Lecture 3: Oscillators, waves

Content:

- harmonic oscillator
- mathematical pendulum
- damped oscillator
- driven oscillator
- waves
- Huygens principle, Doppler effect

basic terms and quantities

The general study of the relationships between motion, forces, and energy is called **mechanics**.

Motion is the action of changing location or position.

Motion may be divided into three basic types - translational, rotational, and oscillatory.

The study of motion without regard to the forces or energies that may be involved is called **kinematics**. It is the simplest branch of mechanics.

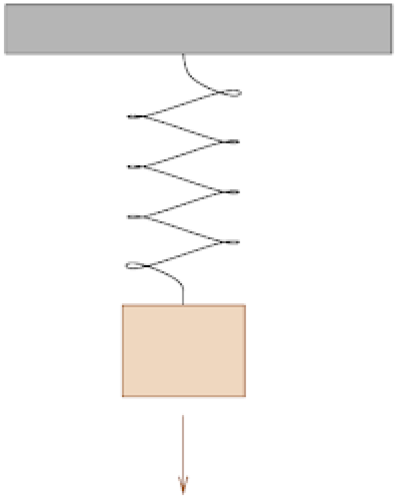
The branch of mechanics that deals with both motion and forces together is called **dynamics** and the study of forces in the absence of changes in motion or energy is called **statics**.

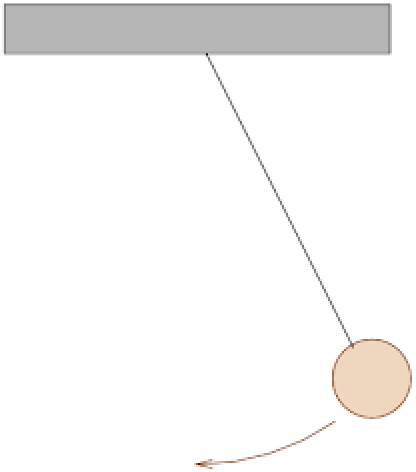
basic terms and quantities

Motion is the action of changing location or position.

Motion may be divided into three basic types - translational, rotational, and oscillatory.

Oscillation is the repetitive variation, typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.



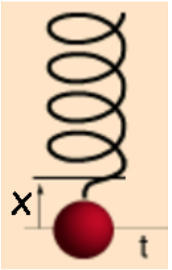


HARMONIC OSCILLATOR

- **Simple harmonic oscillator -** F is the only acting force
- **Damped oscillator –** Friction (damping) occurs
- **Driven oscillator –** Damped oscillator further affected by external force

1. Simple harmonic oscillator

An oscillating spring with a mass



x – distance,
t – time,

Balance of the system is given by (with help of Newton's second law and Hooke's law):

a) $$F = ma = m\frac{d^2x(t)}{dt^2}$$

Newton's second law („Law of Power“)

b) $$F = kx(t)$$

so called Hooke's law:
it states that the force (F) needed to extend or compress a spring by some distance (x) scales linearly with respect to that distance,
(k – so called Hooke's constant)

Equality of these 2 forces gives us the basic equation for a simple harmonic oscillator:

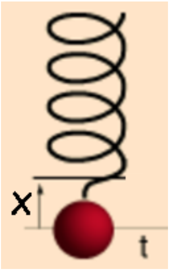
$$m\frac{d^2x(t)}{dt^2} = -kx(t)$$

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

a linear differential equation (LDE)

1. Simple harmonic oscillator

An oscillating spring with a mass



$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

Homogenous LDE with constant coefficients

Solution:

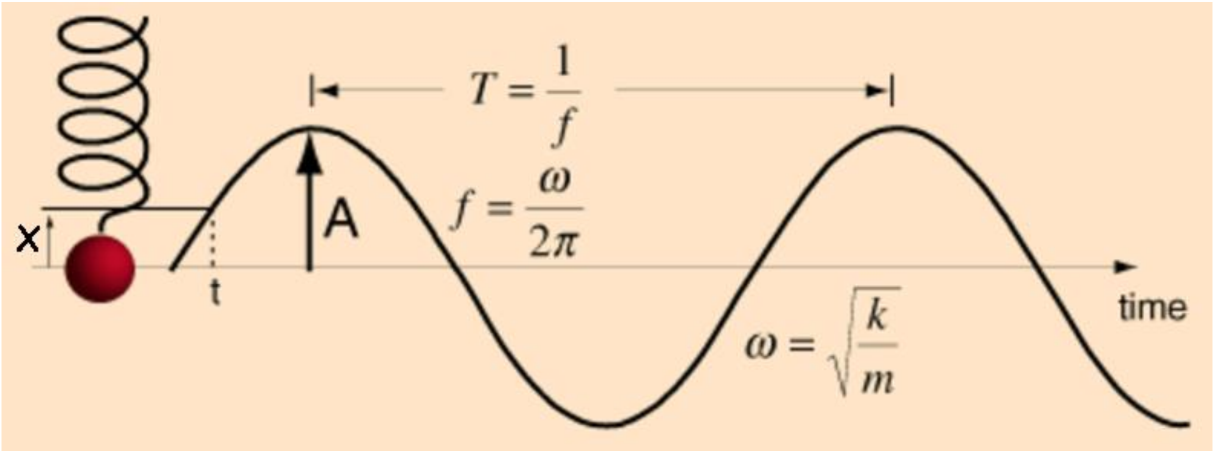
$$x(t) = C_1\cos(\omega t) + C_2\sin(\omega t) = A\cos(\omega t - \varphi)$$

Periodic motion

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f$$

$$A = \sqrt{C_1^2 + C_2^2}$$

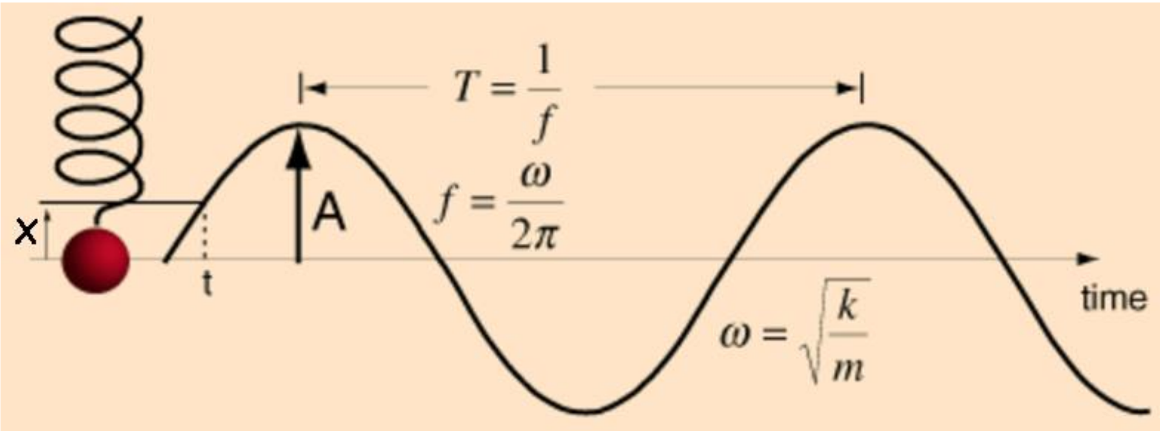
$$\tan\varphi = \frac{C_1}{C_2}$$

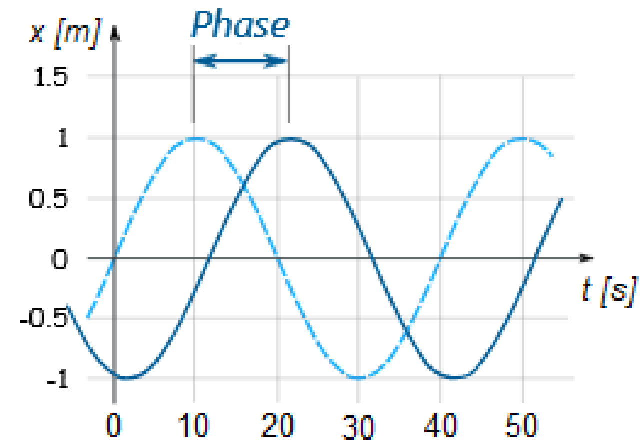


A – amplitude, φ – phase , ω – angular frequency , f – frequency , t – time

1. Simple harmonic oscillator

An oscillating spring with a mass





Velocity $$\vec{v}(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t - \varphi)$$

