Special Theory of Relativity

Newtonian (Classical) Relativity

Assumption

- It is assumed that Newton's laws of motion must be measured with respect to (relative to) some reference frame.

Inertial Reference Frame

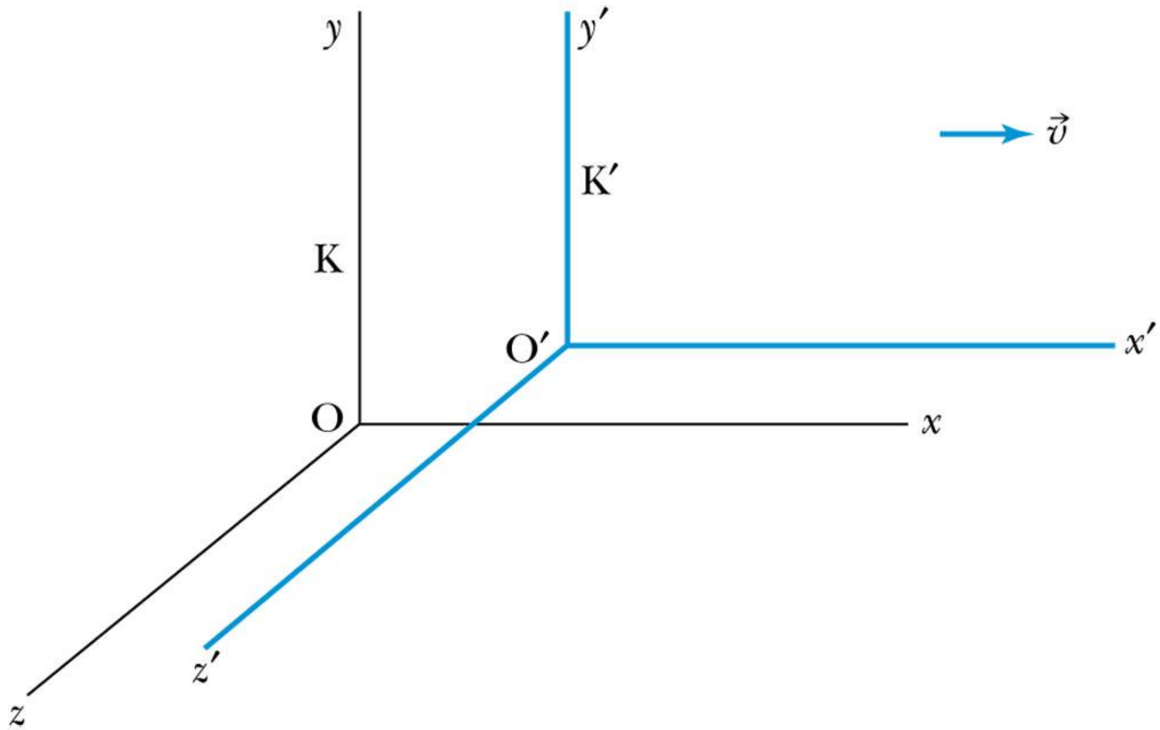
- A reference frame is called an **inertial frame** if Newton laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, is observed to move in rectilinear motion at constant velocity.

Newtonian Principle of Relativity

- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**

Inertial Frames K and K’



- K is at rest and K’ is moving with constant velocity $\vec{v}$
- Axes are parallel
- K and K’ are *INERTIAL COORDINATE SYSTEMS*

Conditions of the Galilean Transformation

- Parallel axes
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Transition to Modern Relativity

- Although Newton’s laws of motion had the same form under the Galilean transformation, Maxwell’s equations did not.
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton’s laws are only an approximation.

Ether

The wave nature of light suggested that there existed a propagation medium called the **ether**.

- ether had to have such a low density that the planets could move through it without loss of energy

- it had to have an elasticity to support the high velocity of light waves

- In Maxwell's theory the speed of light, in terms of the permeability and permittivity of free space, was given by

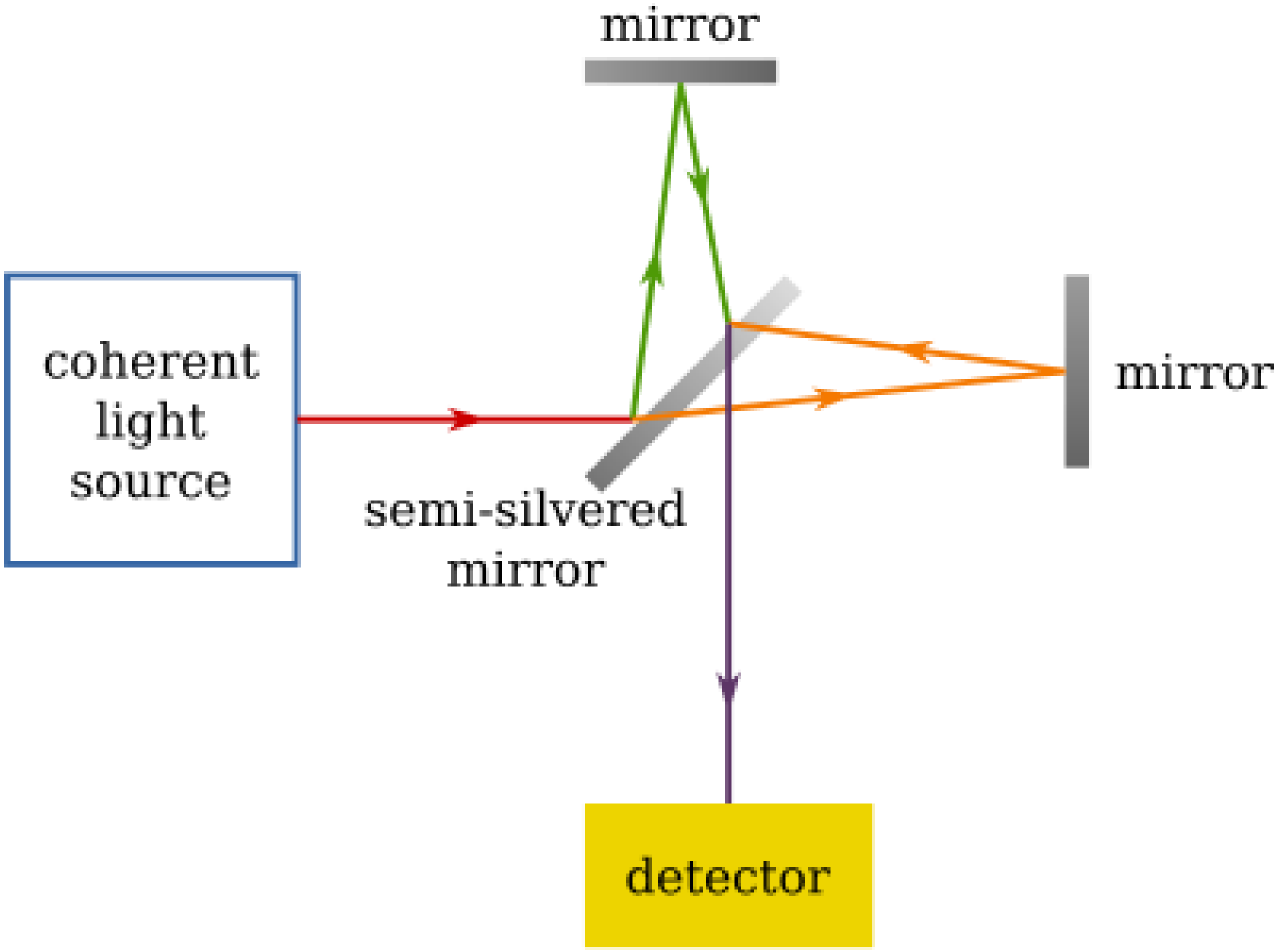
$$\mathrm{v} = \mathrm{c} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

- Thus the velocity of light between moving systems must be a constant.

