic terms and quantities

Moment of force: is the product of a force and its distance from an axis, which measures the rotation effect of the force (about that axis).

In general it is a combination of a physical quantity and a distance.



$$\vec{M} = \vec{F} \times \vec{r} \quad [N \cdot m] = [kg \cdot m^2 \cdot s^{-2}]$$

where **F** is force [N] and

r is distance vector [m].

It is in general a vector quantity.

basic terms and quantities

Momentum: is the product of the mass and velocity of an object. It is connected with the kinematic energy of the moving object.

It is in general a vector quantity.

$$\vec{p} = m\vec{v} \quad \left[\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-1}\right]$$

where m is mass [kg] and **v** is velocity [m·s^{-1}].

basic terms and quantities

Pressure: Pressure is the amount of force acting per unit area.

$$p = \frac{|\vec{F}|}{s} = \frac{F}{s}$$

where F is the size of normal force [N] and s is the area of the surface on contact [m^2].

Unit is pascal [Pa] = [N/m^2] = [$kg \cdot s^{-2} \cdot m^{-1}$].

Old unit was bar (1 bar = 100000 Pa).

It is in a scalar quantity.

Joke: Newton, Laplace and Pascal play hide-and-go-seek. Laplace starts to count, Pascal jumps behind a bush, but Newton stays on his original place and plots around him with a stick a small square into the soil. Laplace stops to count, see immediately Newton and screams “Newton”! But Newton answers: “No, no my dear friend, Newton over m squared is Pascal !!!”.

basic terms and quantities

Mechanical work:

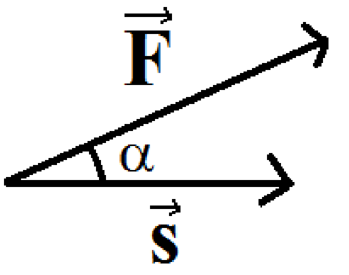
In mechanics, a force is said to do work W if, when acting on a body, there is a displacement of the point of application in the direction of the force.

It size is given by the product of force and distance.

Unit of work is joule [J] = [N·m] = [kg·m^2·s^{-2}].

Mathematically it is a scalar product of force and distance (vectors):

$$W = \vec{F} \cdot \vec{s}$$



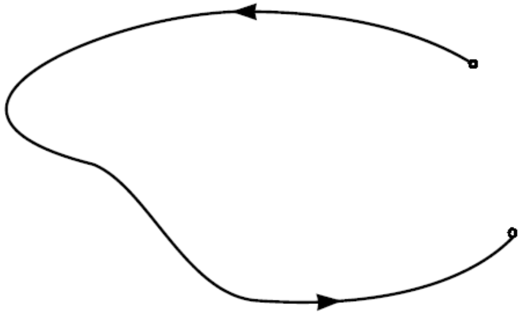
Size of scalar product is given:

$$W = \left|\vec{F}\right| \left|\vec{s}\right| \cos\alpha = F\, s \cos\alpha$$

It is a scalar quantity!

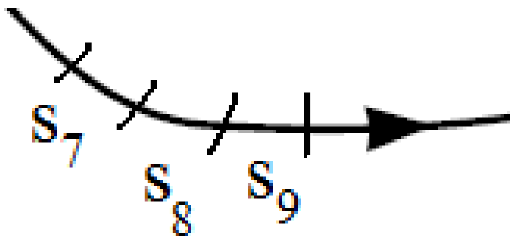
basic terms and quantities

Mechanical work:



But what to do, when the trajectory is not straight, but of irregular shape?

We can divide it into N small parts and evaluate work for each of them:



$$W = \sum_{i=1}^{N} \vec{F}_i \cdot \vec{s}_i$$

… and when the size of these small parts will be very small…?

$$W = \int_S \vec{F} \cdot d\vec{s}$$

where S is the path and ds its differential.

basic terms and quantities

Power:

Power is defined as the rate at which work is done upon an object. Like all rate quantities, power is a time-based quantity.

It is evaluated as the ration of work and time:

$$P = \frac{W}{t}$$

where W is work [J] and t is time [s].

Its unit is watt [W] = [J·s^{-1}] = [kg·m^{2}·s^{-3}].

It is a scalar quantity.

basic terms and quantities

Mechanical energy:

It is the energy associated with the motion and position of an object:

- kinetic energy (E_k),
- potential energy (E_p).

In so called conservative fields the sum of potential energy and kinetic energy is constant.

Additional energies in mechanics:

- energy of rotation body,
- elastic energy.

Unit of energy is identical with the unit of mechanical work (joule) [J].

basic terms and quantities

Mechanical energy:

kinetic energy (E_k) - the energy that it possesses due to its motion.

$$E_k = \frac{1}{2} mv^2$$

where m is the mass [kg] and

v the velocity [$m \cdot s^{-1}$].

This is valid in classical mechanics.

In relativistic mechanics, this is a good approximation only when v is much less than the speed of light.

kinetic energy - derivation:

Energy is connected with work: $\Delta E = F\Delta s$

for: $F = ma$ a $\Delta s \approx v\Delta t$

is valid: $\Delta E \approx mav\Delta t$

and: $a\Delta t = \Delta v$

is valid: $\Delta E \approx mv\Delta v$

But where we got the ½ in the result?:

$$\Delta(v^2) = (v + \Delta v)^2 - v^2 = 2v\Delta v + (\Delta v)^2 \approx 2v\Delta v \quad \Longrightarrow \quad v\Delta v = \frac{1}{2}\Delta(v^2)$$

We have ignored the term $(\Delta v)^2$, because it is a very small number (e.g. $0.01^2 = 0.0001$).

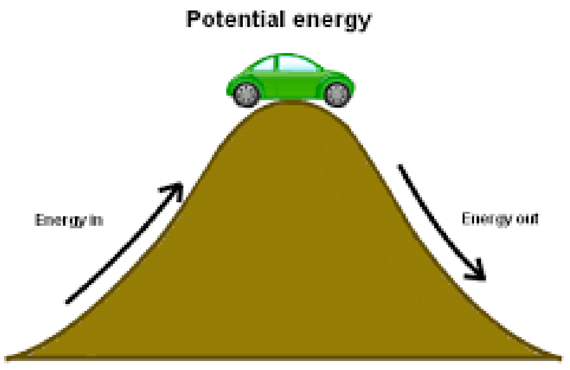
Final expression:

$$\boxed{\Delta E \approx \frac{1}{2}m\Delta(v^2) = \Delta(\frac{1}{2}mv^2)}$$

basic terms and quantities

Mechanical energy:

potential energy (E_p) - the energy that an object has due to its position in a force field (mostly gravitational field).



$$E_p = mgh$$

where m is the mass [kg],

g the gravitational acceleration [$m \cdot s^{-2}$]

and h the height [m].

The change of potential energy is dependent only from the height difference between two points and not from the trajectory of the motion between them.

basic terms and quantities

Moment of inertia: is a measure of an object's resistance to changes in the rotation direction.