Speed $$|\vec{v}(t)| = \omega\sqrt{A^2 - x^2(t)}$$

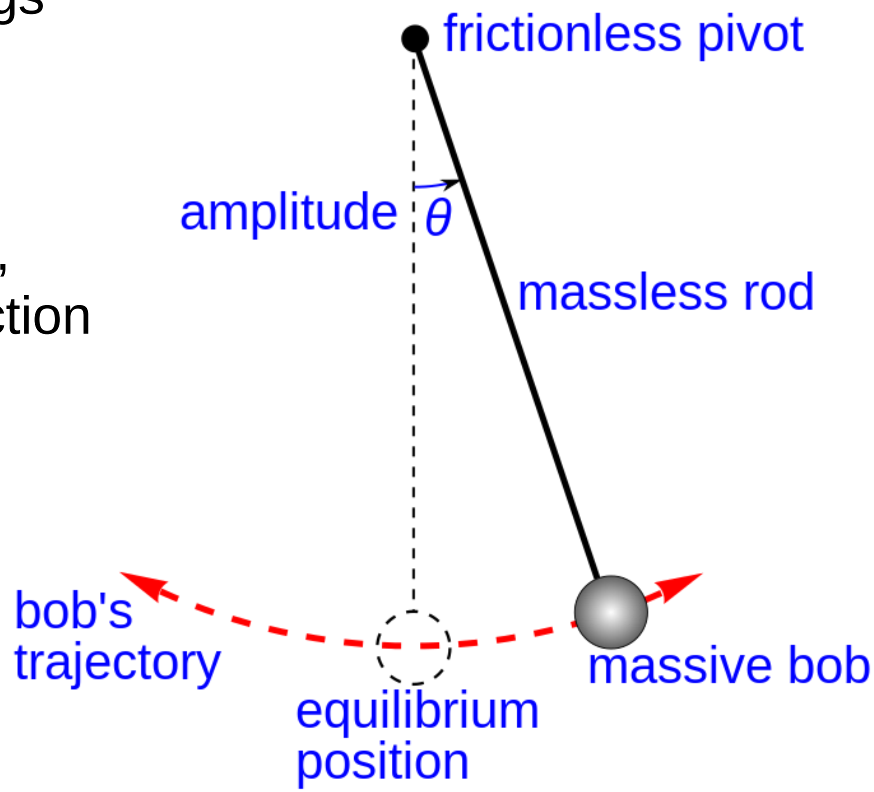
Acceleration $$\vec{a}(t) = \frac{d^2x(t)}{dt^2} = -A\omega^2\cos(\omega t - \varphi)$$

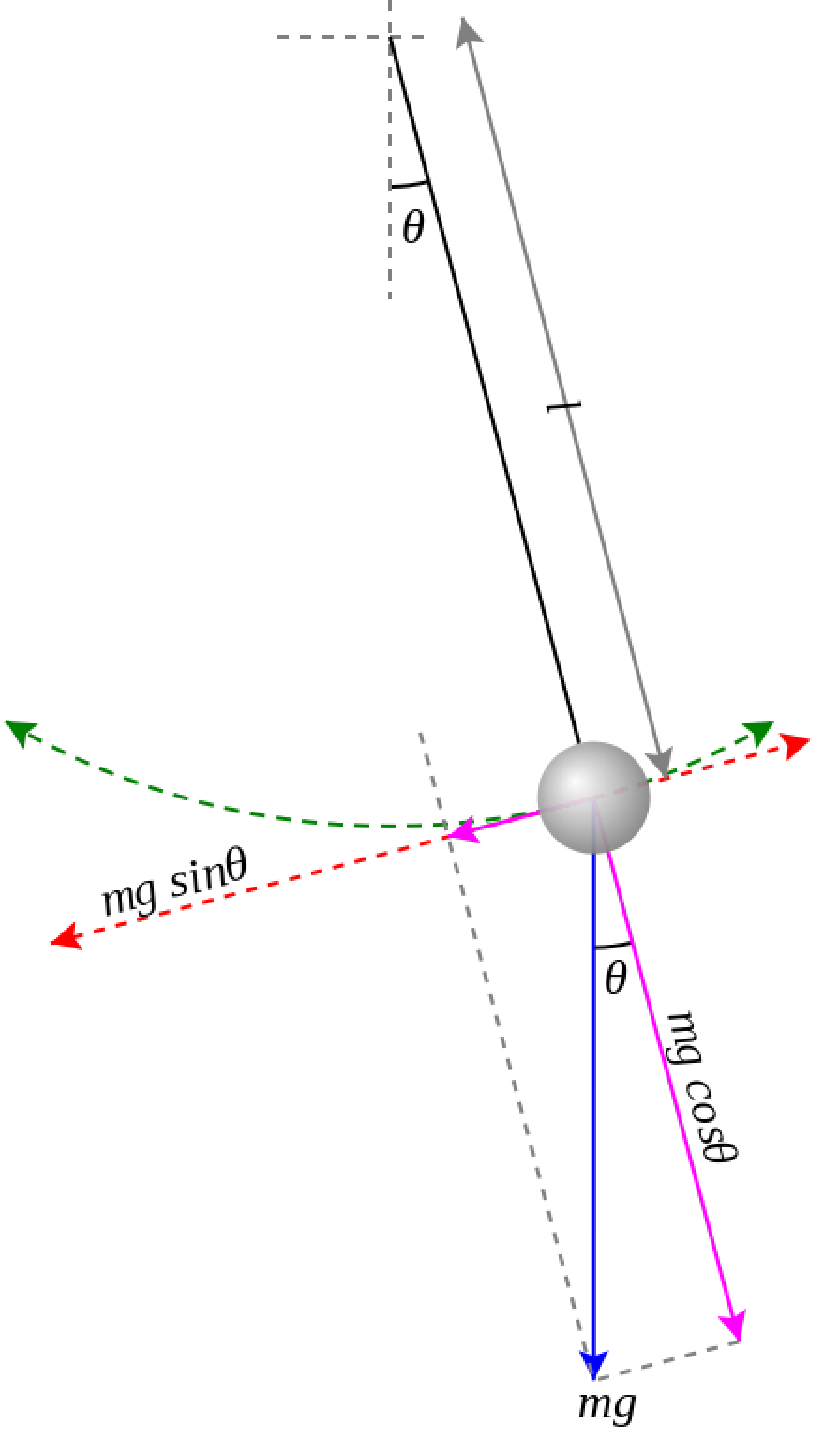
A – amplitude, φ – phase , ω – angular frequency , f – frequency , t – time