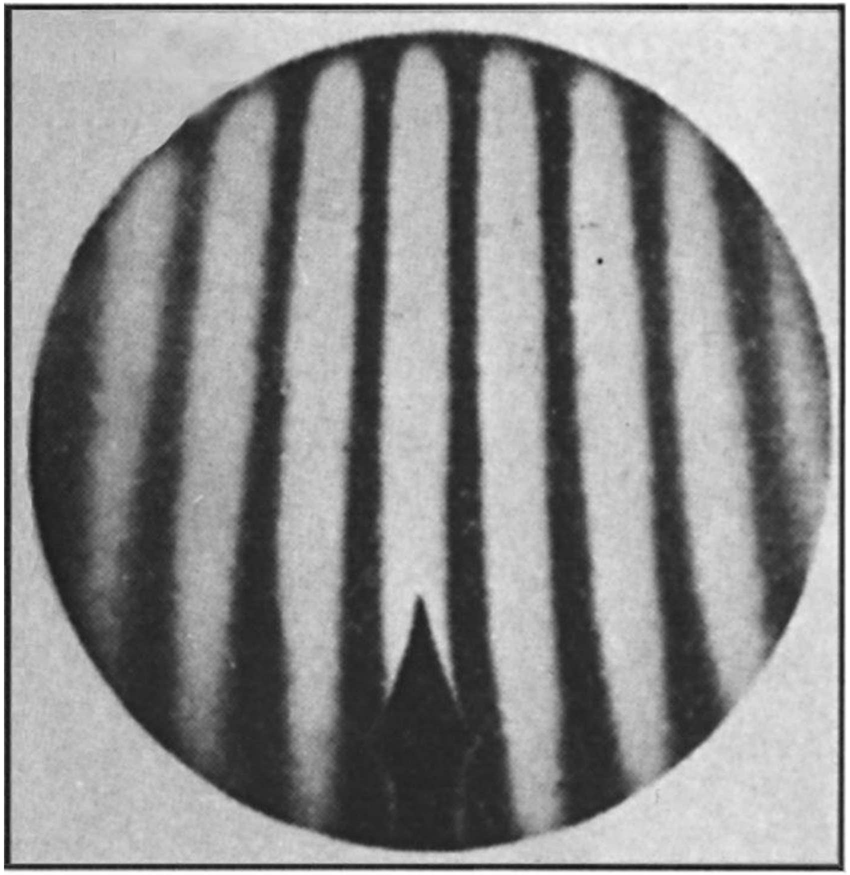
An Absolute Reference System

- Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.
- The Michelson-Morley experiment was an attempt to show the existence of ether.

The Michelson Interferometer



Typical interferometer fringe pattern expected when the system is rotated by 90°



Michelson’s Conclusion

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He thus concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***

Possible Explanations

- Many explanations were proposed but the most popular was the *ether drag* hypothesis.
  - This hypothesis suggested that the Earth somehow “dragged” the ether along as it rotates on its axis and revolves about the sun.
  - This was contradicted by *stellar abberation* wherein telescopes had to be tilted to observe starlight due to the Earth’s motion. If ether was dragged along, this tilting would not exist.

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1-\frac{v^2}{c^2}}$$

…thus making the path lengths equal to account for the zero phase shift.

  - This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Two Postulates

With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

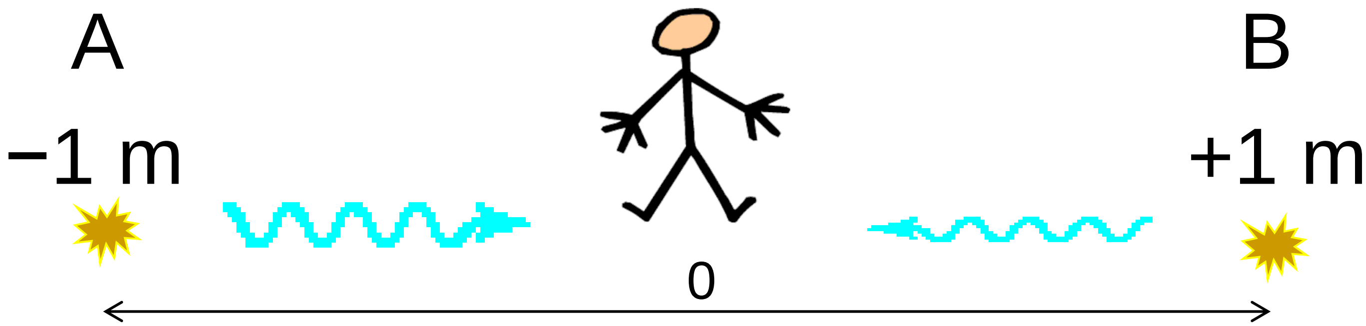
1) **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2) **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

Re-evaluation of Time

- In Newtonian physics we previously assumed that $t = t'$
  - *Thus with "synchronized" clocks, events in K and K' can be considered simultaneous*
- Einstein realized that each system must have its own observers with their own clocks and meter sticks
  - Thus events considered simultaneous in K may not be in K'

The Problem of Simultaneity

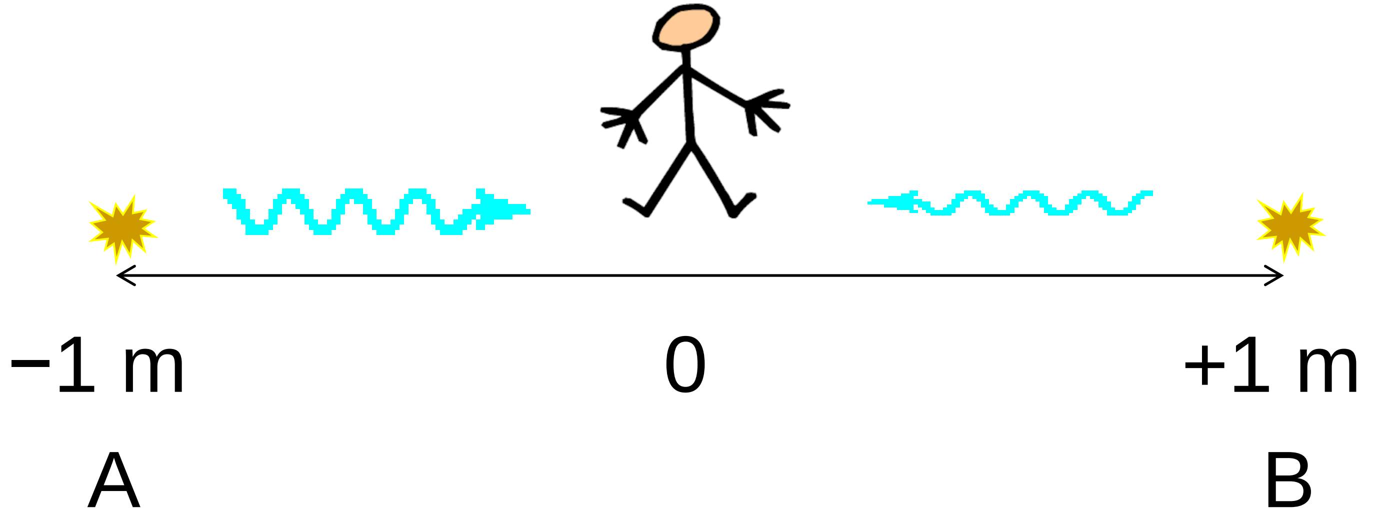
Observer at rest is equidistant from A and B:



Observer “sees” both actions simultaneously

The Problem of Simultaneity

Observer is moving to the right with speed v, "sees" actions A and B in different order:



Observer "sees" event B, then A.