For a point mass it can be expressed as:

$$I = mr^2 \quad \left[kg \cdot m^2\right]$$

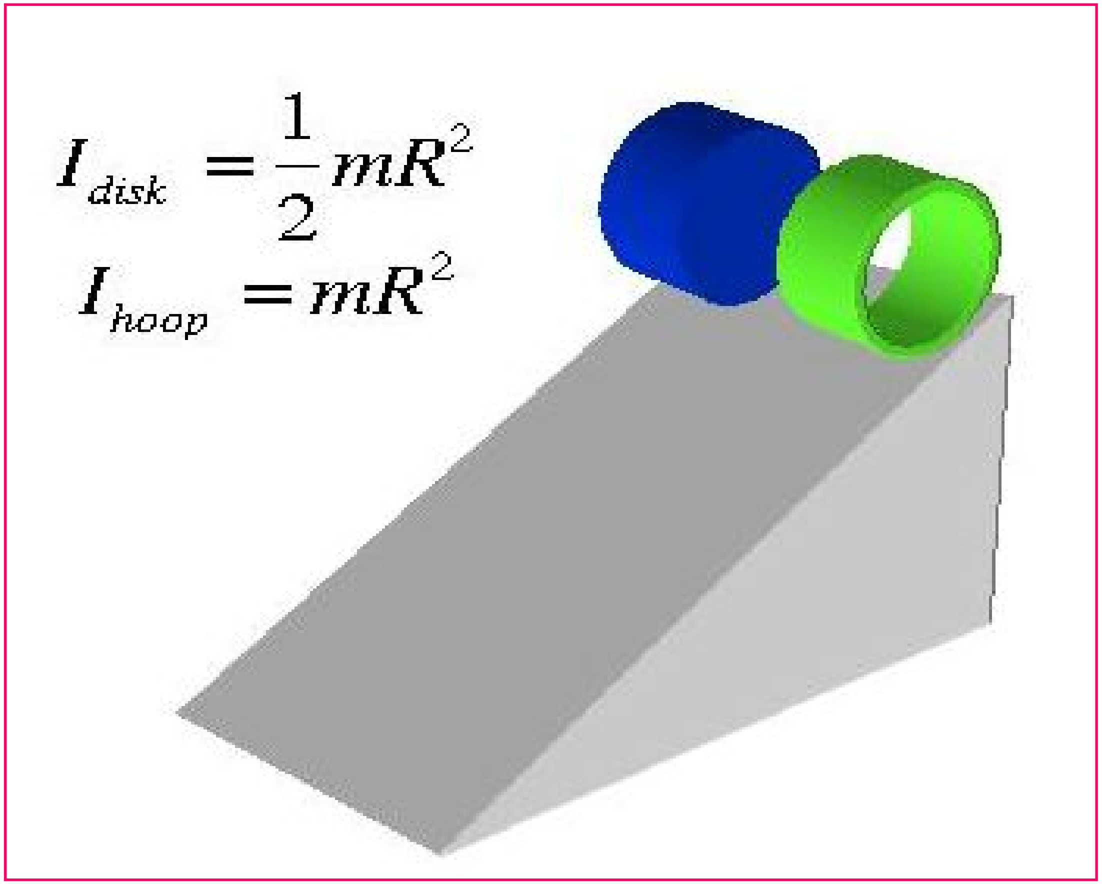
where r is the distance of the point mass from the rotation axis.

Energy of rotating body:

$$E_r = \frac{1}{2} I\omega^2$$

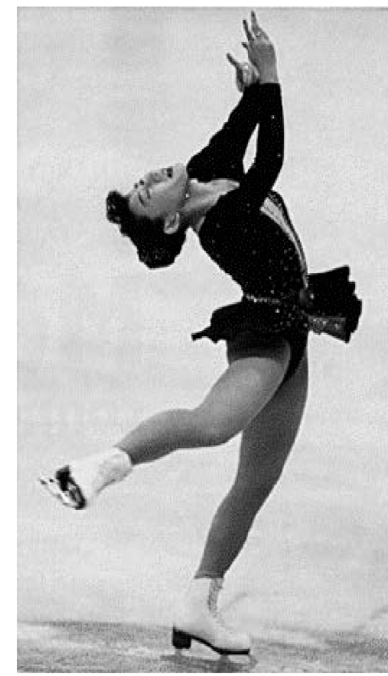
where ω is the size of angular velocity.

Energy of rotating body: **a trial**



$$E_r = \frac{1}{2} I\omega^2$$

$$I = mr^2$$



a) identical masses

b) different distances of masses from the centre

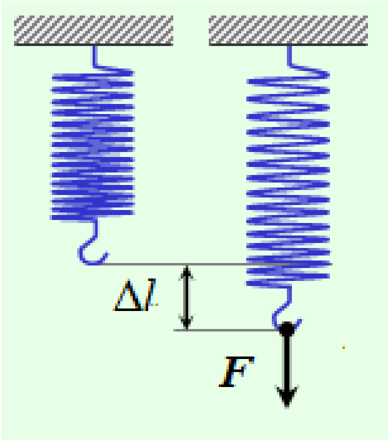
c) which one will move faster (will have higher ω)?

basic terms and quantities

Mechanical energy:

Elastic energy (E_{El}) - is the potential mechanical energy stored in the configuration of a material or physical system as work is performed to distort its volume or shape. Elastic energy occurs when objects are compressed and stretched, or generally deformed in any manner.

$$E_{El} = \frac{1}{2} k \Delta l^2$$

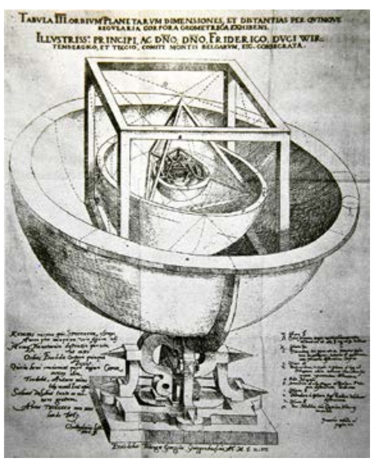


where *k* is so called spring constant and Δl is the length change of the spring.

gravitation

Kepler's Work

- Tycho Brahe led a team which collected data on the position of the planets (1580-1600 with no telescopes).
- mathematician Johannes Kepler was hired by Brahe to analyze the data.

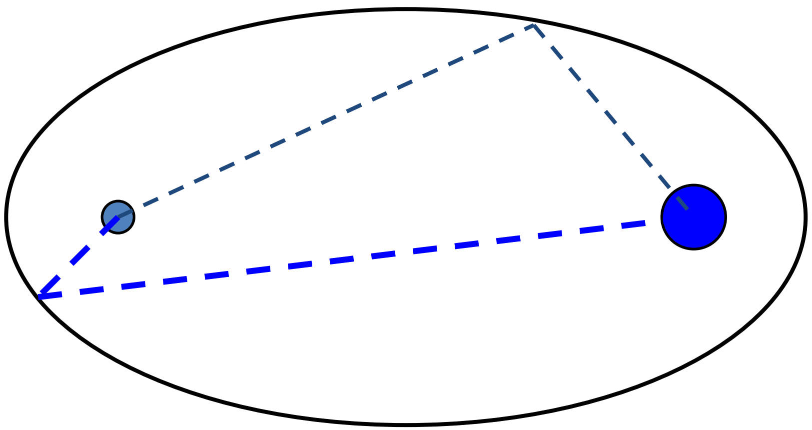


Johannes Kepler
1571 - 1630

- he took 20 years of data on position and relative distance.
- no calculus, no graph paper, no log tables.
- both Ptolemy and Copernicus were wrong.
- he determined three laws of planetary motion (1600-1630).