1. Simple harmonic oscillator

Next example – simple gravity pendulum (or mathematical pendulum)

Basic assumptions:
- the rod or cord on which the bob swings is massless,
- the bob is a point mass,
- motion occurs only in two dimensions (i.e. the bob does not trace an ellipse),
- the motion does not lose energy to friction or air resistance,
- the gravitational field is uniform,
- the support does not move.





simple gravity pendulum

Equality of 2 equations for accelerations gives us the basic equation for the pendulum:

$$F = m \cdot a$$

$$F = -mg\sin\theta = m \cdot a \Rightarrow a = -g\sin\theta$$

$$s = l \cdot \theta \rightarrow v = \frac{ds}{dt} = l\frac{d\theta}{dt} \rightarrow a = l\frac{d^2\theta}{dt^2} \quad \textbf{a)}$$

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0 \quad \textbf{b)}$$

l – length of the pendulum
θ – angular displacement (amplitude)

1. Simple harmonic oscillator (simple gravity pendulum)

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\sin\theta=0$$

assumption: θ << 1 $\rightarrow \sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2}+\frac{g}{l}\theta=0$$

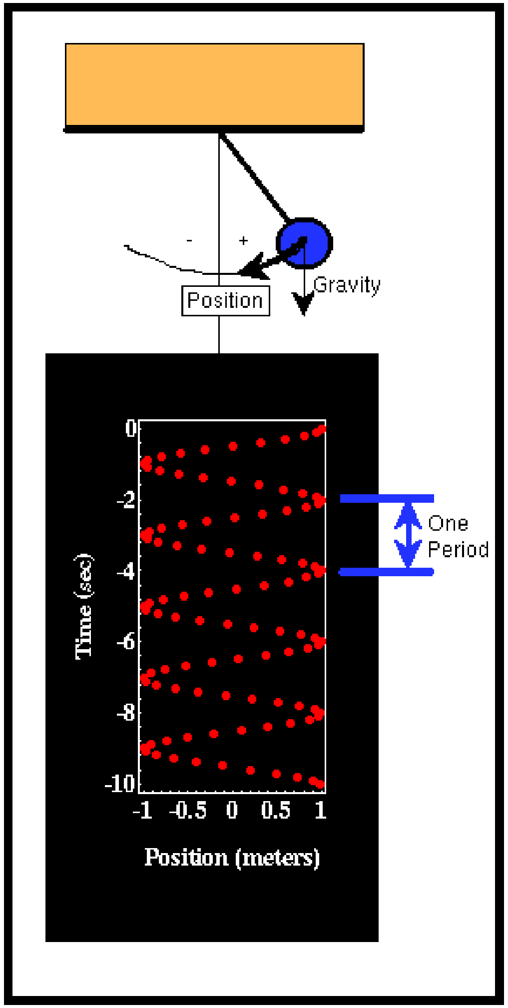
boundary conditions $\theta(0)=\theta_0; \quad \left.\frac{d\theta}{dt}\right|_{t=0}=0$

Solution: $$\theta(t)=\theta_0\cos\left(\sqrt{\frac{g}{l}}t\right)$$ $\theta_0 << 1$

Period: $$T=2\pi\sqrt{\frac{l}{g}}$$

$$\omega=2\pi f\ ,\ f=\frac{1}{T}$$

period of a simple gravity pendulum (T):



Is <u>independent on the mass of the object</u>, it is a function of the length l and gravitational acceleration g:

$$T = 2\pi\sqrt{\frac{l}{g}}\ [s]$$

Walter Lewin – lecture MIT (video)
a trial with pendulum
parameters: L = 5.21 m, g = 9.8 m/s^2,
estimation of pendulum period: 4.58 s

http://www.youtube.com/watch?v=KXys_mymMKA

(important timings: 11:40 pendulum, 16:25 with added mass)

2. Damped oscillator

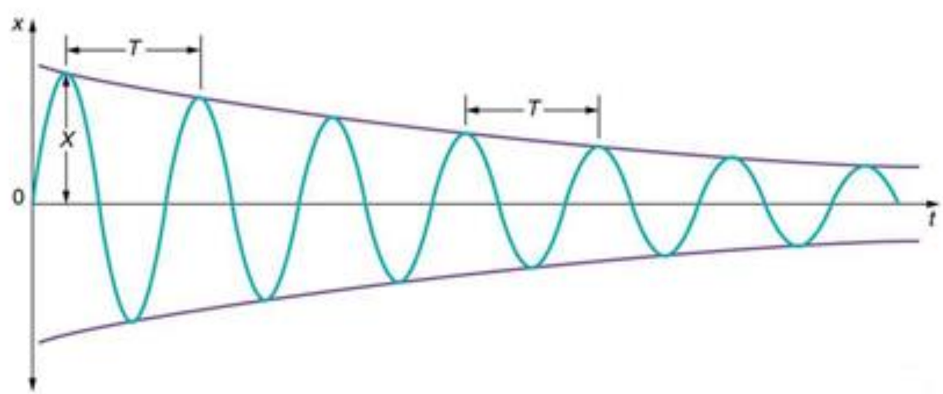
Friction (damping) force: $F_f = -cv$

$$F = F_{ext.} - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$ if $F_{ext.} = 0$

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_0\frac{dx(t)}{dt} + \omega_0^2 x = 0 \qquad \omega_0 = \sqrt{\frac{k}{m}};\ \zeta = \frac{c}{2\sqrt{mk}}$$

$\zeta > 1$ overdamped $\zeta = 1$ critically damped $\zeta < 1$ underdamped

Solution: $x(t) = Ae^{-\zeta\omega_0 t}\sin\left(\sqrt{1-\zeta^2}\,\omega_0 t + \varphi\right)$



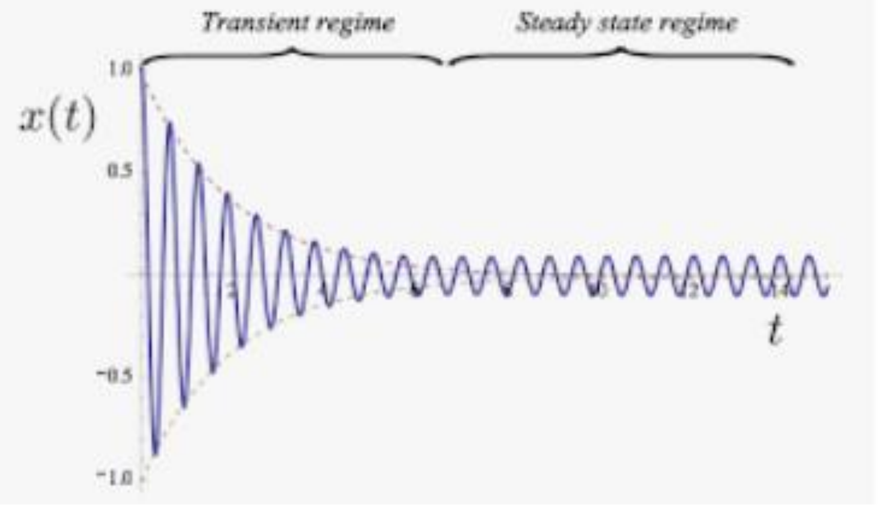
3. Driven oscillator

Externally applied force $F(t)$

$$F = F_{ext.} - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x(t)}{dt^2} + 2\zeta\omega_0\frac{dx(t)}{dt} + \omega_0^2 x = \frac{F(t)}{m}$$

