Electrics

1. The two same size balls have electric charge $Q_1 = 24\cdot 10^{-6}\,C$ and $Q_2 = -18\cdot 10^{-6}\,C$. Find the attracting force, if they are separated by distance $r = 6\,cm$ in the vacuum. Next, find the repulsive force at the same distance, if the balls touch each other before separating.

Solution

a) According to the Coulomb's law we have:

$$\left|F_e\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = \left[\frac{24\cdot 10^{-6}\cdot 18\cdot 10^{-6}}{4\pi\cdot 8.859\cdot 10^{-12}\cdot 0.06^2}\right]N \approx \underline{\underline{1.078\cdot 10^3\,N}}.$$

b) When the balls touch each other, the charges are balanced, so the total charge of two balls will be:

$$Q_{1,2} = \left[24\cdot 10^{-6} - 18\cdot 10^{-6}\right]C = 6\cdot 10^{-6}\,C.$$

After the balls are separated, both will be of the same charge: $Q_1 = Q_2 = 3\cdot 10^{-6}\,C$. The repulsive force is then:

$$\left|F_e\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2} = \left[\frac{3\cdot 10^{-6}\cdot 3\cdot 10^{-6}}{4\pi\cdot 8.859\cdot 10^{-12}\cdot 0.06^2}\right]N \approx \underline{\underline{22.46\,N}}$$

2. There are two fixed charges separated by distance L. The charges are: $Q_1 = QC$ and $Q_2 = 4QC$. Find the position of the charge $\bar{Q}$ (on the abscissa connecting Q_1 and Q_2) that there will be no force acting on it.

Solution

The given condition will be fulfilled if the forces acting on the $\bar{Q}$ will be the same size but opposite sign. Let the distance between Q and $\bar{Q}$ be "x". Then, we have:

$$\frac{Q\bar{Q}}{4\pi\varepsilon x^2} = \frac{4Q\bar{Q}}{4\pi\varepsilon (L-x)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(L-x)^2} \Rightarrow 3x^2 - 2Lx - L^2 = 0 \rightarrow \begin{array}{l} x_1 = \frac{L}{3} \\ x_2 = -L \end{array}$$

While we are looking for the solution between Q_1 and Q_2 our solution is $\underline{\underline{x_1 = \frac{L}{3}}}$. So, the searched position is in the one third of the distance L measured from the lower charge

3. Find the intensity of the electric field in the point lying directly in the center between two charges $Q_1 = 50\,\mu C$ and $Q_2 = 70\,\mu C$ separated by distance $r = 20\,cm$. The charges are placed in the petroleum ($\varepsilon = 2\varepsilon_0$).

Solution

The total intensity **E** equals to the sum of the intensities **E1** and **E2**:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 = \frac{1}{4\pi \cdot 2\varepsilon_0} \frac{Q_1}{r_1^3} \boldsymbol{r}_1 + \frac{1}{4\pi \cdot 2\varepsilon_0} \frac{Q_2}{r_2^3} \boldsymbol{r}_2 \,,$$

where $r_1 = r_2 = \frac{r}{2} = 10\,cm;$ $\boldsymbol{r}_1 = -r_1 \boldsymbol{i};$ $\boldsymbol{r}_2 = r_2 \boldsymbol{i}$, where **i** is the unit vector in the x-direction. According to this, we can write:

$$\boldsymbol{E} = \frac{1}{4\pi \cdot 2\varepsilon_0} \left(-\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2} \right) = \frac{1}{8\pi \cdot 8.859} \left(\frac{70 \cdot 10^{-6}}{0.1^2} - \frac{50 \cdot 10^{-6}}{0.1^2} \right) \approx \underline{\underline{8.983 \cdot 10^6\, Vm^{-1}}}$$

4. Find the force acting on the pointed electric charge placed in the electrostatic field of the infinite metal plate with the constant surface charge density, if it is surrounded by vacuum.

Solution

The force we are looking for we have:

$$\boldsymbol{F_e} = Q\boldsymbol{E}$$

The electrostatic field in vicinity of infinitely large charged plate is homogenous, the intensity **E** can be searched as limit of the formula for the electrostatic field's intensity for circular plate with the radius enlarged to infinity ($R \to \infty$). Such formula is:

$$\boldsymbol{E} = \frac{Q}{2\pi\varepsilon_0 R^2}\left(1 - \frac{a}{\sqrt{R^2 + a^2}}\right)\boldsymbol{a}_0 ,$$

where a is the normal distance of the charge Q from the circular plate, $\mathbf{a}_0$ is the unit vector of the normal. Now, the surface density of the charge on the circular plate can be written as:

$$\sigma = \frac{Q}{\pi R^2} ,$$

so we have:

$$\boldsymbol{E} = \lim_{R \to \infty}\left[\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{R^2 + a^2}}\right)\boldsymbol{a}_0\right] = \frac{\sigma}{2\varepsilon_0}\boldsymbol{a}_0 .$$

The desired force is then:

$$\boldsymbol{F_e} = Q\boldsymbol{E} = \underline{\underline{\frac{\sigma Q}{2\varepsilon_0}\boldsymbol{a}_0}}$$