Thermodynamics

1. The steel made ball is falling from the height $h = 20\,m$ with the initial speed of $v_0 = 4\,m\,s^{-1}$ and after impact is bounced back to the height $h_0 = 4\,m$. What will be the increasing of the ball's temperature, if the 60% of the work (carried out by ball's deformation) is added to the total inner energy of the ball?

Solution

According to the law of the conservation of energy, the total energy of the ball instantly after its releasing was:

$$E = mgh + \frac{1}{2}mv_0^2 .$$

While $h_0 < h$ part of the energy is transmitted to the surround through deformation work and increased the inner energy of the ball. The deformation work is, according to conservation of energy is given by:

$$A = mgh + \frac{1}{2}mv_0^2 - mgh_0 .$$

Part of this work (60% of it: $A' = 0.6A$) contributes to the increasing of the ball's inner energy, what results into increasing of its temperature. The same increasing of the temperature could be made by accommodation of the heat Q from outside source:

$$Q = mc\Delta t ,$$

where Δt is increasing of the temperature and c is specific heat capacity. While the work A' and heat Q are equivalent, we can write:

$$A' = 0.6A = Q = mc\Delta t .$$

According to the second equation we have:

$$mc\Delta t = 0.6\left(mgh + \frac{1}{2}mv_0^2 - mgh_0\right) \Rightarrow \Delta t = \frac{0.6\left(mgh + \frac{1}{2}mv_0^2 - mgh_0\right)}{mc} = \frac{0.6\left[2g\left(h - h_0\right) + v_0^2\right]}{2c}$$

$$\Delta t = \frac{0.6\left[2g\left(h - h_0\right) + v_0^2\right]}{2c} = \left[\frac{0.6\left[2 \cdot 9.81(20-4) + 16\right]}{2 \cdot 450}\right]{}^{\circ}C = \underline{\underline{0.22^{\circ}C}}$$

2. The gas ($p_0 = 1013\cdot 10^2\ Pa;\quad t_0 = 20°C;\quad V_0 = 830\,dm^3$) was compressed. The needed work was $A = 166\,770\ J$. Find the final volume, pressure and temperature of the gas if the state change was controlled by equation $pV^n = const.$, where $n = 1.25$?

Solution

The work made by gas in the change of the volume is given:

$$A' = \int_{V_0}^{V_1} p\,dV\,.$$

While we are dealing with the compression, the work made by gas is negative, because the work necessary for the compression came from outside, so:

$$A = -A' = -\int_{V_0}^{V_1} p\,dV\,.$$

While:

$$p_0V_0^n = pV^n = const. \Rightarrow p = \frac{p_0V_0^n}{V^n}\,,$$

so we have:

$$A = -\int_{V_0}^{V_1} \frac{p_0V_0^n}{V^n}\,dV = -p_0V_0^n\int_{V_0}^{V_1}\frac{1}{V_1^n}\,dV = \frac{p_0V_0^n}{n-1}\left(V_1^{1-n} - V_0^{1-n}\right) \Rightarrow V_1^{1-n} = \frac{(n-1)A + p_0V_0}{p_0V_0^n}$$

$$V_1^{1-n} = \frac{(n-1)A + p_0V_0}{p_0V_0^n} \to \left[\frac{0.25\cdot 166\,770 + 1.013\cdot 10^5\cdot 0.83}{1.013\cdot 10^5\cdot (0.83)^{1.25}}\right] m^3 = V_1^{-0.25} \Rightarrow V_1 = \left[\frac{0.25\cdot 166\,770 + 1.013\cdot 10^5\cdot 0.83}{1.013\cdot 10^5\cdot (0.83)^{1.25}}\right]^{-4} m^3 = \underline{\underline{0.166\ m^3}}$$

The resultant pressure came from the equation:

$$p_0V_0^n = p_1V_1^n \Rightarrow p_1 = p_0\frac{V_0^n}{V_1^n} \to p_1 = 1.013\cdot 10^5\left(\frac{0.83}{0.166}\right)^{1.25} \approx 757\,394\,Pa\,.$$

The final temperature is obtained with help of state equation (pV = nRT):

$$\frac{p_0V_0}{T_0} = \frac{p_1V_1}{T_1} \Rightarrow V_1 = \frac{p_0V_0}{T_0}\frac{T_1}{p_1}\,.$$

We substitute the last equation into $p_0V_0^n = p_1V_1^n$ and we have:

$$T_1 = \left(\frac{p_0}{p_1}\right)^{\frac{1-n}{n}} T_0 \to T_1 = \left[\left(\frac{1013\cdot 10^2}{757\,394}\right)^{\frac{1-1.25}{1.25}} 293\right] K = 438.13K \approx \underline{\underline{165°C}}$$

3. Find the heat which has to be taken by cooling from the $m = 45\,g$ of CO_2 $\left(t_1 = -15°C,\, p_1 = 22.563 \cdot 10^4\,Pa\right)$ in the isothermal compression to pressure $p_2 = 56.898 \cdot 10^4\,Pa$!