Solutions depend on external force (e.g. can be sinusioidal)



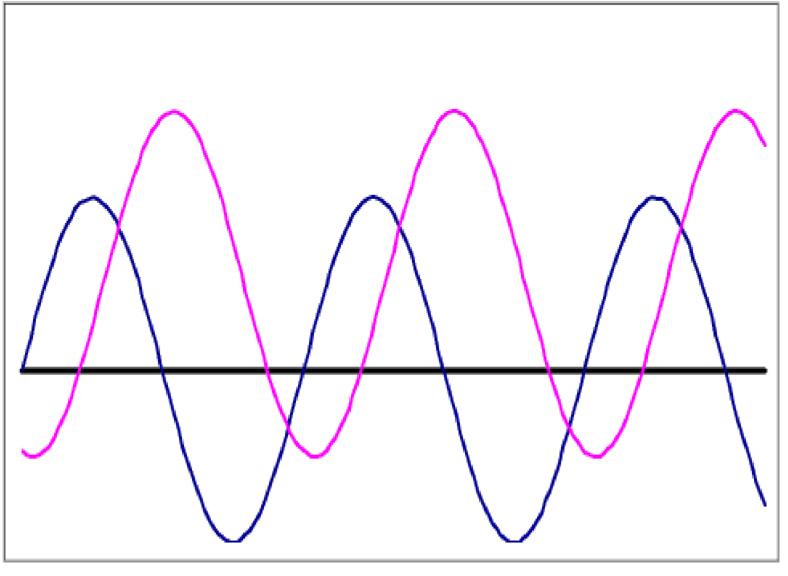
Combinations of oscillations

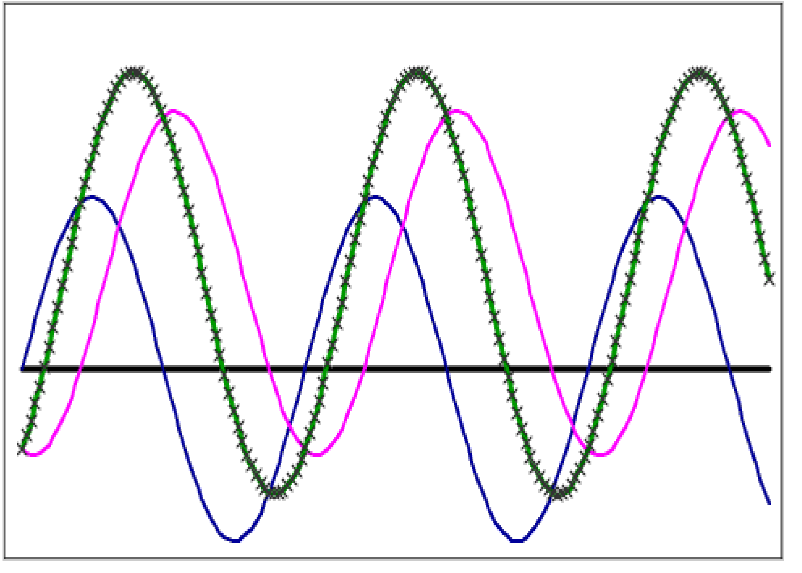
Oscillation 1 $x_1(t) = A_1 \cos(\omega_1 t - \varphi_1)$

Oscillation 2 $x_2(t) = A_2 \cos(\omega_2 t - \varphi_2)$

$$x = x_1 + x_2 = A_1 \cos(\omega_1 t - \varphi_1) + A_2 \cos(\omega_2 t - \varphi_2)$$

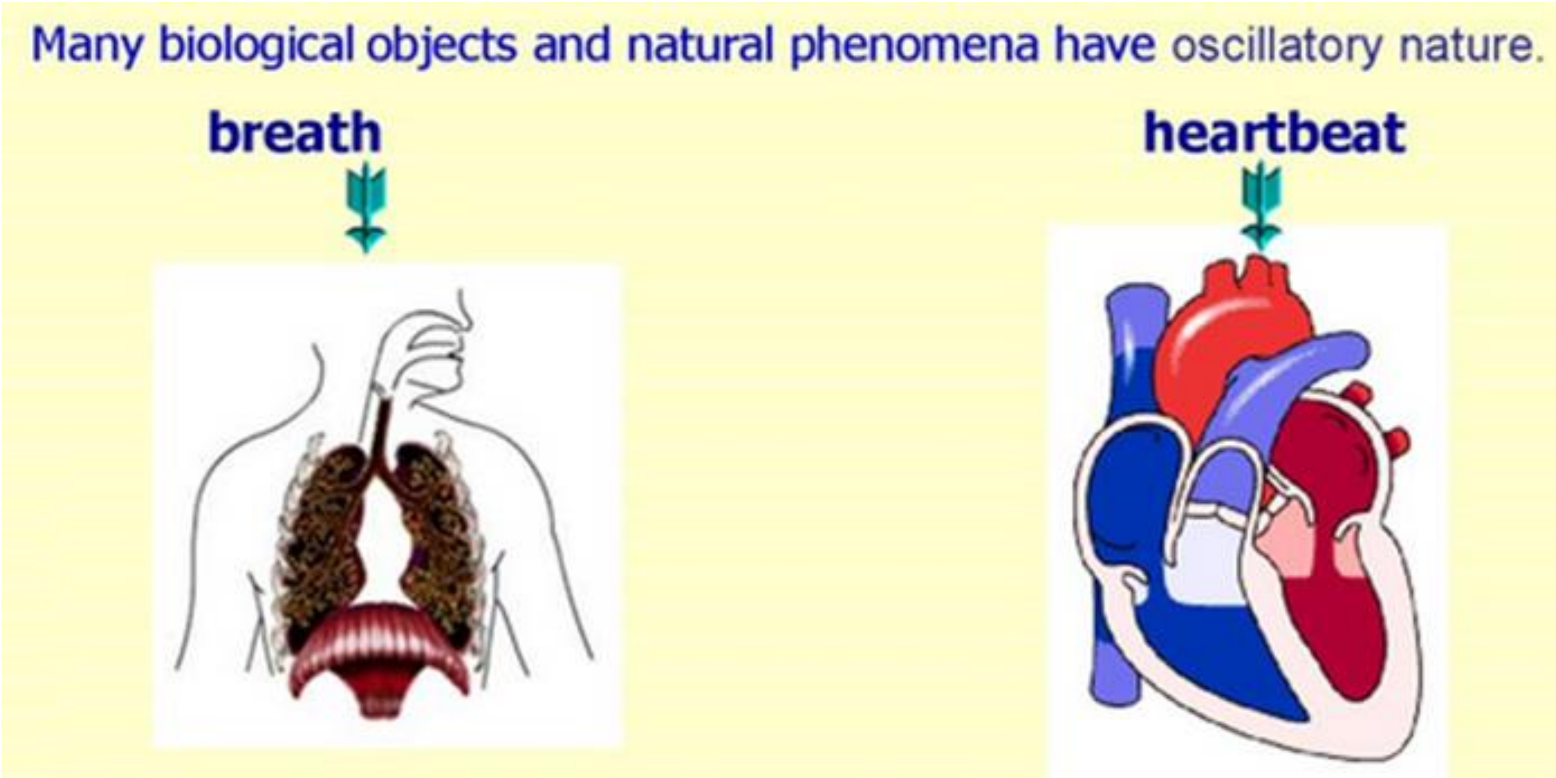
Maximum possible displacement $x_{max} = A_1 + A_2$





Comment: We will come back to this topic during the so called interference of waves.

oscillators in biology



Comment: In biology, there is very important so called circardian rhytm.