We thus observe…

- *Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K’) moving with respect to the first frame.*

The Lorentz Transformations

The special set of linear transformations that:

1) preserve the constancy of the speed of light (*c*) between inertial observers;

and,

2) account for the problem of simultaneity between these observers

known as the **Lorentz transformation equations**

Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformation Equations

A more simple form:

$$\beta = \frac{v}{c}$$

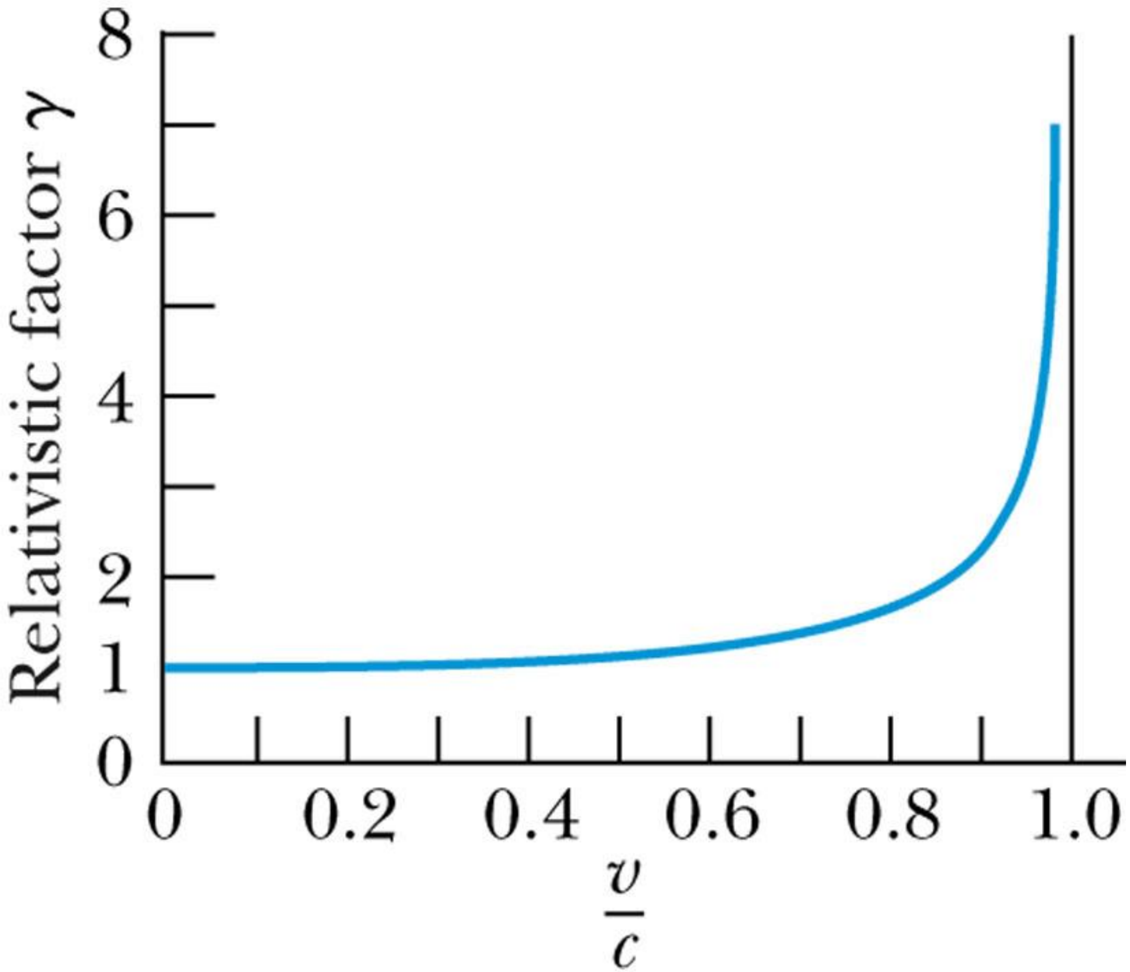
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \gamma\left(x - \beta ct\right)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$


Relativistic factor γ
8
6
4
2
1
0
0
0.2
0.4
0.6
0.8
1.0
$\frac{v}{c}$

Remarks

1) If $v \ll c$, i.e., $\beta \approx 0$ and $\gamma \approx 1$, we see these equations reduce to the Galilean transformation.

2) Space and time are now not separated.

3) For non-imaginary transformations, the frame velocity cannot exceed c.

Time Dilation and Length Contraction

Consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in K' run slow with respect to stationary clocks in K.

- **Length Contraction**:

  Lengths in K' are contracted with respect to the same lengths stationary in K.

Time dilation equation

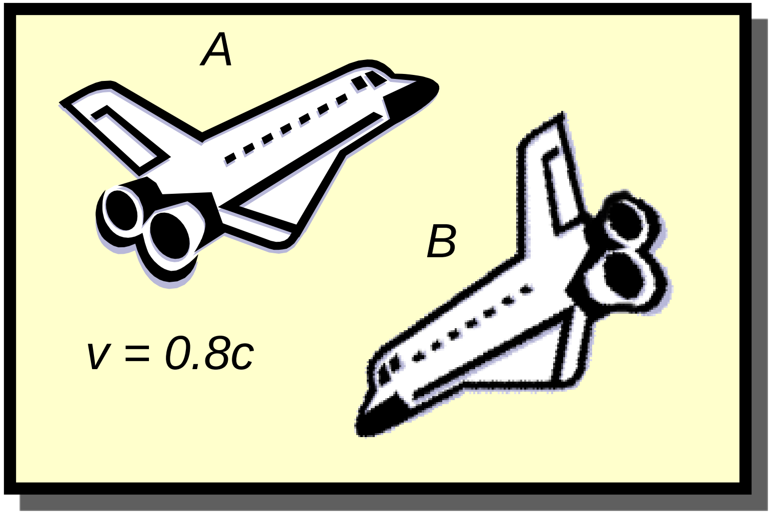
$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Δt = Relative time (Time measured in frame moving relative to actual event).

Δt_o= Proper time (Time measured in the same frame as the event itself).

v = Relative velocity of two frames.

Example: Ship *A* passes ship *B* with a relative velocity of 0.8c. A person aboard Ship *B* takes 4s to walk the length of his ship. What time is recorded by the person in Ship A?



$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

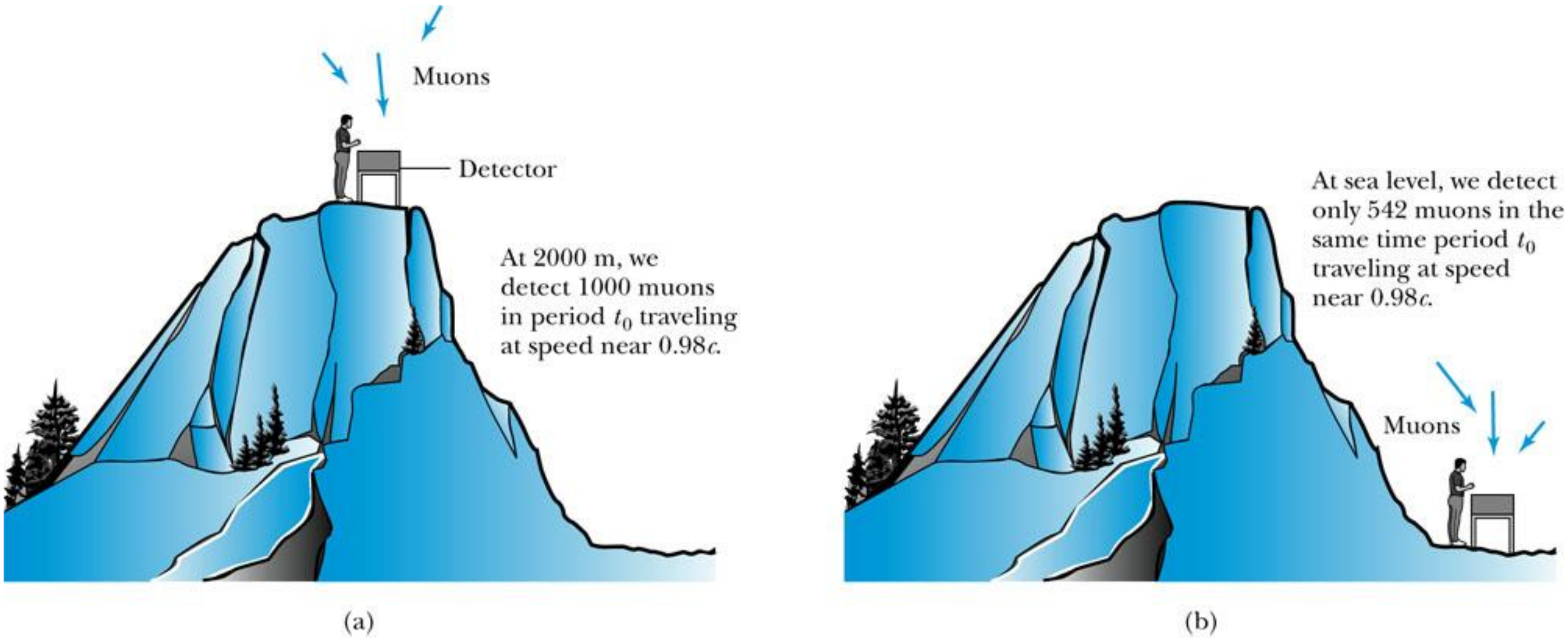
Proper time Δt_o = 4 s

$$\Delta t = \frac{4.00\ \text{s}}{\sqrt{1 - (0.8c)^2 / c^2}} = \frac{4.00\ \text{s}}{\sqrt{1 - 0.64}}$$