Kepler's First Law

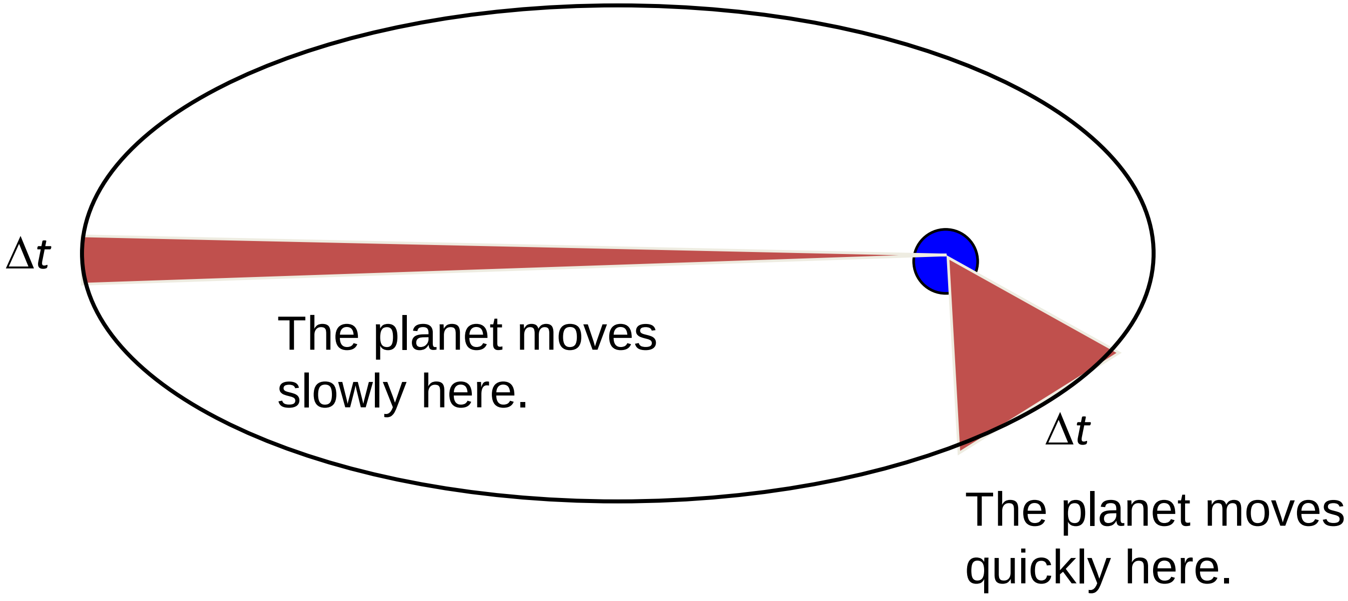
- The orbit of a planet is an ellipse with the sun at one focus.



A path connecting the two foci to the ellipse always has the same length.

Kepler’s Second Law

- The line joining a planet and the sun sweeps equal areas in equal time.



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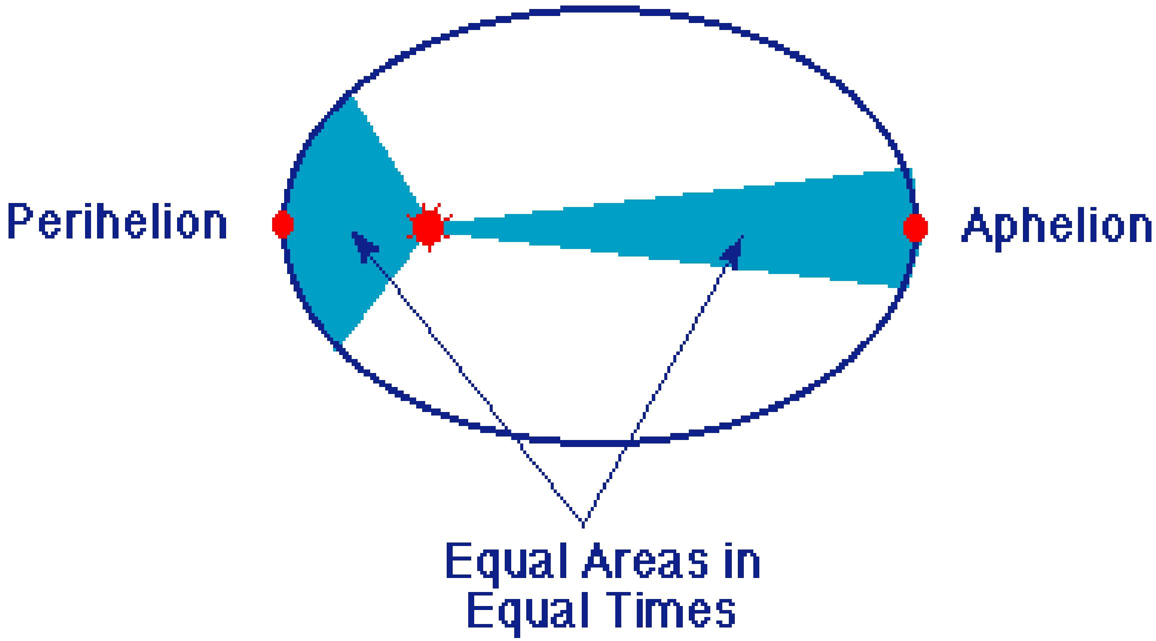
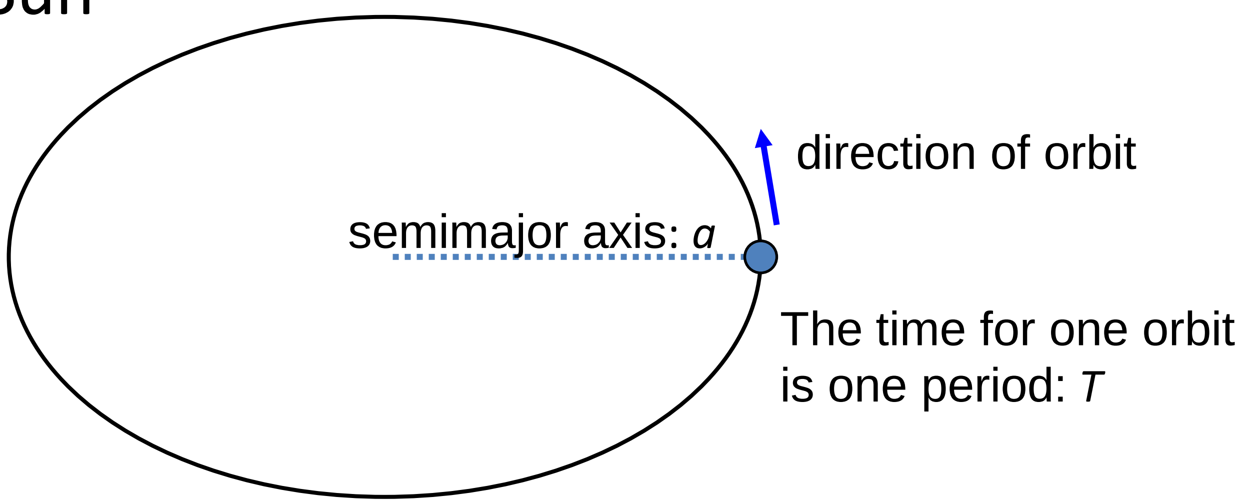


Kepler's Third Law

- The square of a planet's period is proportional to the cube of the length of the orbit's semimajor axis.
  - T^2/a^3 = constant
  - The constant is the same for all objects orbiting the Sun



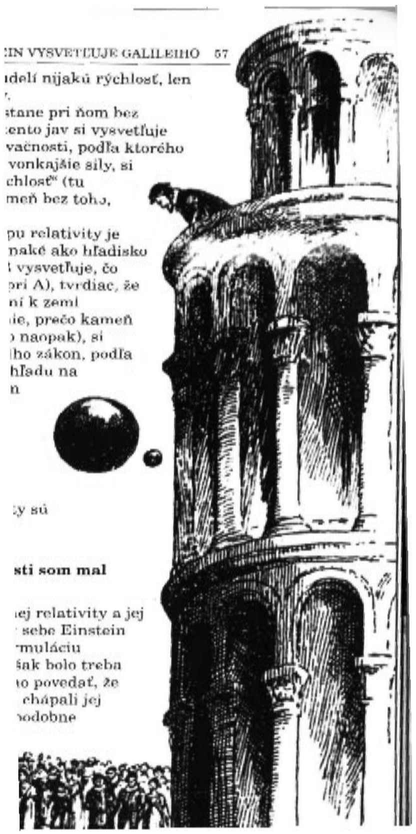
Kepler's Third Law

Example: *planets* ***Earthem*** *and* ***Jupiter***.
Jupiter's period is 11.86 year (11,86-times period of Earth), semimajor axis (compared to Earth) is 5.2-times larger.
So, it should be valid:

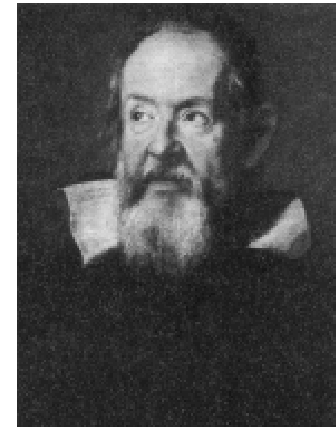
$$(11.86)^2/1^2 = (5.2)^3/1^3$$

$$140.66 \approx 140.61$$

Work of Galileo Galilei



- various contributions to
  the concept of modern science,
- mathematical derivations,
- astronomical observations,
- engineering experiments,
- free fall experiments
  (velocity is independent from the body mass
  - a contradiction to Aristotelian physics).