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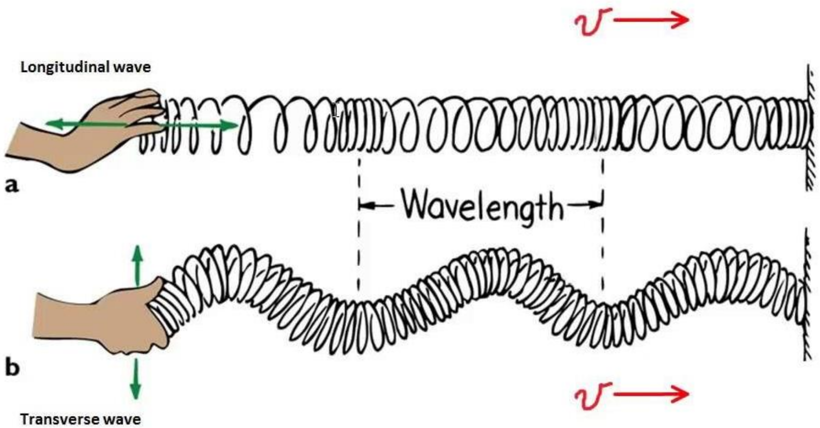
Waves

In physics, a wave is a propagating dynamic disturbance (change from equilibrium) of one or more quantities, sometimes as described by a wave equation.

– oscillation accompanied by the transfer of energy which travels through mass or space
– little or no associated mass transport.

waves
- mechanical – through medium which is deformed, e.g. sound waves, body waves
- electromagnetic – no medium, periodic oscillations of electric and magnetic fields, e.g. visible light, radio waves, ...

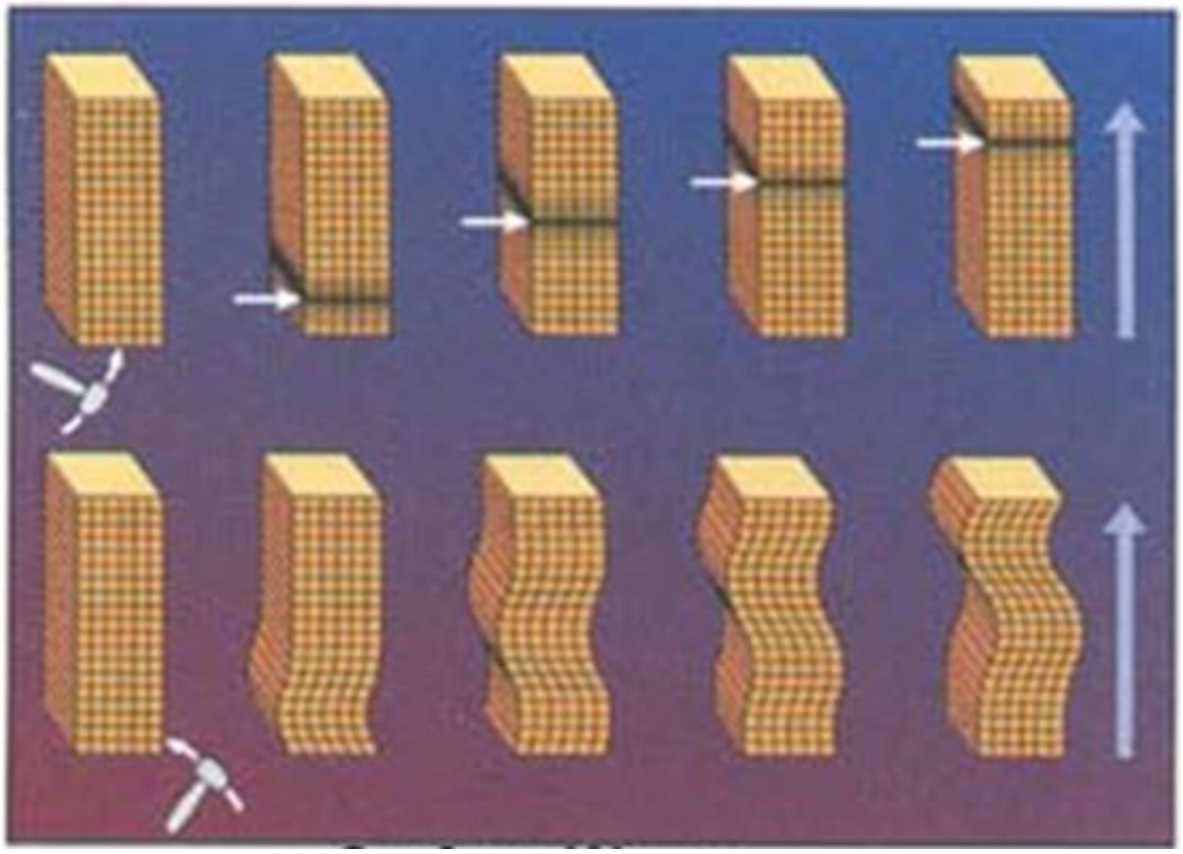
waves
- transverse – oscillations are perpendicular to the energy transfer
- longitudinal – oscillations are parallel to the energy transfer



Waves

waves

- transverse – oscillations are perpendicular to the energy transfer
- longitudinal – oscillations are parallel to the energy transfer



longitudinal

transverse

Wave equation

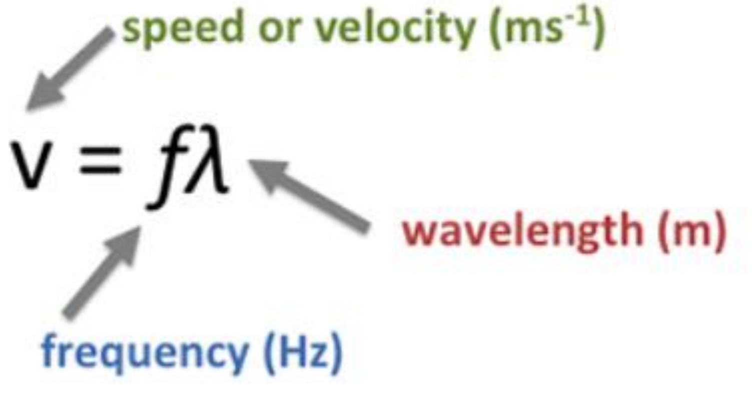
$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

