Atomic physics, radioactivity

1. Find the radius of the first energy level of the Bohr's model of the hydrogen atom and find the speed of the electron on this level!

Solution

The electron is moving on the circle trajectory around the core. The Coulomb's attracting force is acting as centrifugal force, so we have:

$$m_0 \frac{v^2}{a} = \frac{e^2}{4\pi\varepsilon_0 a^2},$$

where a is the radius, v is the speed, e is elementary charge, m_0 is the mass of the electron. According to the Bohr's axiom we have:

$$2\pi a m_0 v = nh,$$

where n is quantum number and h is Planck's constant. The radius of the nth energy level is than given by:

$$a = \frac{\varepsilon_0 h^2}{\pi m_0 e^2} n^2 .$$

While we are searching for the radius of the first level (n = 1) we have:

$$a_1 = \frac{8.859 \cdot 10^{-12} \left(6.62 \cdot 10^{-34}\right)^2}{\pi \cdot 9.109 \cdot 10^{-31} \left(1.602 \cdot 10^{-19}\right)^2} \doteq \underline{\underline{0.53 \cdot 10^{-10} m}}$$

2. Find the total energy of the electron in the Bohr's model of hydrogen atom on the second energy level!

Solution

The total energy is sum of potential and the kinetic energy. The potential energy on the n[th] level with radius *a* with respect to the infinity we have:

$$E_p = \frac{1}{4\pi\varepsilon_0}\int_{\infty}^{a}\frac{e^2}{r^2}dr = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{a}$$

The kinetic energy is found with help of:

$$m_0\frac{v^2}{a} = \frac{e^2}{4\pi\varepsilon_0 a^2},$$

so we have:

$$E_k = \frac{1}{2}m_0 v^2 = \frac{e^2}{8\pi\varepsilon_0 a}.$$

The total energy is then:

$$E = E_p + E_k = -\frac{e^2}{4\pi\varepsilon_0 a} + \frac{e^2}{8\pi\varepsilon_0 a} = -\frac{e^2}{8\pi\varepsilon_0 a}.$$

While

$$a = \frac{\varepsilon_0 h^2}{\pi m_0 e^2}n^2$$

we have:

$$E = -\frac{m_0 e^4}{32\varepsilon_0^2 h^2}.$$

Finally, for n = 2 we have:

$$E_2 = -\frac{9.109\cdot10^{-31}\left(1.602\cdot10^{-19}\right)^4}{32\cdot\left(8.859\cdot10^{-12}\right)^2\left(6.62\cdot10^{-34}\right)^2} \approx -5.45\cdot10^{-19}\,J = \underline{\underline{-3.4\,eV}}$$

3. The half-life of the radium is $T = 1622$ years. Find the time in which the total amount of not decayed atoms equals 0.1% of original amount!

Solution

$$N = 0.1\% N_0 = 0.001 N_0 \,,$$

The exponential decay is given by:

$$N = N_0 e^{-\lambda t} = N_0 e^{-\frac{ln2}{T}t} \,,$$

so we have:

$$0.001 = e^{-\frac{ln2}{T}t} \Rightarrow t = -\frac{ln 10^{-3} \cdot T}{ln 2} \approx 16164 \text{ years.}$$

4. The measurements in the dating of the Shroud of Turin gave following values – the concentration of ^{14}C was approximately $0.919 \cdot N_0$ where N_0 is its amount in today's living organisms. Find the age of the Shroud of Turin if the half-life of ^{14}C is 5730 years!

Solution

$$N = N_0 e^{-\lambda t} = N_0 e^{-\frac{ln 2}{T} t}$$

$$\frac{N}{N_0} = N_0 e^{-\frac{ln 2}{T} t} \Rightarrow t = \frac{T \, ln \frac{N}{N_0}}{ln 2} = \frac{5730 \cdot ln(0.919)}{ln 2} \approx \underline{\underline{698}} \; years$$