Δt = 6.67 s

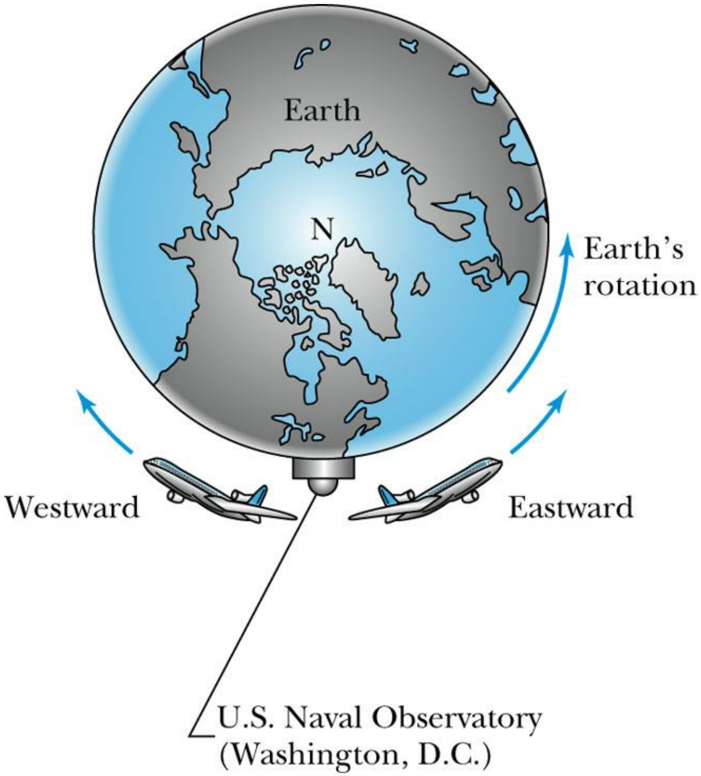
Experimental Verification

Time Dilation and Muon Decay



The number of muons detected with speeds near 0.98*c* is much different (a) on top of a mountain than (b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Atomic Clock Measurement



Two airplanes took off (at different times) from Washington, D.C., where the U.S. Naval Observatory is located. The airplanes traveled east and west around Earth as it rotated. Atomic clocks on the airplanes were compared with similar clocks kept at the observatory to show that the moving clocks in the airplanes ran slower.

Twin Paradox

The Situation

Twins Jožo and Fero at age 30 decide on two career paths: Jožo decides to become an astronaut and to leave on a trip from the Earth at a great speed and to return; Fero decides to reside on the Earth.

The Problem

Upon Jožo's return, Fero reasons that Jožo's clocks measuring his age must run slow. As such, he will return younger. However, Jožo claims that it is Fero who is moving and consequently his clocks must run slow.

The Paradox

Who is younger upon Jožo's return?

The Resolution

1) Fero’s clock is in an **inertial system** during the entire trip; however, Jožo’s clock is not. As long as Jožo is traveling at constant speed away from Fero, both of them can argue that the other twin is aging less rapidly.

2) When Jožo slows down to turn around, he leaves his original inertial system and eventually returns in a completely different inertial system.

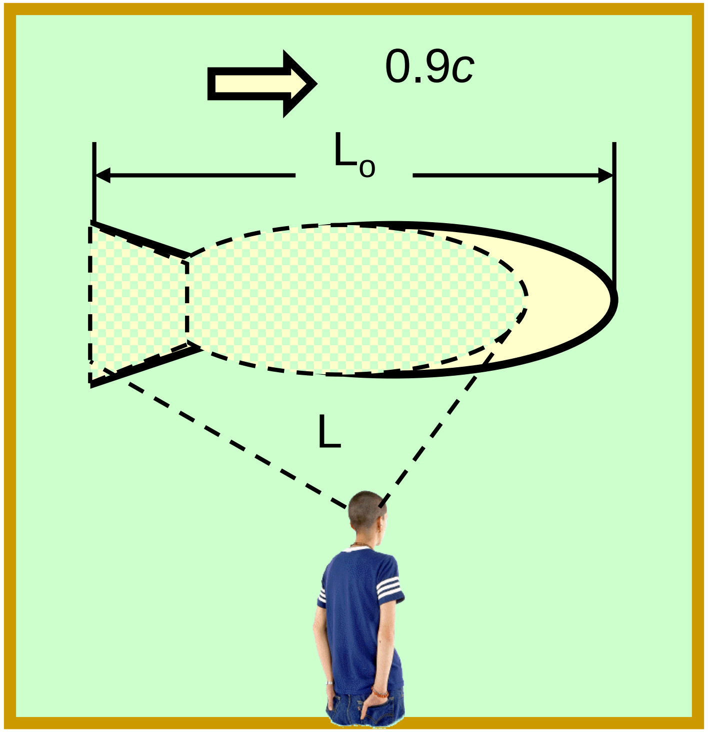
3) Jožo’s claim is no longer valid, because he does not remain in the same inertial system. There is also no doubt as to who is in the inertial system. Fero feels no acceleration during Jožo’s entire trip, but Jožo does.

Length Contraction

$$L = L_0\sqrt{1 - v^2/c^2}$$

L_o is proper length

L is relative length



Example: A meter stick moves at 0.9c relative to an observer. What is the relative length as seen by the observer?

$$L = L_0\sqrt{1 - v^2/c^2}$$

$$L = (1\ \mathrm{m})\sqrt{1 - 0.81} = 0.436\ \mathrm{m}$$

L = 43.6 cm

Relativistic Momentum

The basic conservation laws for momentum and energy can not be violated due to relativity.

Newton's equation for momentum (*mv*) must be changed as follows to account for relativity:

Relativistic momentum:
$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

m_o is the proper mass, often called the rest mass. Note that for small values of v, this equation reduces to Newton's equation.

Relativistic Mass

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

Note that as an object is accelerated by a resultant force, its mass increases, which requires even more force.

Mass and Energy

Prior to the theory of relativity, scientists considered mass and energy as separate quantities, each of which must be conserved.

Now mass and energy must be considered as the same quantity. We may express the mass of a baseball in joules or its energy in kilograms! The motion adds to the mass-energy.

Total energy *E* of a particle of is given by

$$E = mc^2 \qquad (m_o c^2 + K)$$

Total energy includes rest energy and energy of motion. If we are interested in just the energy of motion, we must subtract $m_o c^2$.