Galileo Galilei

(1564 - 1642)

Very nice trial (Brian Cox, vacuum chamber):
https://www.youtube.com/watch?v=E43-CfukEgs&feature=youtu.be
Experiment on the Moon (Apollo 15):
https://upload.wikimedia.org/wikipedia/commons/transcoded/e/e8/Apollo_15_feather_and_hammer_drop.ogv/Apollo_15_feather_and_hammer_drop.ogv.240p.webm

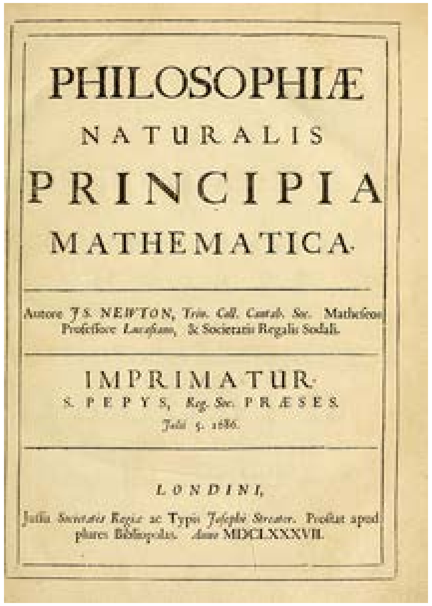
Newton’s Work

- laws of motion
- universal law of gravity
- mathematical derivation of Kepler’s laws
- introduction of calculus (derivatives)
- most important work:

  Philosophiæ Naturalis Principia Mathematica ("Mathematical Principles of Natural Philosophy"), first published 5 July 1687

  (later edited versions: 1713 and 1726)
- he spent the second half of his life in Royal mint

Isaac Newton
1643 - 1727



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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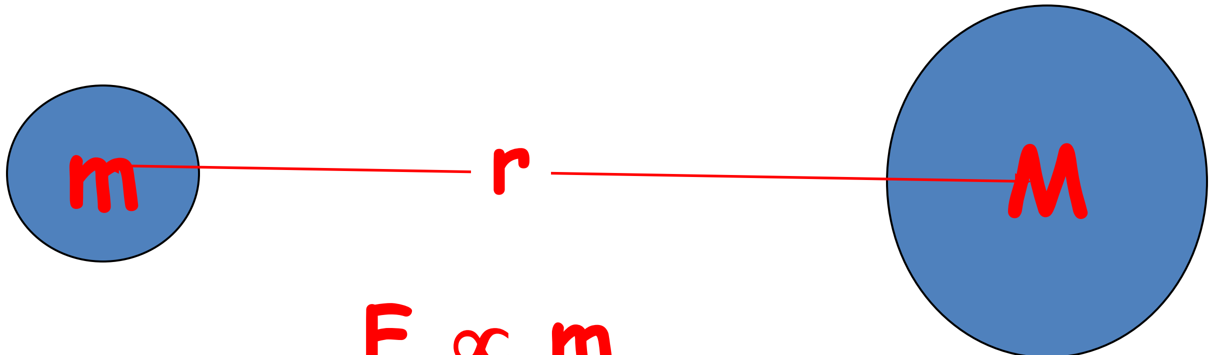
Universal law of gravity



$$F \propto m$$

$$F \propto M$$

$$F \propto \frac{1}{r^2}$$

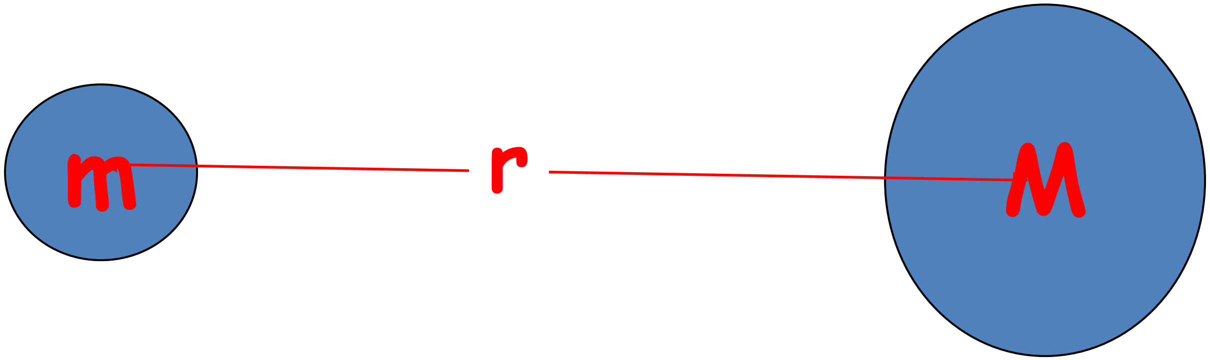
combine: $$F \propto \frac{mM}{r^2}$$

Proportionality constant:

"Newton's Constant"

$$F = G\,\frac{mM}{r^2}$$

Universal law of gravity



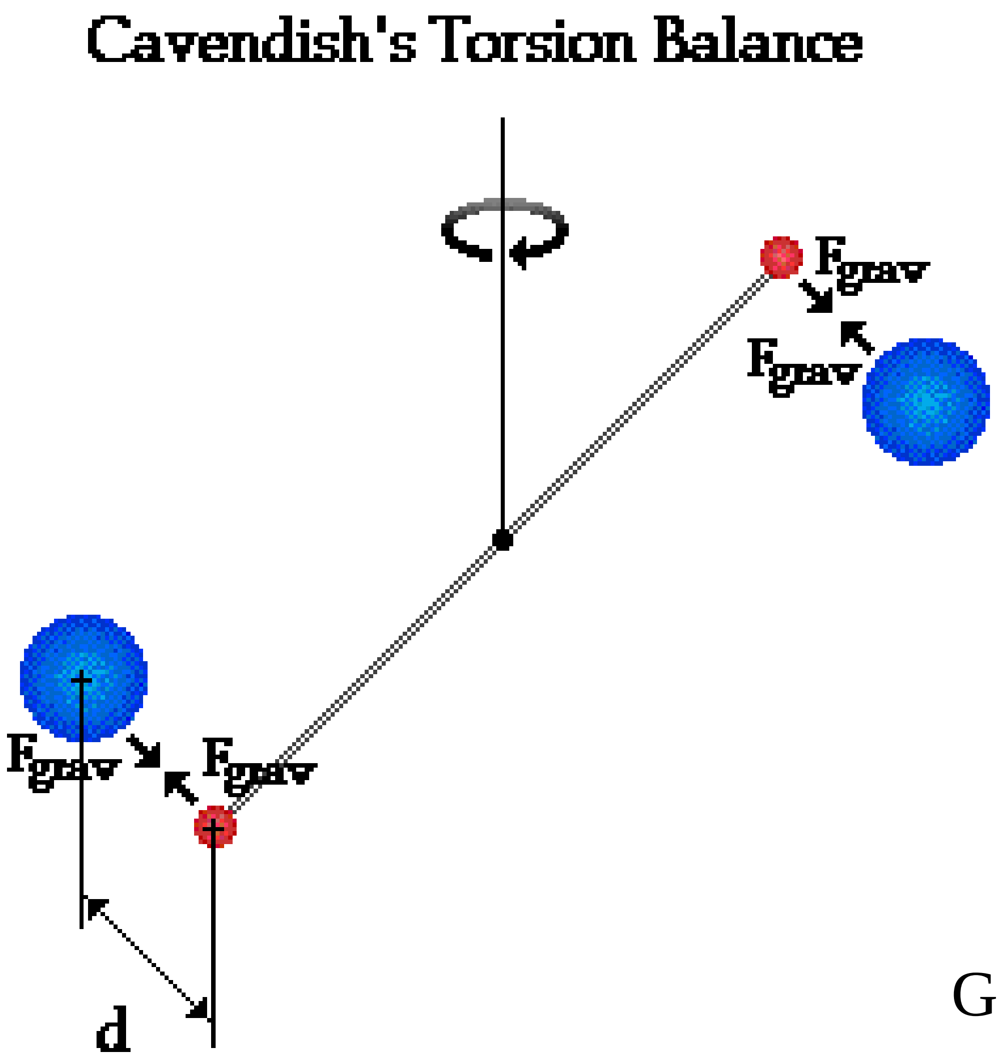
Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is <u>directly proportional to the product of their masses (*m*, *M*)</u> and <u>inversely proportional to the square of the distance between them (*r*)</u>.