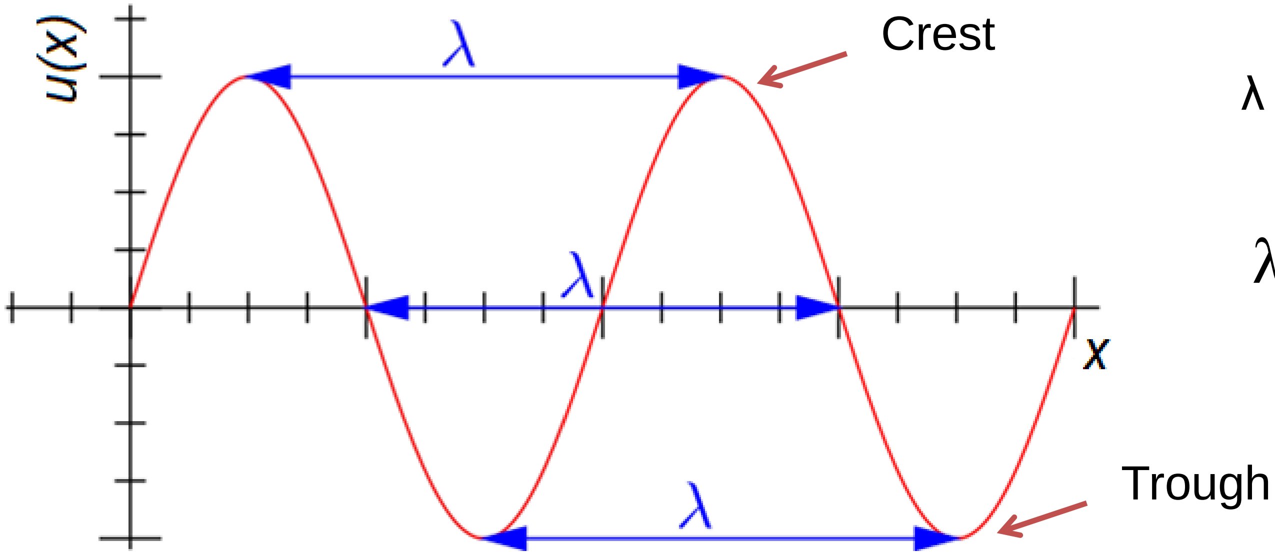
$u = u(x_1, x_2, ..., x_n, t)$ - scalar function whose values could model, for example, the mechanical displacement of a wave

One space dimension case: $$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Now the searched solution (u) is not only a function of time (t), but also function of space (x).

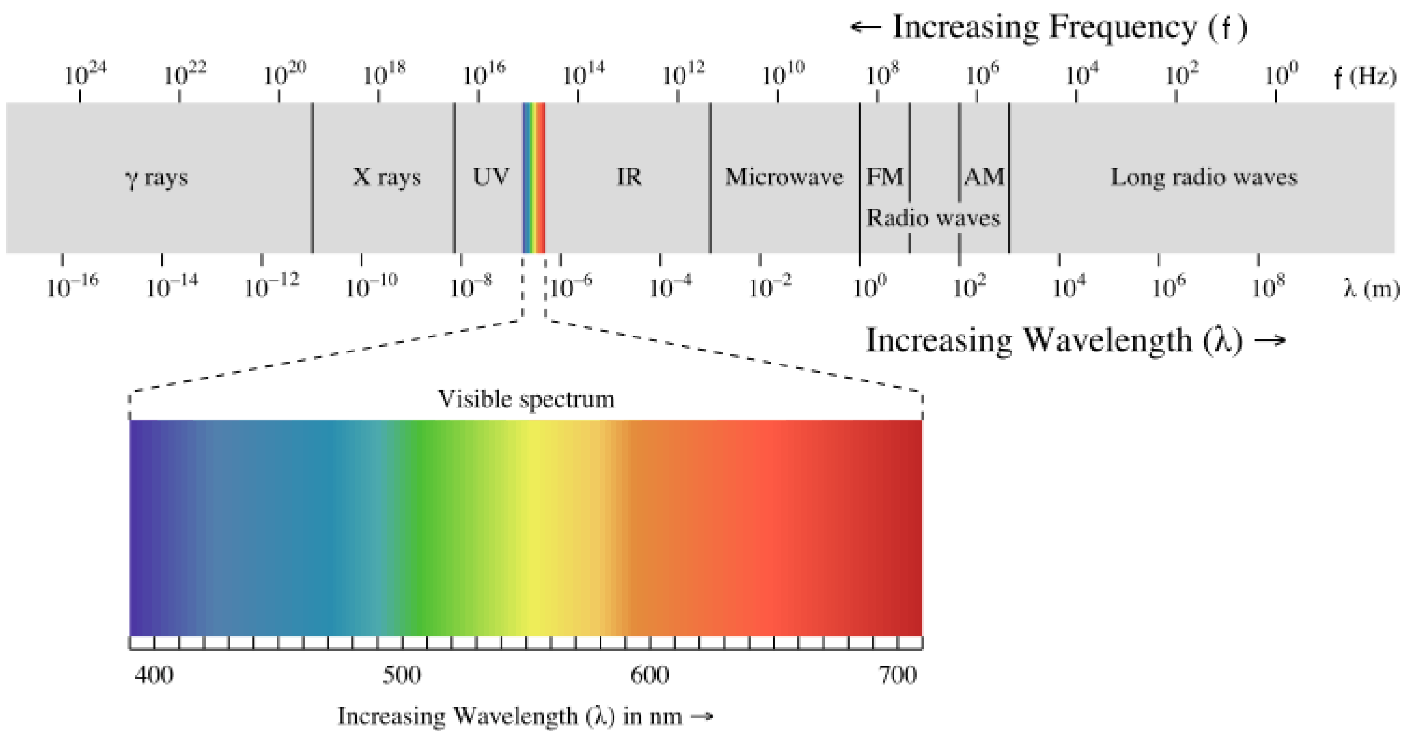
One important consequence:





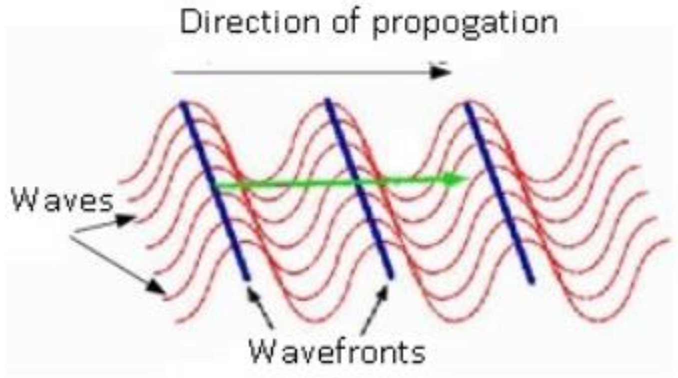
λ - wavelength

$$\lambda = vT = \frac{v}{f}$$

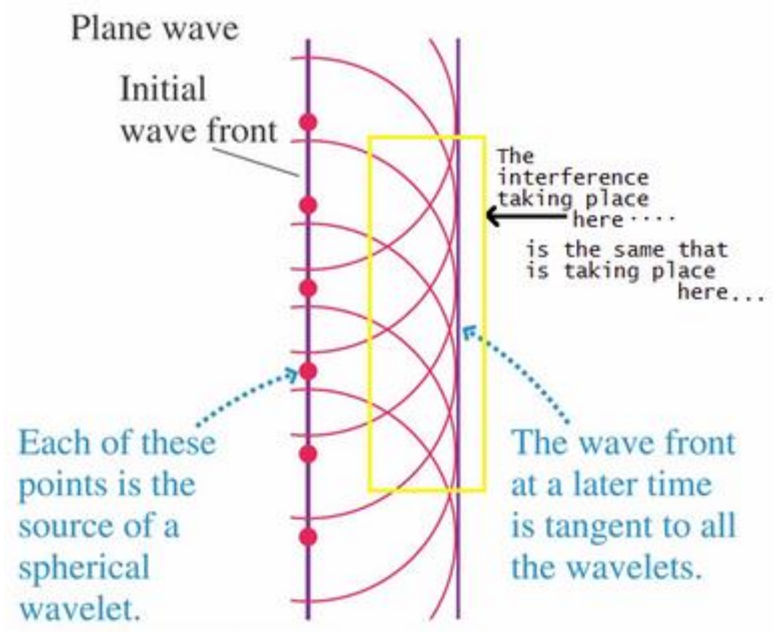


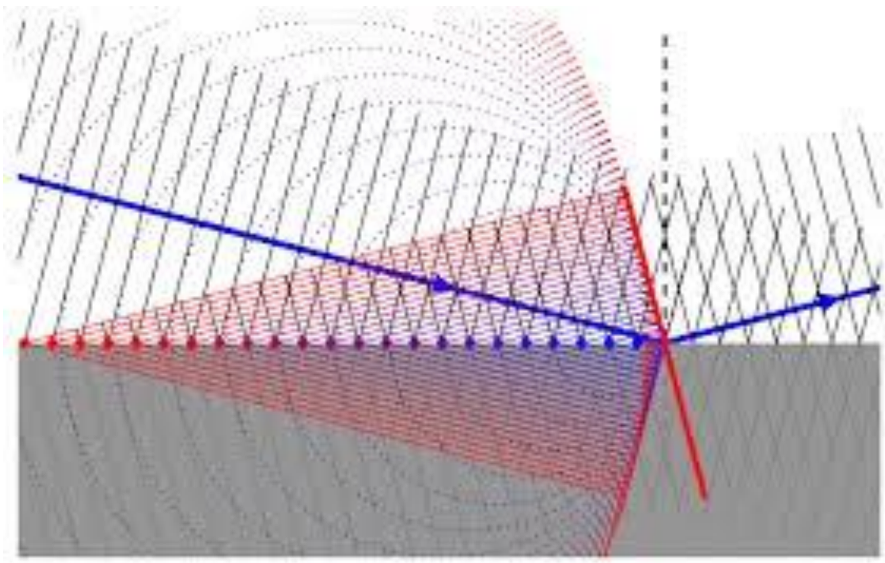
Wavefronts, Huygens principle

Waves can propagate in packages, called as wavefronts.



<u>Huygens principle:</u> Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction. The new wave-front is the tangential surface to all of these secondary wavelets.





Wavefronts, Huygens principle

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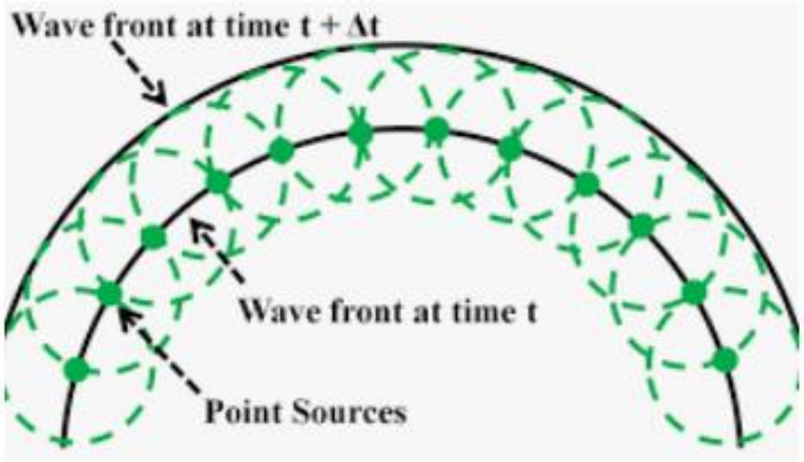
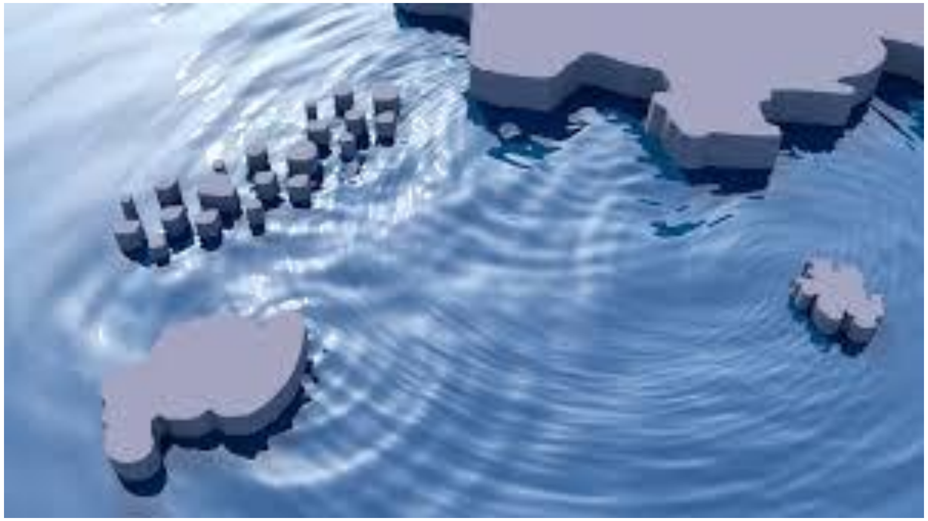
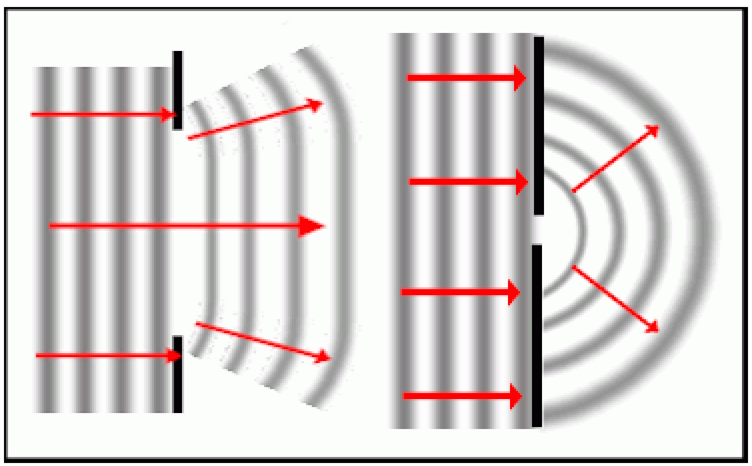


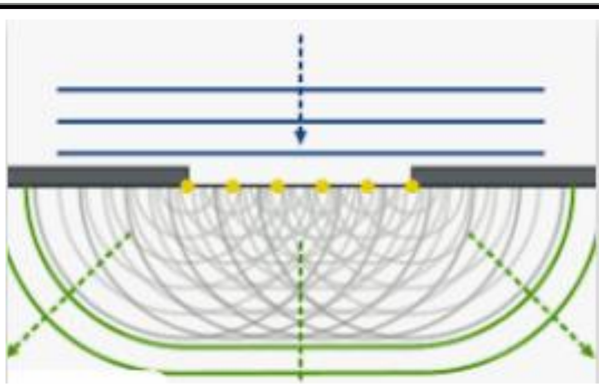


Huygens principle – experiments with slits

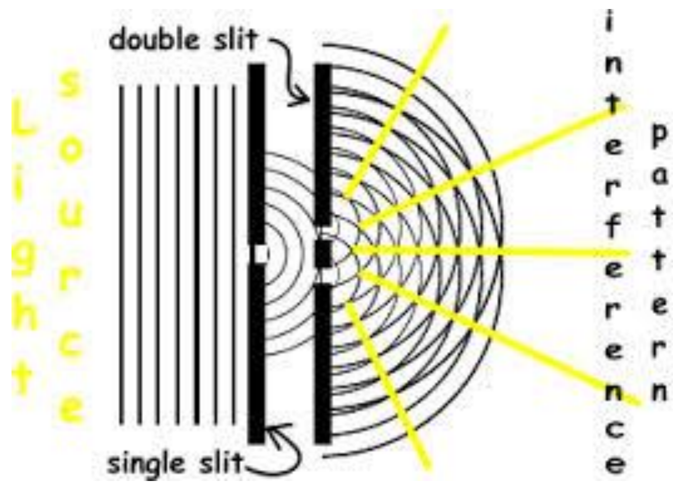
<u>Huygens principle:</u> Every point on a wave-front may be considered a source of secondary spherical wavelets which spread out in the forward direction. The new wave-front is the tangential surface to all of these secondary wavelets.