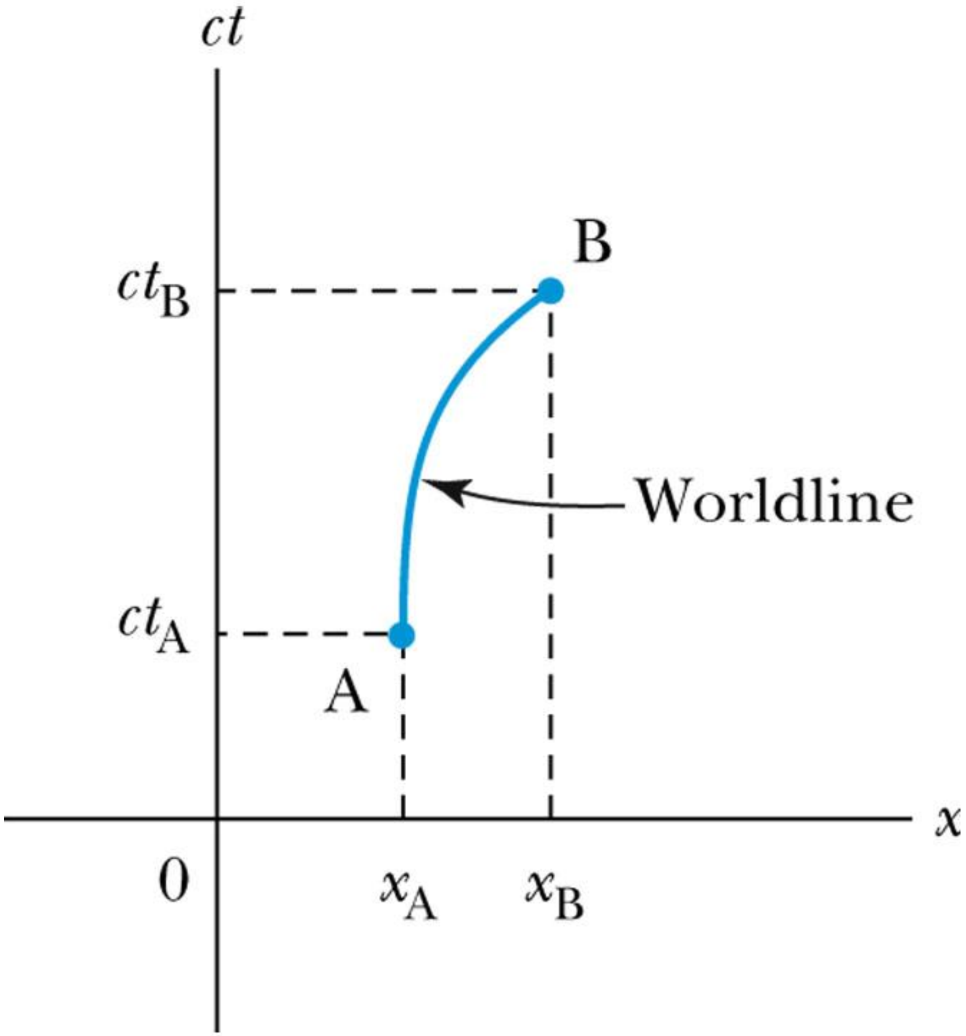
Kinetic Energy: $K = (m - m_o)c^2$

Spacetime

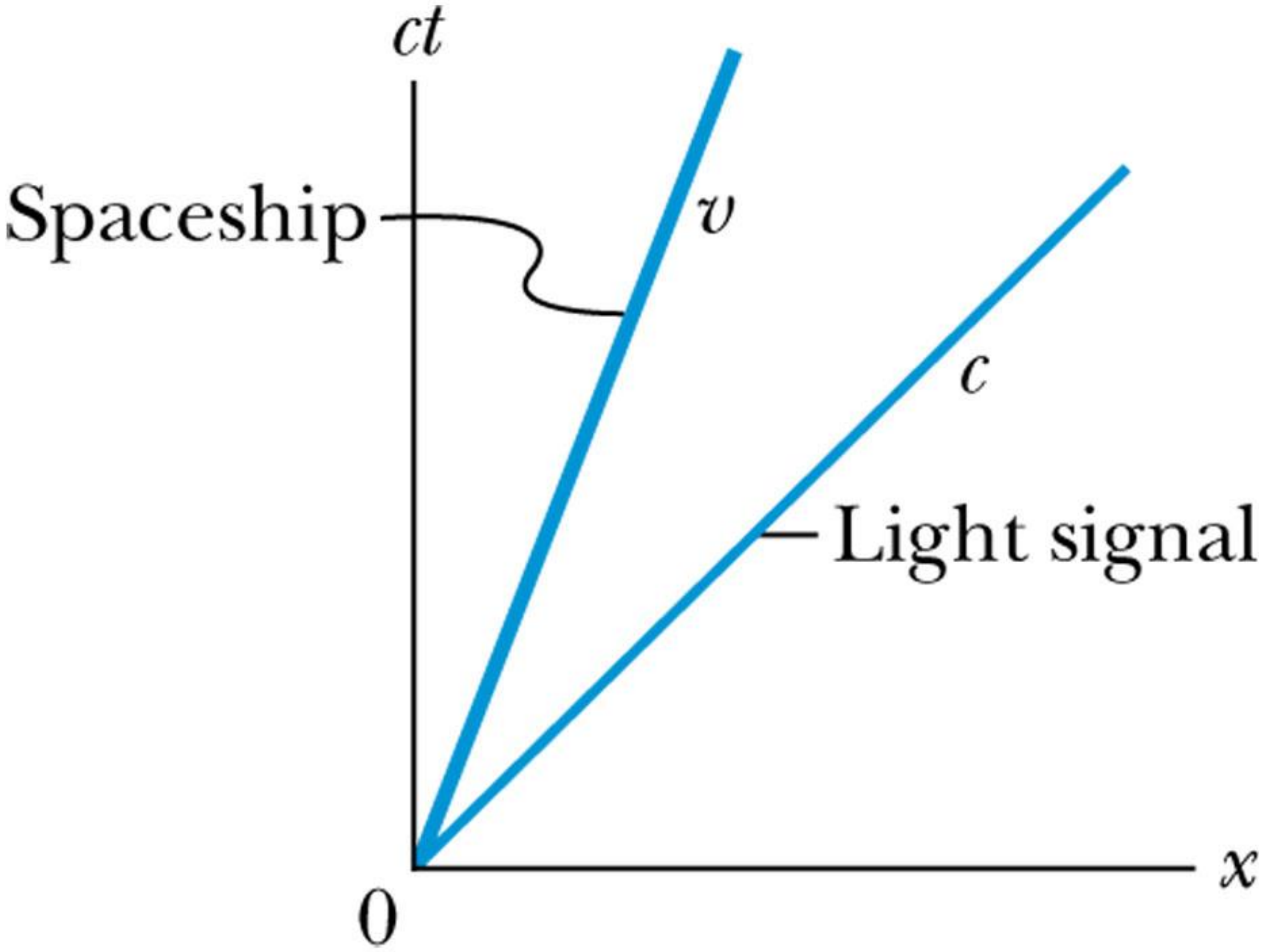
- When describing events in relativity, it is convenient to represent events on a **spacetime** diagram.

- In this diagram one spatial coordinate x, to specify position, is used and instead of time t, ct is used as the other coordinate so that both coordinates will have dimensions of length.