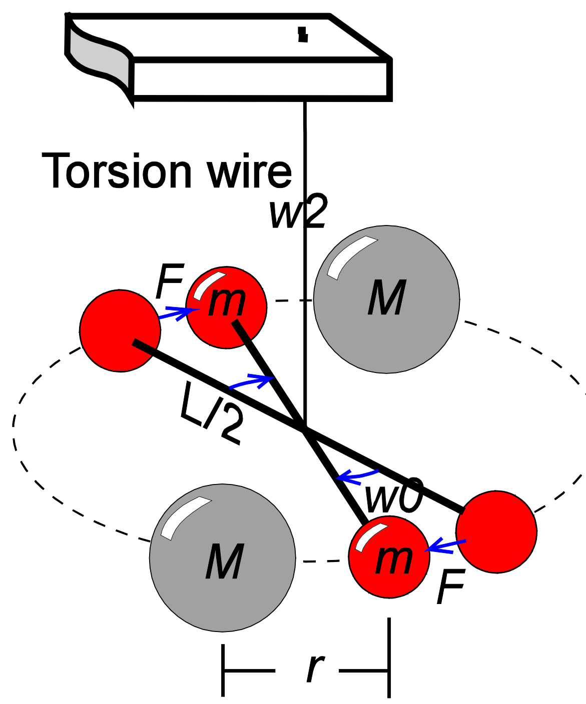
$$\left|\vec{F}_G\right| = G\frac{mM}{r^2}$$

$$G = \kappa = 6.67\cdot 10^{-11}\left[N\cdot m^2\cdot kg^{-2}\right]$$

G – is universal gravitational constant
(estimated for the first time by H. Cavendish in 1797-1798)

Measuring gravity force between "ordinary-sized" objects is very hard





$$G = \kappa = 6.74 \cdot 10^{-11} \left[N \cdot m^2 \cdot kg^{-2} \right]$$

gravitational acceleration (g):

Newton's gravity law: $$F = G\frac{mM}{r^2}$$

2. Newton's motion law: $$F = mg \Rightarrow g = \frac{F}{m} \Rightarrow \boxed{g = G\frac{M}{r^2}} \quad [\mathrm{m \cdot s^{-2}}]$$

Value of g?

In our country approx. 9.81 $\mathrm{m \cdot s^{-2}}$ (rounded 10 $\mathrm{m \cdot s^{-2}}$).

It is not a constant! Its value is influenced by many factors (rotation of Earth, distance from Earth center, large masses on the surface or below it).

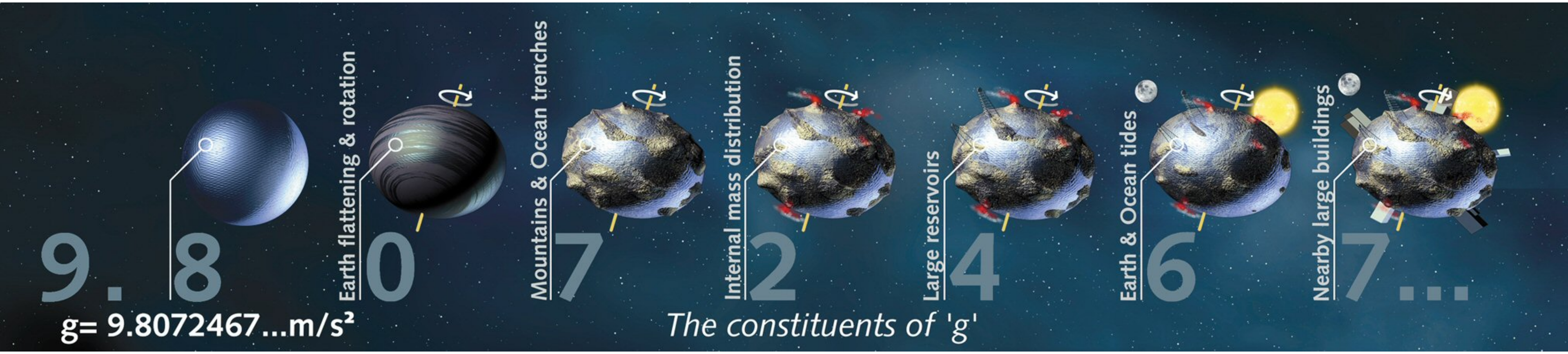
But in the same place on the Earth it acts on falling object independently on their mass (in vacuum).

gravitational acceleration (g):

Estimation of the g value for our Earth:
(mass of the Earth ~ $5.97 \cdot 10^{24}$ kg; radius ~ 6371000 m,
G ~ $6.74 \cdot 10^{-11}$ $N \cdot m^2/kg^2$)

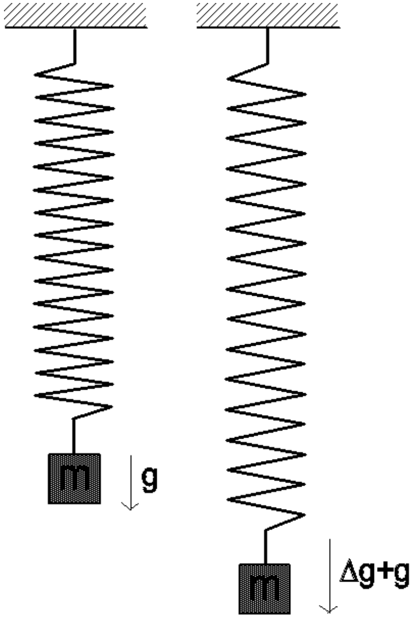
$$g = G\frac{M}{r^2} \approx 9.8 \quad [m \cdot s^{-2}]$$

In one point at the Earth surface g value is constant (independent from the mass), but it changes with the change of position (!)