Solution

The inner energy of ideal gas is not changed in the isothermal process. When the volume is changed from V_1 to V_2, the gas acts the external work A' by transformation of the received heat Q:

$$A' = Q = \int_{V_1}^{V_2} p\,dV \,.$$

The state equation is used:

$$pV = \frac{m}{M} RT \Rightarrow p = \frac{m}{M}\frac{RT}{V} \,.$$

Then:

$$A' = Q = \int_{V_1}^{V_2} \frac{m}{M}\frac{RT}{V} dV = \frac{m}{M} RT\, ln\left(\frac{V_2}{V_1}\right) .$$

The Boyle-Marriot law is applied in the isothermal state change:

$$\frac{V_2}{V_1} = \frac{p_1}{p_2} \,.$$

While $p_1 < p_2$ this work is negative number. The positive work must be provided from outside in the isothermal compression $A = -A'$, which is completely changed into heat Q', which is pass by the gas (and taken away by cooling). So, we have:

$$Q = A = -A' = -\frac{m}{M} RT\, ln\left(\frac{p_1}{p_2}\right) \rightarrow Q = -\left[\frac{45 \cdot 10^{-3} kg}{44 kg \cdot kmol^{-1}} \cdot 8314 JK^{-1} kmol^{-1} \cdot 258K \cdot ln\left(\frac{22.563 \cdot 10^4\,Pa}{56.898 \cdot 10^4\,Pa}\right)\right] J = \underline{\underline{2029 J}}$$

4. The ideal thermodynamic cycle engine with 1 mol of ideal gas as working medium, works in the cycle made of 3 reversible steps:
a) Isobaric warm up from original volume V_1 and original temperature T_1 to temperature T_2.
b) The adiabatic increasing of the volume until the temperature drops to the original value T_1.
c) The isothermal compression to the original volume V_1
What is the efficiency of this engine?

Solution

The efficiency is found by equation $\eta = \frac{A'}{Q}$, where A' is total work made by engine and Q is the heat taken by engine from hot reservoir.

a) The isobaric expansion – the gas increasing its volume to $V_2 = V_1 \frac{T_2}{T_1}$ and the work made by it is:

$$A'_a = \int_{V_1}^{V_2} p\, dV .$$

From the state equation we have: $p\,dV + V\,dp = R\,dV$. If the pressure is not changed $(dp = 0)$ we have $p\,dV = R\,dV$ and so:

$$A'_a = \int_{V_1}^{V_2} p\, dV \rightarrow \int_{T_1}^{T_2} R\, dT = R(T_2 - T_1).$$

The inner energy is changed by:

$$\Delta U = U_1 - U_2 = \int dU = \int_{T_1}^{T_2} C_V\, dT = C_V (T_2 - T_1),$$

where C_V is heat capacity at constant volume.
The heat taken by gas is found by I. thermodynamics law:

$$Q_a = A'_a + \Delta U = R(T_2 - T_1) + C_V (T_2 - T_1) = C_p (T_2 - T_1),$$

where C_p is heat capacity at constant pressure.
b) In the adiabatic expansion the volume is increased from V_2 to V_3 and its temperature drops from T_2 to $T_3 = T_1$. The relationship between temperature and volume is known: $TV^{\kappa-1} = const.$, where κ is the Poisson constant. The V_3 is then:

$$T_3 V_3^{\kappa-1} = T_2 V_2^{\kappa-1} = T_2 \left(V_1 \frac{T_2}{T_1} \right)^{\kappa-1} \Rightarrow V_3 = V_1 \left(\frac{T_2}{T_1} \right)^{\frac{\kappa}{\kappa-1}} .$$

The work made by engine in the adiabatic expansion equals to the decreasing of its inner energy:

$$A'_b = \int_{V_2}^{V_3} p\, dV = -\int dU = -\int_{T_2}^{T_3=T_1} C_V\, dT = C_V (T_2 - T_1).$$

The heat was not taken nor obtained: $Q_b = 0$.

c) The isothermal compression results into original state of the gas with volume V_1, temperature T_1 and pressure $p_1 = \frac{RT_1}{V_1}$. The work made by engine is:

$$A_c' = \int_{V_3}^{V_1} p\, dV = \int_{V_3}^{V_1} \frac{RT_1}{V}\, dV = RT_1 \ln \frac{V_1}{V_3}$$. (this work is negative)

While we already have (from step **b)**) $\frac{V_3}{V_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}}$ we can write:

$$A_c' = -RT_1 \ln \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}}.$$

The gas release the positive heat $Q_c' = A_c = -A_c'$:

$$Q_c' = RT_1 \ln \left(\frac{V_3}{V_1}\right).$$

Finally, after all state changes the gas made total work:

$$A' = A_a' + A_b' + A_c' = R(T_2 - T_1) + C_V (T_2 - T_1) - RT_1 \ln \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}},$$

and receive the heat:

$$Q = Q_a = C_p (T_2 - T_1)$$

The efficiency of the engine is then:

$$\eta = \frac{A'}{Q} = \frac{(R + C_V)(T_2 - T_1) - RT_1 \ln \left(\frac{T_2}{T_1}\right)^{\frac{\kappa}{\kappa-1}}}{C_p (T_2 - T_1)}.$$

While $\frac{\kappa}{\kappa - 1} = \frac{C_p}{R}$ and $R = C_p - C_V$ we have:

$$\underline{\underline{\eta = 1 - \frac{T_1}{T_2 - T_1} \ln \frac{T_2}{T_1}}}.$$