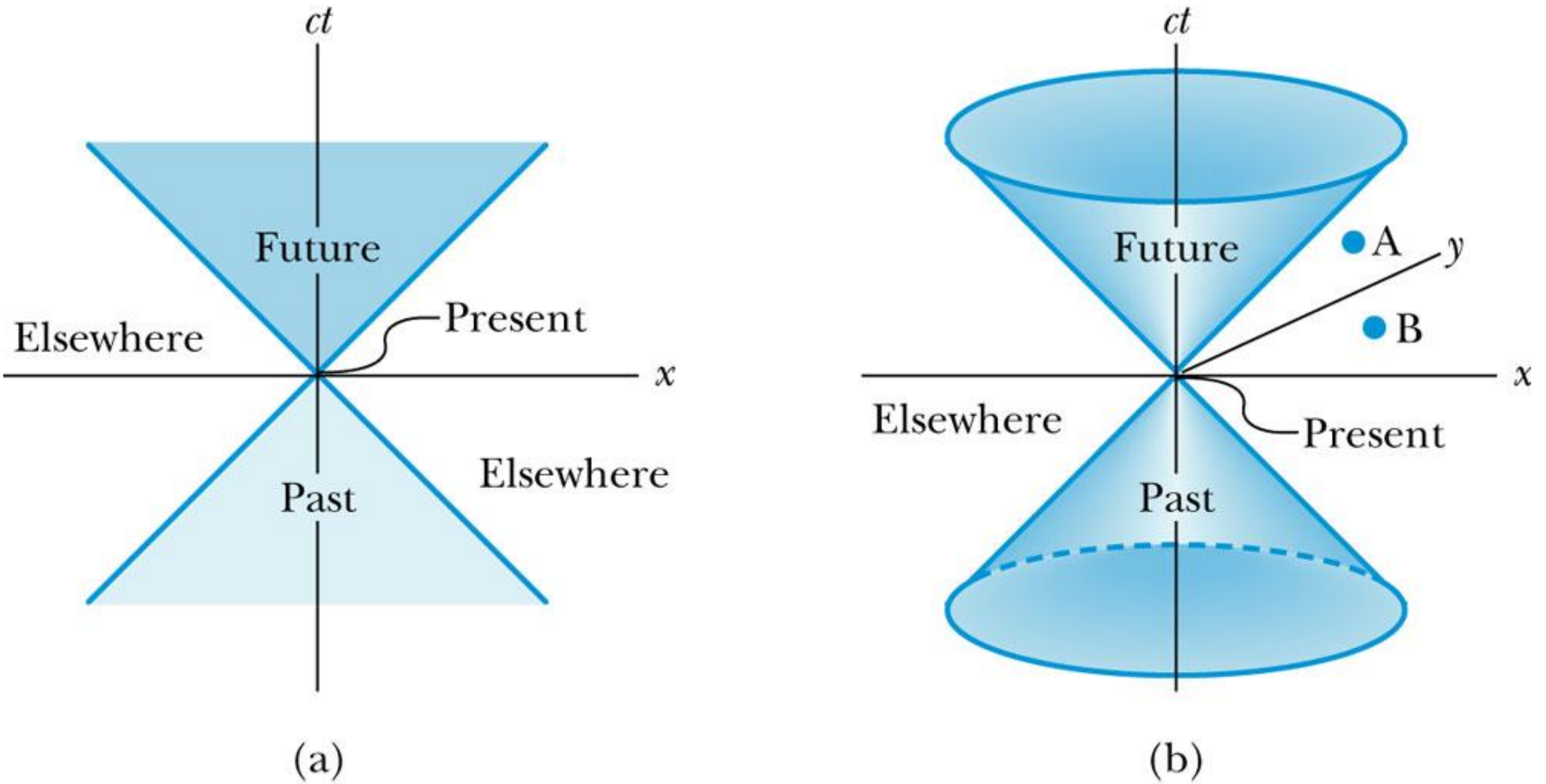
Spacetime Diagram



Particular Worldlines



The Light Cone



General Relativity

Tenets of General Relativity

- General relativity is the extension of special relativity. It includes the effects of accelerating objects and their mass on spacetime.
- As a result, the theory is an explanation of gravity.
- It is based on two concepts: (1) the principle of equivalence, which is an extension of Einstein’s first postulate of special relativity and (2) the curvature of spacetime due to gravity.

Principle of Equivalence

A uniform gravitational field in some direction is indistinguishable from a uniform acceleration in the opposite direction

Keep in mind that an accelerating frame introduces pseudo-forces in the direction opposite to the true acceleration of the frame (e.g. inside a car when brakes are applied)

Inertial Mass and Gravitational Mass

- Recall from Newton's 2nd law that an object accelerates in reaction to a force according to its inertial mass:

$$\vec{F} = m_I \vec{a}$$

- Inertial mass measures how strongly an object resists a change in its motion.

$$\vec{F} = m_G \vec{g}$$

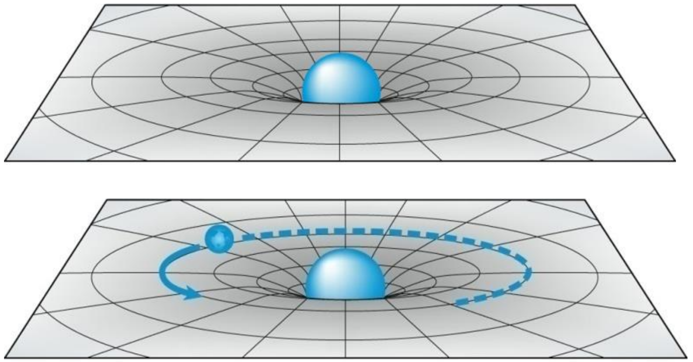
- Gravitational mass measures how strongly it attracts other objects.

- For the same force, we get a ratio of masses: $\vec{a} = \left(\frac{m_G}{m_I}\right)\vec{g}$

- According to the principle of equivalence, the inertial and gravitational masses are equal.

Spacetime Curvature of Space

- Light bending for the Earth observer seems to violate the premise that the velocity of light is constant from special relativity. Light traveling at a constant velocity implies that it travels in a straight line.
- Einstein recognized that we need to expand our definition of a *straight line.*
- The shortest distance between two points on a flat surface appears different than the same distance between points on a sphere. The path on the sphere appears curved. We shall expand our definition of a *straight line* to include any minimized distance between two points.
- Thus if the spacetime near the Earth is not flat, then the straight line path of light near the Earth will appear curved.



The Unification of Mass and Spacetime

- Einstein mandated that the mass of the Earth creates a dimple on the spacetime surface. In other words, the mass changes the geometry of the spacetime.
- The geometry of the spacetime then tells matter how to move.
- Einstein’s famous field equations sum up this relationship as:

  * *mass-energy tells spacetime how to curve*

  * *Spacetime curvature tells matter how to move*

- The result is that a standard unit of length such as a meter stick increases in the vicinity of a mass.

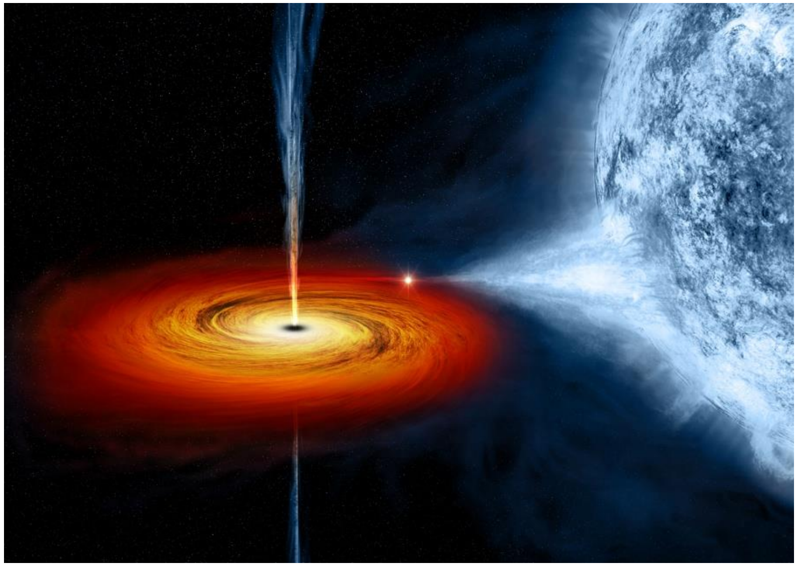
$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

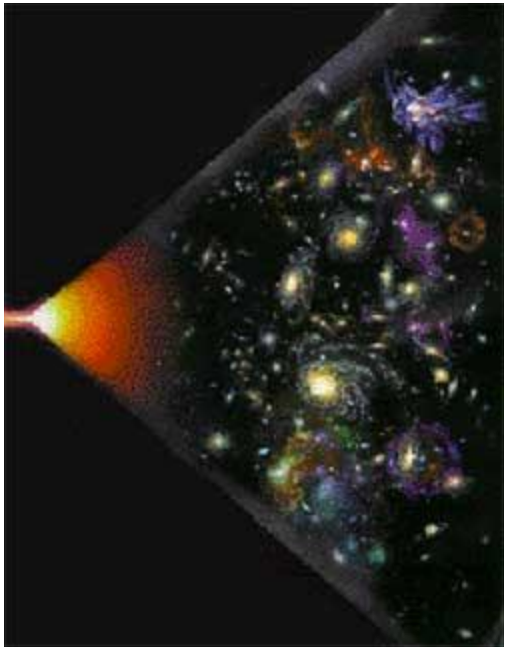
where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, G is Newton's gravitational constant, c is the speed of light in vacuum, and $T_{\mu\nu}$ is the stress–energy tensor.

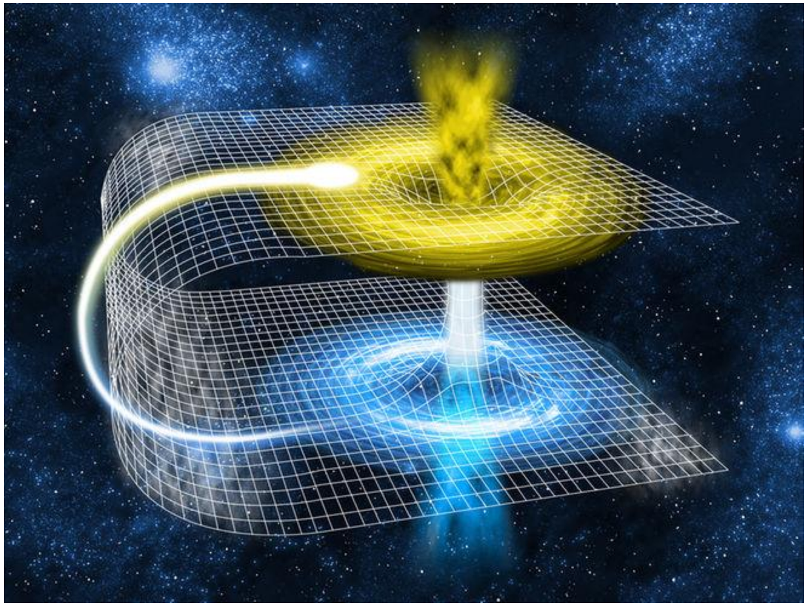
Despite the simple appearance of the equations they are actually quite complicated. Given a specified distribution of matter and energy in the form of a stress–energy tensor, the EFE are understood to be equations for the metric tensor $g_{\mu\nu}$, as both the Ricci tensor and scalar curvature depend on the metric in a complicated nonlinear manner. In fact, when fully written out, the EFE are a system of 10 coupled, nonlinear, hyperbolic-elliptic partial differential equations.

