Special relativity

1. Two electron escaping the source in the opposite directions, each one with the speed $v_1 = v_2 = 0.8c$ with respect to the source. What is their relative speed?

Solution

The relativistic combination of the speed is given by:

$$w = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}.$$

Our case is then: $w = \frac{0.8c + 0.8c}{1 + \frac{0.8c \cdot 0.8c}{c^2}} = \underline{\underline{0.976c}}$

2. There α particle is escaping the accelerator with the speed v and moving straight forward into the tube of length $l = 12\,cm$. It makes this distance in 1 ns. Find the length of the tube in the reference frame connected with the tube.

Solution

The speed of the particle is: $v = \frac{s}{t} = \left[\frac{0.12}{10^{-9}}\right] ms^{-1} = 1.2 \cdot 10^8 ms^{-1}$. The length of the tube from its reference frame is then: $l = l_0\sqrt{1 - \frac{v^2}{c^2}} = \left[12\sqrt{1 - \left(\frac{1.2 \cdot 10^8}{3 \cdot 10^8}\right)^2}\right] m = \underline{\underline{0.11m}}$

3. The unstable particles are escaping the accelerator with speed $v = 0.8c$. The half-life of them is $t_0 = 1.8 \cdot 10^{-8} s$. Find the time in which the half of them will decay and what distance they will travel within that time.

Solution

$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \left[\frac{1.8 \cdot 10^{-8}}{\sqrt{1 - \left(\frac{0.8c}{c}\right)^2}}\right] s = \underline{\underline{3 \cdot 10^{-8} s}}$; $s = vt = \left[0.8c \cdot 3 \cdot 10^{-8}\right] m = \underline{\underline{7.2\,m}}$

4. The density of the iron is $\rho_0 = 7400\ kg \cdot m^{-3}$ in the reference frame which is still. What will be the change of the density in the reference frame which is moving with the speed $v = 0.5c$.

$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \wedge V = V_0\sqrt{1 - \frac{v^2}{c^2}} \rightarrow \rho = \frac{m}{V} = \frac{m_0}{V_0}\frac{1}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2} = \rho_0\frac{1}{1 - \frac{v^2}{c^2}} = \left[7400 \cdot \frac{1}{1 - \left(\frac{0.5c}{c}\right)^2}\right] kg \cdot m^{-3} = \underline{\underline{9866.7\ kg \cdot m^{-3}}}$