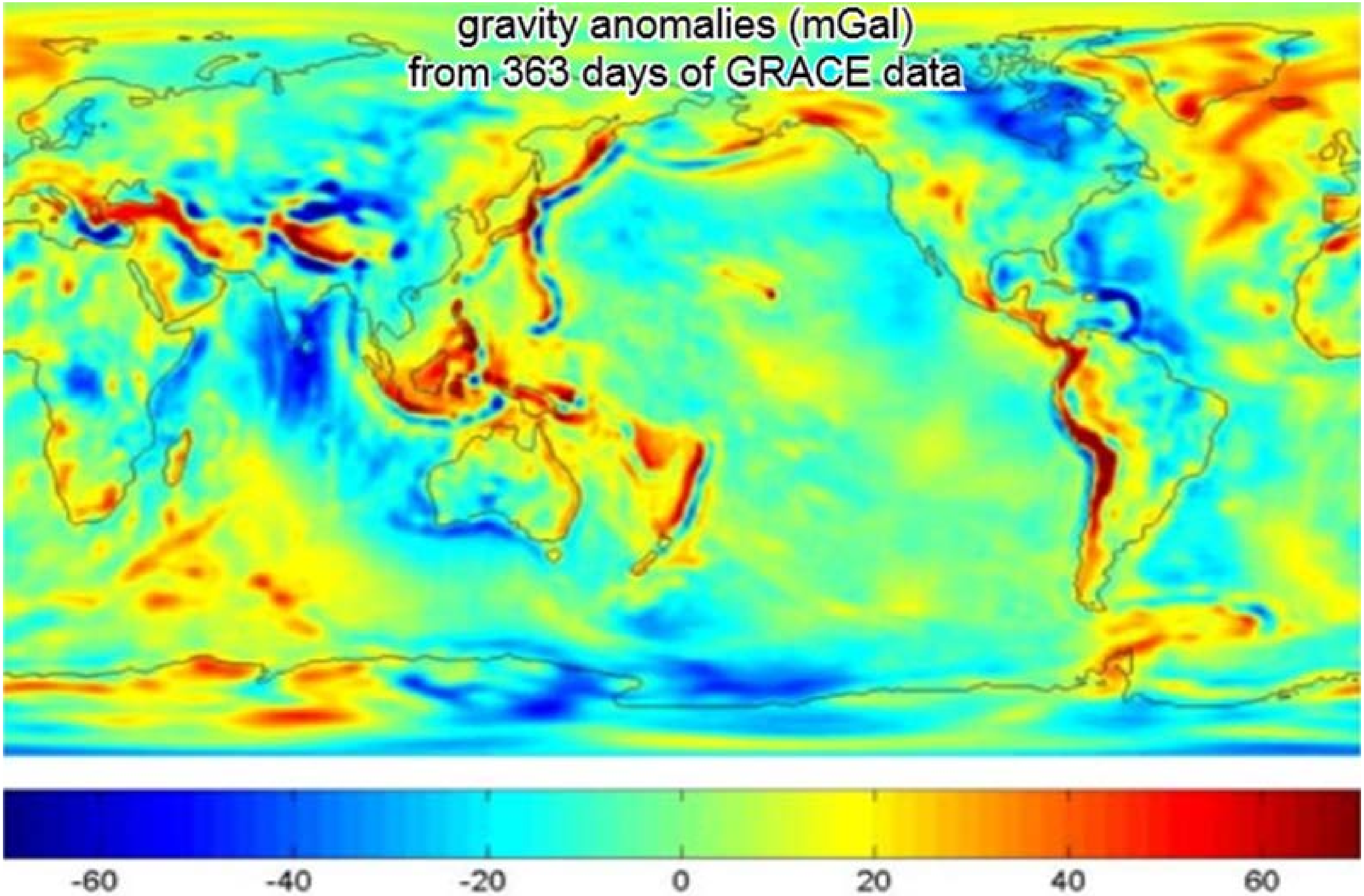
gravitational acceleration (g) - measurement:

free-fall instrument
(absolute gravimeter)

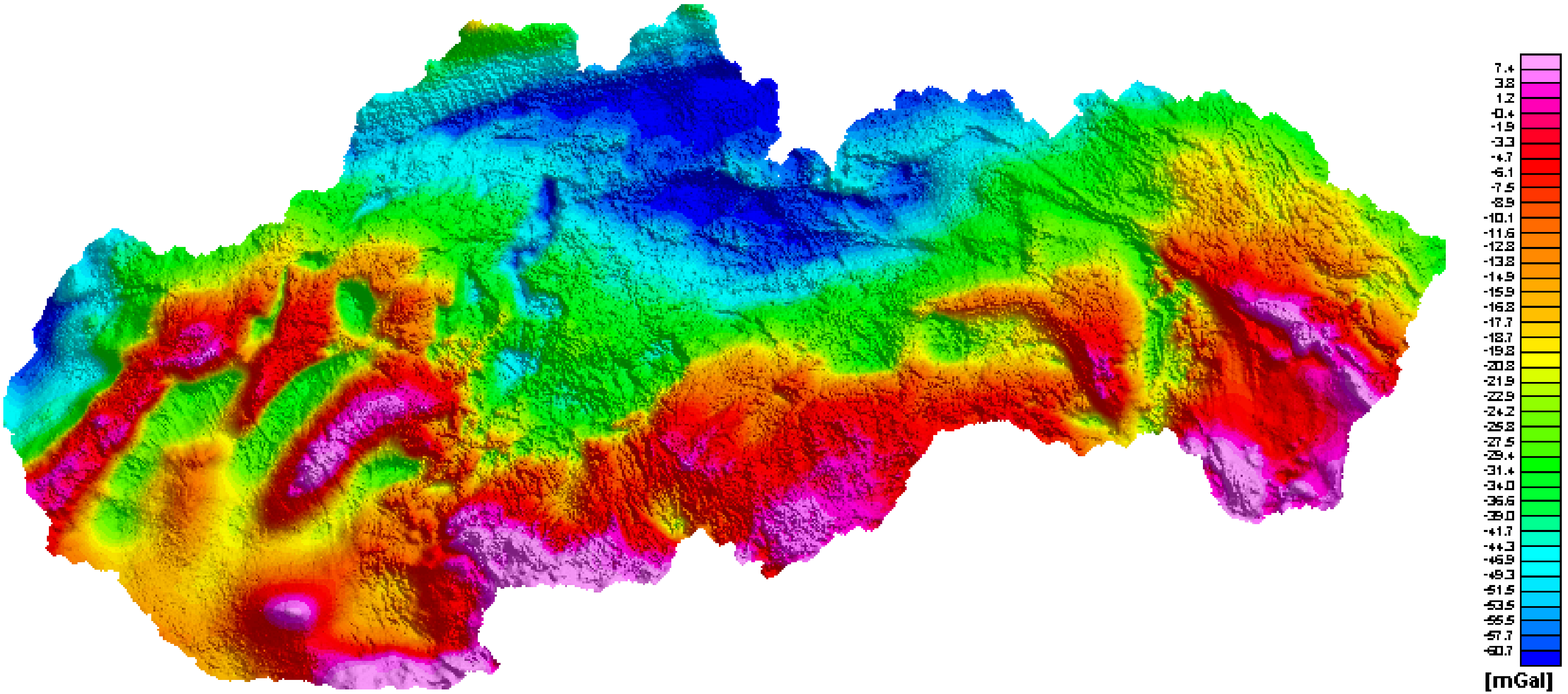


string instrument
(relative gravimeter)

gravity anomalies - worldwide



gravity anomalies - Slovakia



free fall – basic equations (1/2)

From 2. Newton's law of motion it follows:

$$mg = m\frac{\partial^2 s}{\partial t^2}$$

$$g = \frac{\partial^2 s}{\partial t^2}$$

Integrating this equation with respect to t, we get:

$$\int g dt = \int \left[\frac{\partial^2 s}{\partial t^2}\right] dt$$

$$g\int dt = \frac{\partial s}{\partial t} + c_1$$

$$gt + c_2 = \frac{\partial s}{\partial t} + c_1$$

$$gt = v + c_3$$

$$v = gt + v_0$$

Accepting the original condition that for the time t = 0 the initial velocity of the object at some level z_0 is v_0, we get: $c_3 = -v_0$.

free fall – basic equations (2/2)

$$v = gt + v_0$$

In further step we integrate this equation again with respect to t:

$$\int v\,dt = \int [gt + v_0]\,dt$$

$$s + c_4 = \int gt\,dt + \int v_0\,dt$$

$$s + c_4 = g\int t\,dt + v_0\int dt$$

$$s + c_4 = g\frac{t^2}{2} + c_5 + v_0 t + c_6$$

$$s = g\frac{t^2}{2} + v_0 t + c_7$$

$$\boxed{s = \frac{1}{2}gt^2 + v_0 t + z_0}$$

Accepting the original condition that for the time $t = 0$ the position of the object is the level z_0 we get: $c_7 = z_0$.