Some predictions of GR

- bending of light
- expanding universe
- black holes
- wormholes



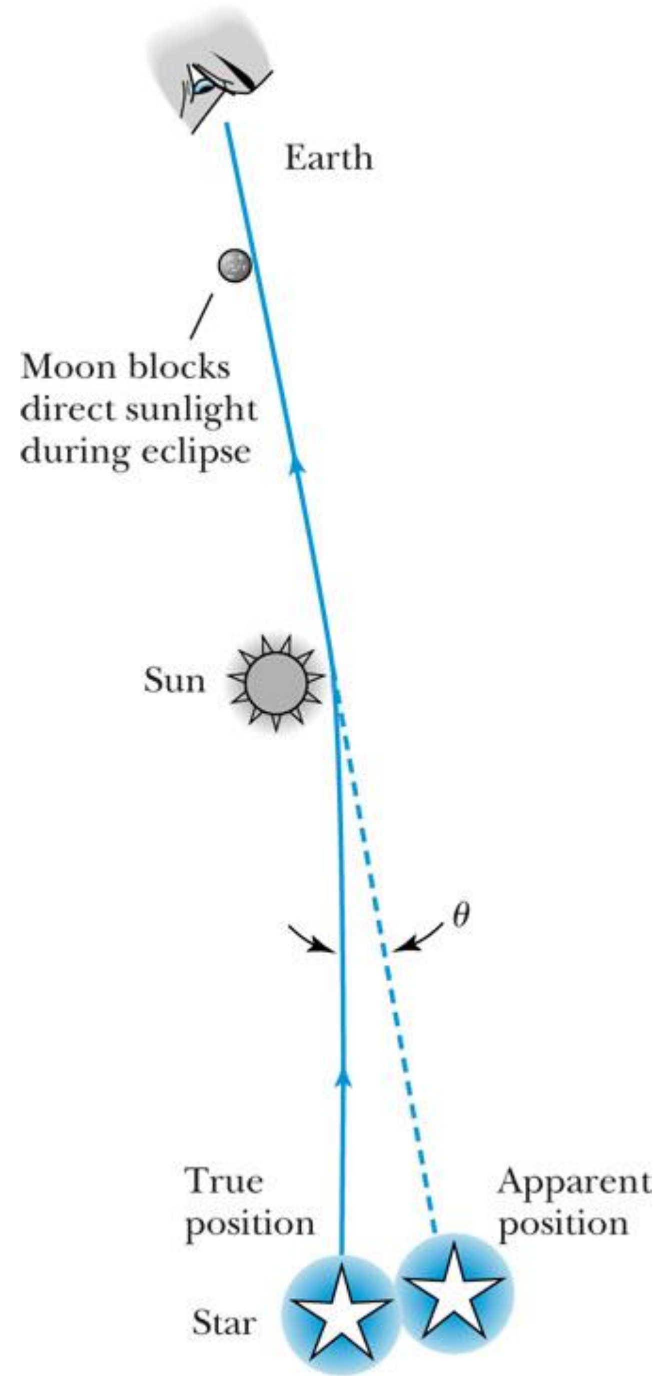




Tests of General Relativity

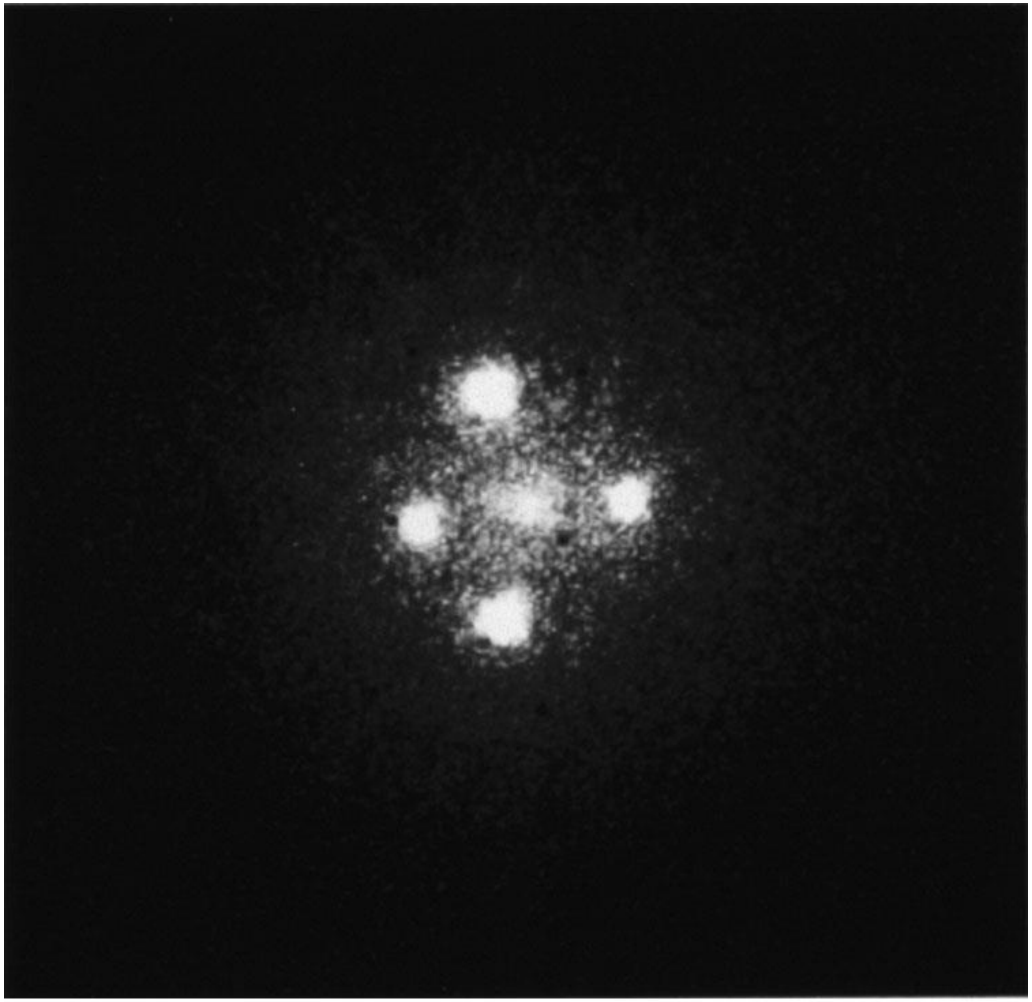
Bending of Light

- During a solar eclipse of the sun by the moon, most of the sun's light is blocked on Earth, which afforded the opportunity to view starlight passing close to the sun in 1919. The starlight was bent as it passed near the sun which caused the star to appear displaced.
- Einstein's general theory predicted a deflection of 1.75 seconds of arc, and the two measurements found 1.98 ± 0.16 and 1.61 ± 0.40 seconds.
- Since the eclipse of 1919, many experiments, using both starlight and radio waves from quasars, have confirmed Einstein's predictions about the bending of light with increasingly good accuracy.



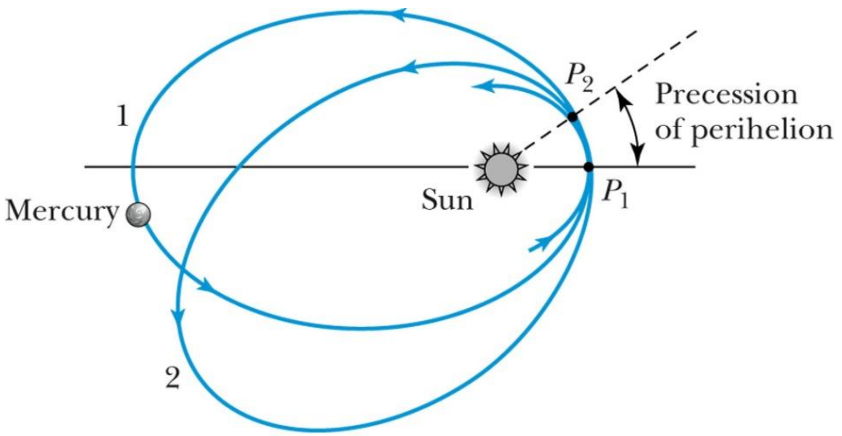
Gravitational Lensing

- When light from a distant object like a quasar passes by a nearby galaxy on its way to us on Earth, the light can be bent multiple times as it passes in different directions around the galaxy.