Experiment with one slit:

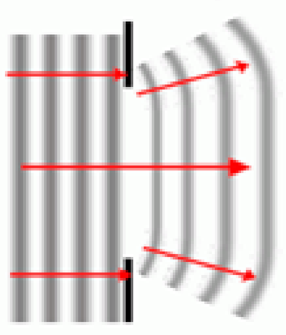




Experiment with double slit:

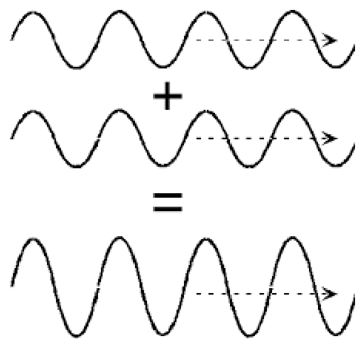


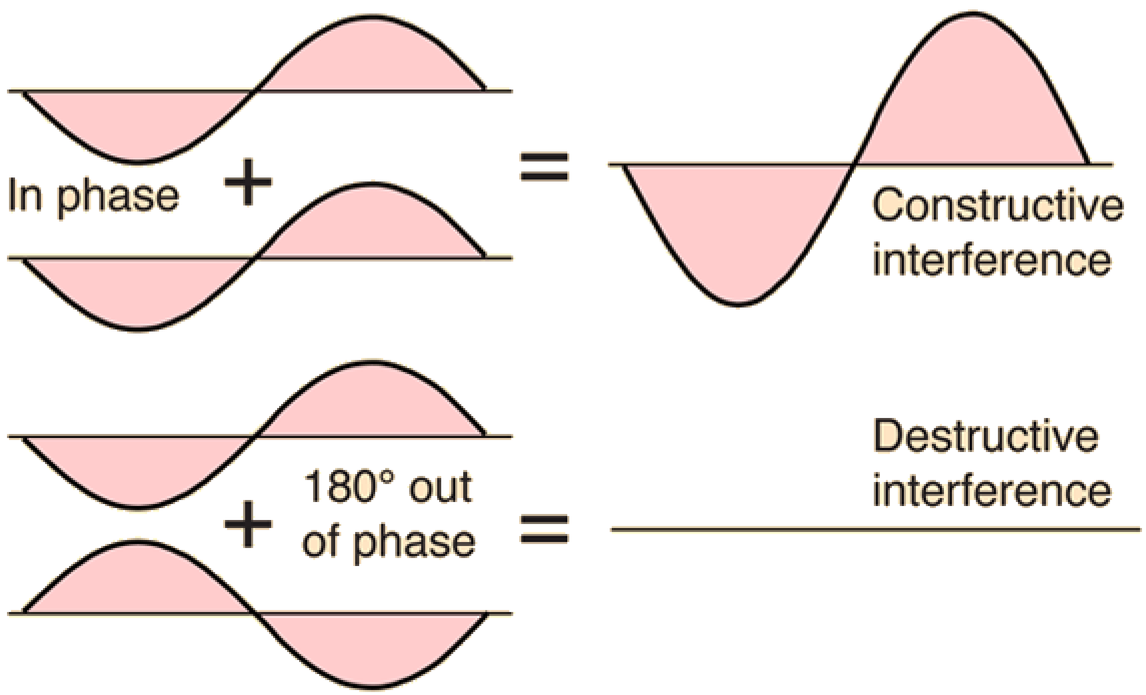
Huygens principle – one slit experiment



Interference

The principle of superposition of waves states that when two or more propagating waves of same type are incident on the same point, the total displacement at that point is equal to the point wise sum of the displacements of the individual waves.





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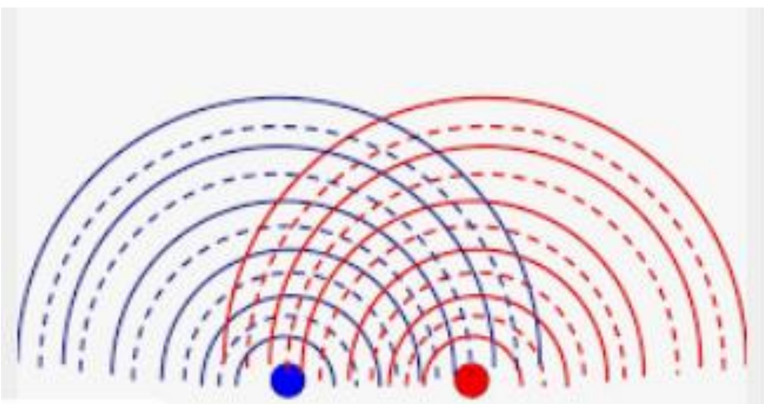

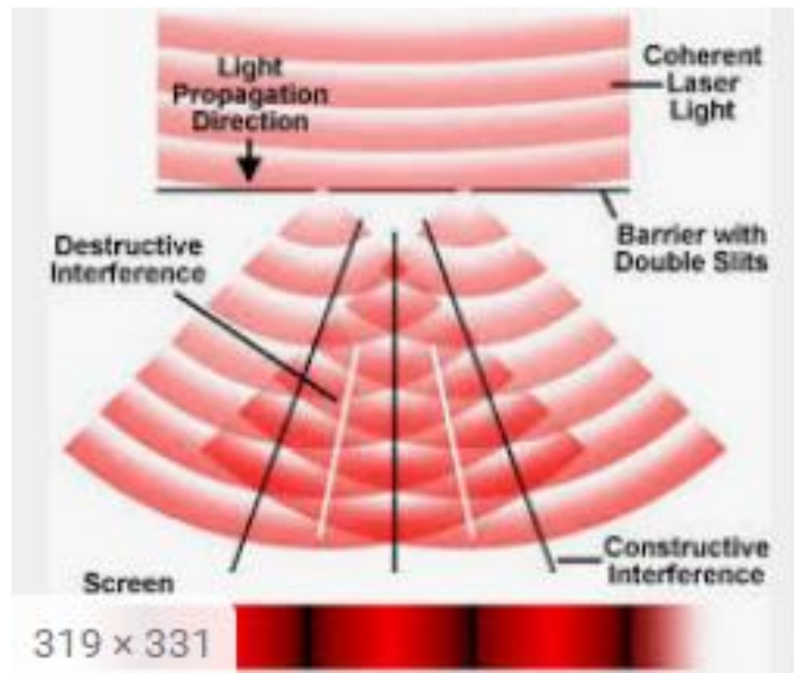

Doppler effect

It is the change in frequency of a wave (or other periodic event) for an observer moving relative to its source. The received frequency is higher during the approach, identical at the instant of passing by, and lower during the recession.