free fall

$$\boxed{s = \frac{1}{2} g t^2 + v_0 t + z_0}$$

When we take the original values for the time $t = 0$:
$z_0 = 0$ and $v_0 = 0$, we get the well known formula:

$$s = \frac{1}{2} g t^2$$

Example:

$t_1 = 1$ sec $\Rightarrow s_1 = 0.5\ g\ t_1^2 = 5 \cdot 1 = 5$ m

$t_2 = 2$ sec $\Rightarrow s_2 = 0.5\ g\ t_2^2 = 5 \cdot 4 = 20$ m

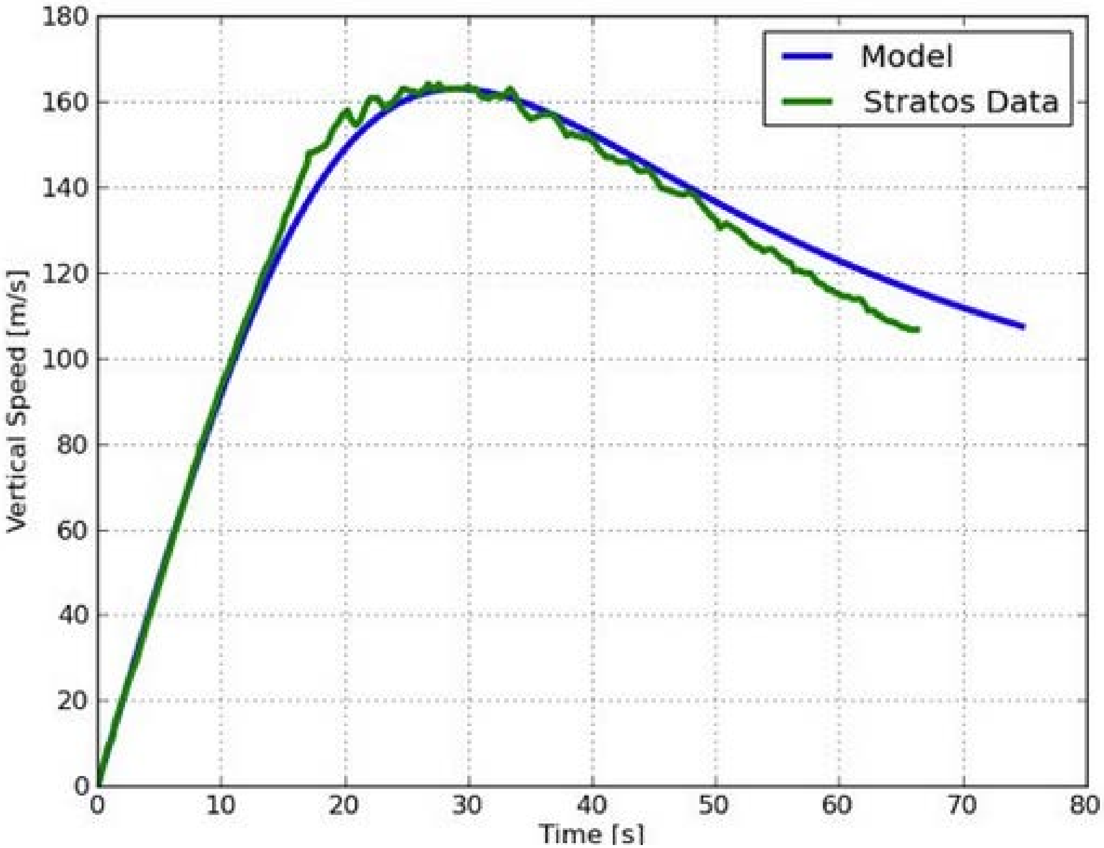
$t_3 = 3$ sec $\Rightarrow s_3 = 0.5\ g\ t_3^2 = 5 \cdot 9 = 45$ m

$t_4 = 4$ sec $\Rightarrow s_4 = 0.5\ g\ t_4^2 = 5 \cdot 16 = 80$ m

free fall

$$s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}}$$

Example: jump of Felix Baumgartner (2012)
height: 38 969 m
time: 4 min 20 sec.
Can we check it by means of free fall formula?



Air resistance:

$$\left|\vec{F}_{AIR}\right| = -k\left|v\right|^2$$

where k is the air resistance coefficient
$k = 0.24$ [kg/m]

motion in gravitational field

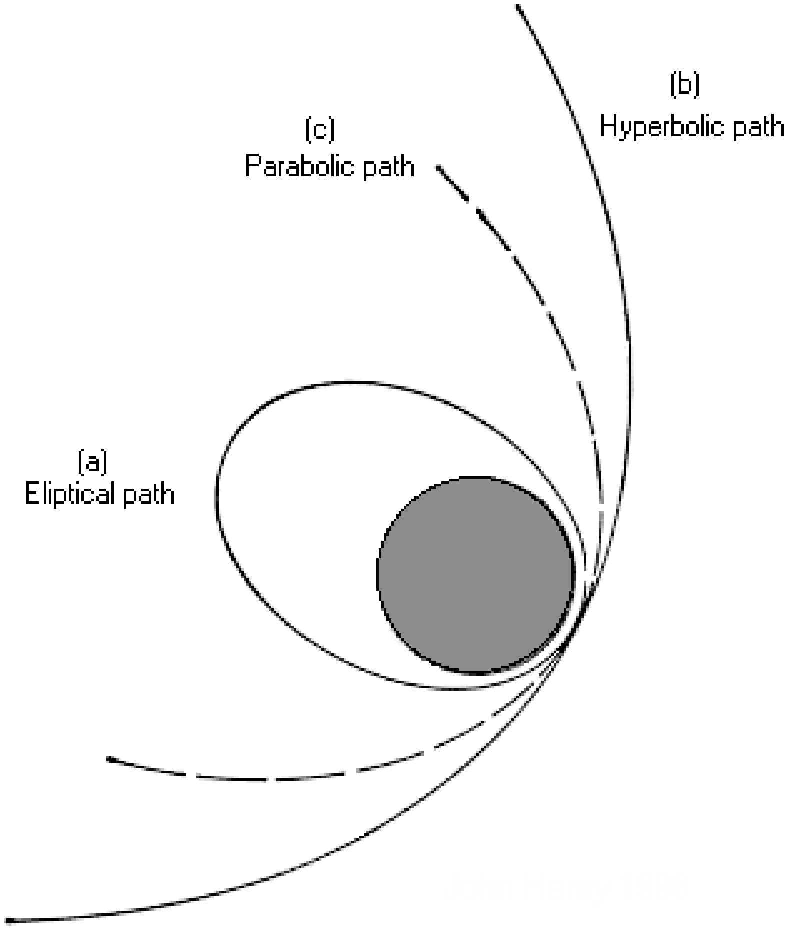
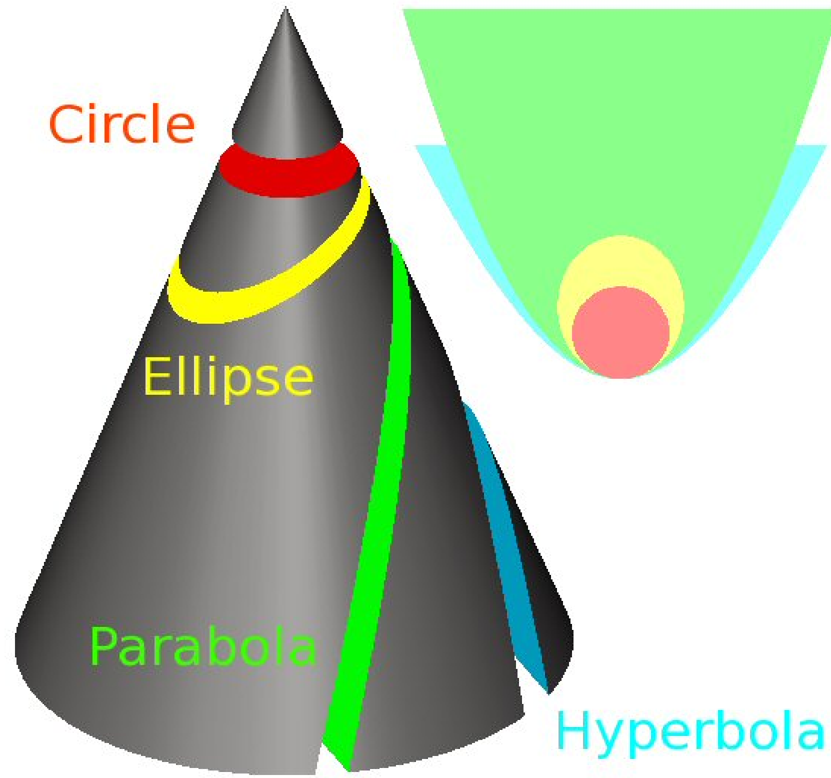