Perihelion Shift of Mercury

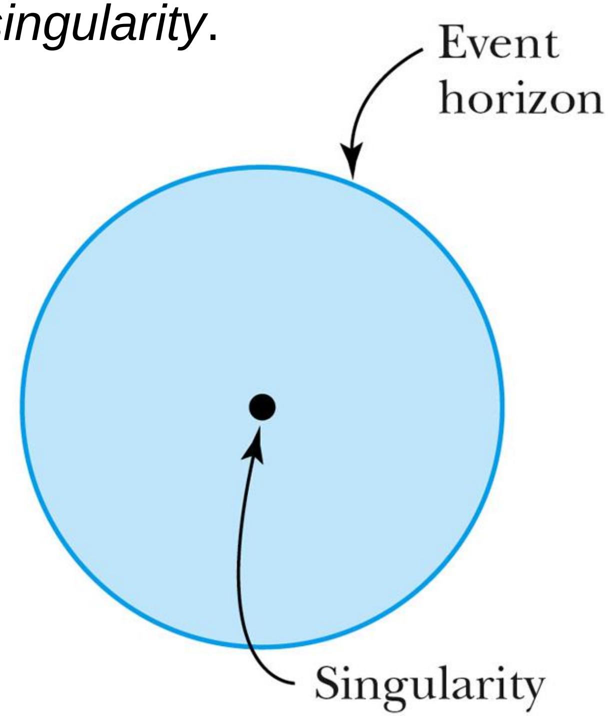
- The orbits of the planets are ellipses, and the point closest to the sun in a planetary orbit is called the perihelion. It has been known for hundreds of years that Mercury's orbit precesses about the sun. Accounting for the perturbations of the other planets left 43 seconds of arc per century that was previously unexplained by classical physics.
- The curvature of spacetime explained by general relativity accounted for the 43 seconds of arc shift in the orbit of Mercury.



Black Holes

- While a star is burning, the heat produced by the thermonuclear reactions pushes out the star's matter and balances the force of gravity. When the star's fuel is depleted, no heat is left to counteract the force of gravity, which becomes dominant. The star's mass collapses into an incredibly dense ball that could wrap spacetime enough to not allow light to escape. The point at the center is called a *singularity*.
- A collapsing star greater than 3 solar masses will distort spacetime in this way to create a black hole.
- Karl Schwarzschild determined the radius of a black hole known as the *event horizon*.

$$r_S = \frac{2GM}{c^2}$$



Black Hole Detection

- Since light can't escape, they must be detected indirectly:
- Severe redshifting of light.
- Hawking radiation results from particle-antiparticle pairs created near the event horizon. One member slips into the singularity as the other escapes. Antiparticles that escape radiate as they combine with matter. Energy expended to pair production at the event horizon decreases the total mass-energy of the black hole.
- Hawking calculated the blackbody temperature of the black hole to be:

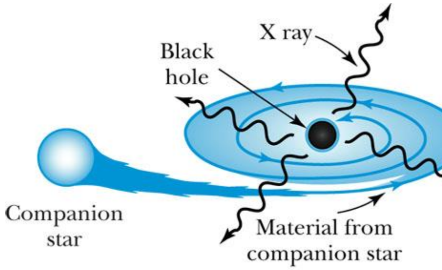
$$T = \frac{\hbar c^3}{8\pi kGM}$$

- The power radiated is: $P(T) = 4\pi\sigma {r_S}^2 \left(\dfrac{\hbar c^3}{8\pi kGM} \right)^4$

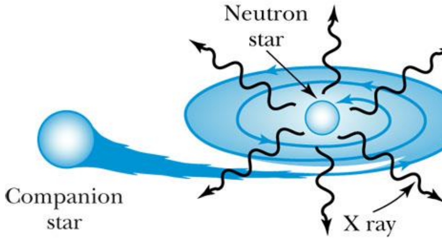
  This result is used to detect a black hole by its Hawking radiation.
- Mass falling into a black hole would create a rotating accretion disk. Internal friction would create heat and emit x rays.

Black Hole Candidates

- Although a black hole has not yet been observed, there are several plausible candidates:
  - Cygnus X-1 is an x ray emitter and part of a binary system in the Cygnus constellation. It is roughly 7 solar masses.
  - The galactic center of M87 is 3 billion solar masses.
  - NGC 4261 is a billion solar masses.



(a)



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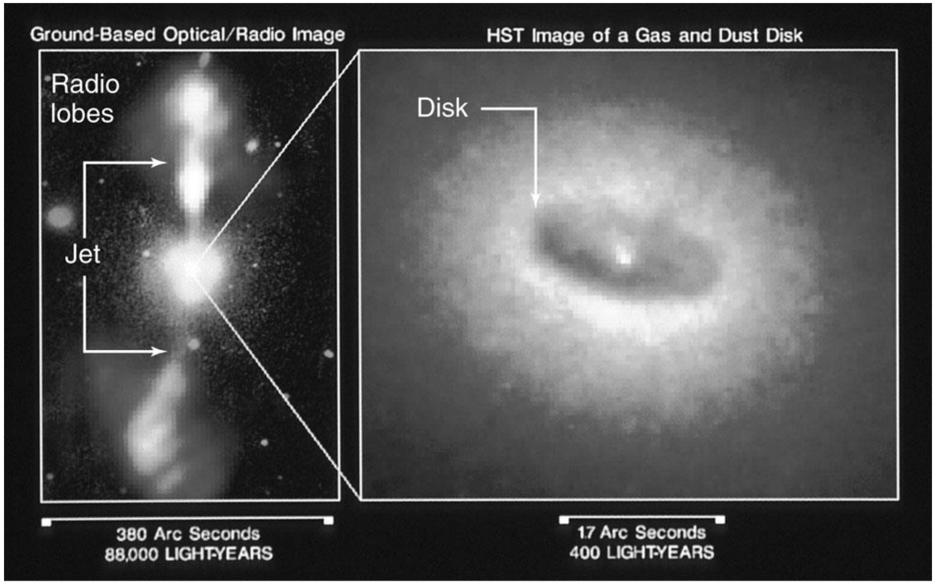

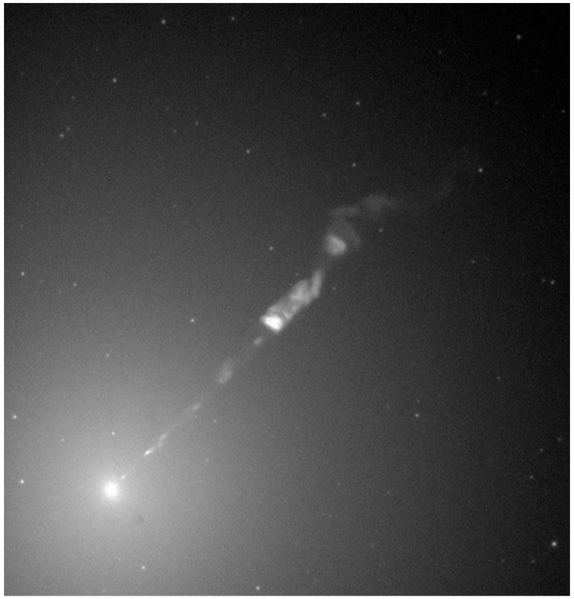

Gravitational Waves

- When a charge accelerates, the electric field surrounding the charge redistributes itself. This change in the electric field produces an electromagnetic wave, which is easily detected. In much the same way, an accelerated mass should also produce gravitational waves.
- Gravitational waves carry energy and momentum, travel at the speed of light, and are characterized by frequency and wavelength.
- As gravitational waves pass through spacetime, they cause small ripples. The stretching and shrinking is on the order of 1 part in 10^{21} even due to a strong gravitational wave source.
- Due to their small magnitude, gravitational waves would be difficult to detect. Large astronomical events could create measurable spacetime waves such as the collapse of a neutron star, a black hole or the Big Bang.