Fig-1: Types of paths



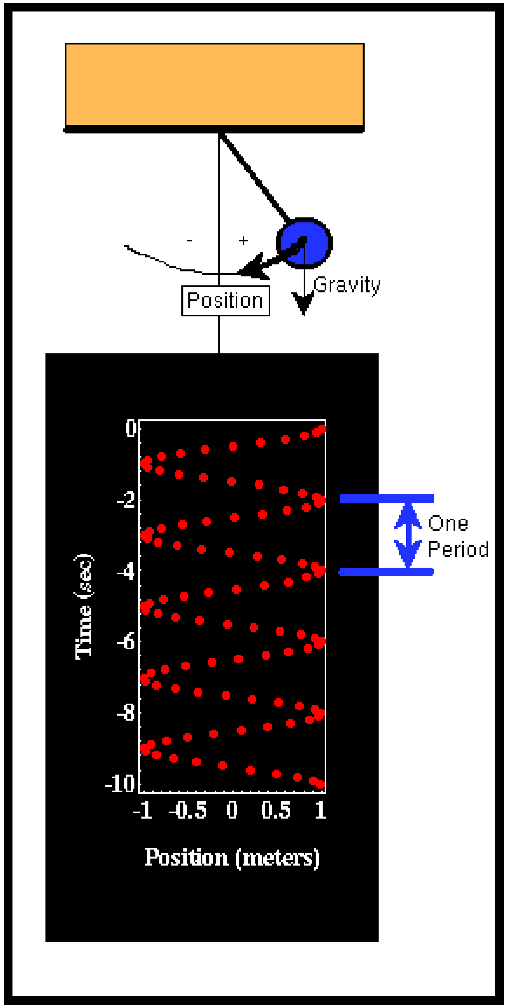
different orbits shapes

motion in gravitational field

When we move under some angle (not a free fall), equations Became little bit more complicated, but they can be solved.

Stroboscopic shoots of a moving ball that trajectory have shapes of parables (in fact parts of ellipses)

period of a mathematic pendulum (T):



Is also <u>independent on the mass of the object</u>, it is a function of the length *L* and gravitational acceleration *g*:

$$T = 2\pi\sqrt{\frac{L}{g}}\ [s]$$

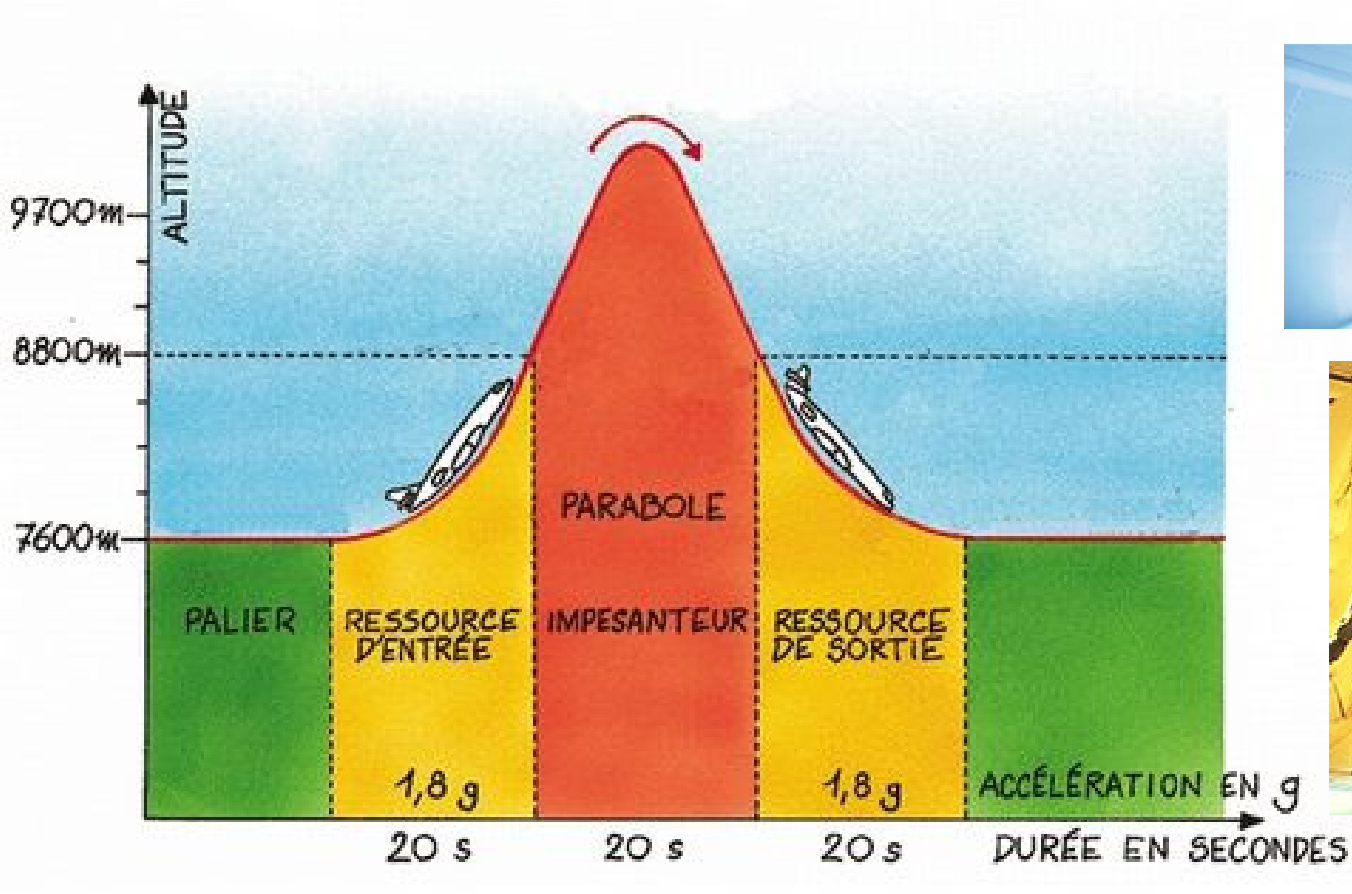
Mathematical derivation, e.g.:
http://dev.physicslab.org/Document.aspx?doctype=3&filename=OscillatoryMotion_PendulumSHM.xml

Walter Lewin – lecture MIT (video),
L = 5.21 m, g = 9.8 m/s^2,
estimation of pendulum period: 4.58 s

http://www.youtube.com/watch?v=KXys_mymMKA

zero G parable

simulation of weightless stage (in an aircraft)



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