Lecture 7: electromagnetism – Maxwell‘s equations

Content:

- summary of known EM-laws and math. formalism
- Maxwell‘s equations in integral and differential form
- EM waves, Poynting vector
- EM spectrum, light
- light – seeing things, reflection vs. refraction
- light diffraction, Huygens–Fresnel principle

- summary of known EM-laws and math. formalism

Lecture Nr. 6 (magnetism): slide nr. 30:

1.

$$\Phi_B = \oiint_S \vec{B} \cdot d\vec{S} = 0$$

Gauss’s law for magnetic field

(flux is zero due to the dipole character of magnetic field)

Lecture Nr. 5 (electricity): slide nr. 54:

2.

$$\Phi_E = \oiint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

Gauss’s law for electric field

(is non-zero due to the monopolar character of electric field)

Lecture Nr. 6 (magnetism): slide nr. 38:

3.

$$\oint_{l(S)} \vec{B} \cdot d\vec{l} = \mu_0 I$$

Ampere‘s law

(integration of magnetic induction along a closed circle – so called circulation)

- summary of known EM-laws and math. formalism

Lecture Nr. 6 (magnetism): slide nr. 40 and nr. 30:

4.
$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\iint_S \vec{B}\cdot d\vec{S}\right]$$

Faraday‘s law of induction

(due to the dipole character of magnetic field)

(here ε is not electric permitivity, but electromotive force is in [V])

Electromotive force is the voltage developed by any source of electrical energy.

It can be also evaluated by means of the circulation integral for the electric field:

$$\varepsilon = \int_A^B \vec{E}\cdot d\vec{l}$$

so, we can write for the Farraday's law of induction:

$$\boxed{\oint_{l(S)} \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{S}}$$

- summary of known EM-laws and math. formalism

These 4 equations together with Lorentz force law (lecture Nr.6, slides nr.32-34) form the foundation of classical electrodynamics, classical optics, and electric circuits.

$$\vec{F} = \underset{\text{Electric force}}{q\vec{E}} + \underset{\text{Magnetic force}}{q\vec{v}\ x\ \vec{B}}$$

Lorentz force law

The 4 basic equations are in general called as Maxwell’s equations.

They are named after the physicist and mathematician **James Clerk Maxwell**, who published an early form of those equations between 1861 and 1862.

With the publication of A Dynamical Theory of the Electromagnetic Field in 1865, Maxwell demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light.

Maxwell's equations for electromagnetism have been called the "second great unification in physics (the first one was from Isaac Newton).

James Clerk Maxwell (1831–1879)

- summary of known EM-laws and math. formalism

Formulation of Maxwell‘s equations (ME) is connected with the development of physics in the end of 19th cent., when the internal structure of matter was not well known (idea about the existence of positive and negative charges in their structure was accepted), so they are based mostly on the description of macroscopic phenomena.

The ME equations have two major variants:

a) The "microscopic" set of Maxwell's equations uses total charge and total current, including the complicated charges and currents in materials at the atomic scale; it has universal applicability, but may be infeasible to calculate.

b) The "macroscopic" set of Maxwell's equations defines two new auxiliary fields that describe large-scale behaviour without having to consider these atomic scale details.

From the point of view of mathematical formalism, we divide ME in their integral and differential form. In the actual stage of this lecture, we will start with the integral form.

Maxwell's equations – integral form

$$\oiint_{S(V)} \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho dV$$ Gauss law

$$\oiint_{S(V)} \vec{B} \cdot d\vec{S} = 0$$ Gauss law for magnetism

$$\oint_{l(S)} \vec{E} \cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{dt}} \iint_S \vec{B} \cdot d\vec{S}$$ Maxwell-Faraday equation

$$\oint_{l(S)} \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{S} + \mu_0 \varepsilon_0 \frac{\mathrm{d}}{\mathrm{dt}} \iint_S \vec{E} \cdot d\vec{S}$$ Maxwell-Ampere equation

where: *V* is volume, enclosed by surface *S(V)*, *t* is time,

$l(S)$ is curve enclosing a surface *S*,

E is electrical field intensity vector, **B** is magnetic induction vector,

ρ is volume density of electrical charge,

J is density of electrical current (vector quantity),

ε_0 is electrical permitivity of vacuum, μ_0 is magnetic permeability of vacuum

Maxwell’s equations

The 4 basic equations are in general called as Maxwell’s equations.

Name	Integral equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega} \rho\, \mathrm{d}V$	The electric field leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{B}\cdot \mathrm{d}\mathbf{S}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0\iint_{\Sigma} \mathbf{J}\cdot \mathrm{d}\mathbf{S} + \mu_0\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{E}\cdot \mathrm{d}\mathbf{S}$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

Instead of current *I* and charge *Q*, there are used integrals of current density (***J***) and charge density (ρ):

$$I = \iint_S \vec{J}\cdot d\vec{S} \qquad Q = \iiint_V \rho\, dv$$

In the last equation (Amper’s law) there is an additional term (added by Maxwell):

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{d}{dt}\iint_S \vec{E}\cdot d\vec{S}$$

called therefore also as Maxwell-Ampere equation

Maxwell's equations in vacuum (free space)

Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_{\Sigma} \mathbf{J}\cdot \mathrm{d}\mathbf{S} + \mu_0\varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{E}\cdot \mathrm{d}\mathbf{S}$

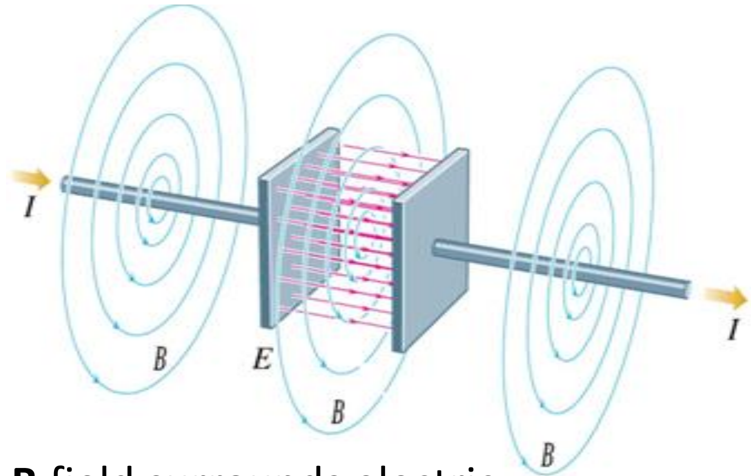
In vacuum there are no free charges and no current ($\rho = 0$, $\mathbf{J} = 0$).

$$\oiint_{S(V)} \vec{E}\cdot d\vec{S} = 0$$

$$\oiint_{S(V)} \vec{B}\cdot d\vec{S} = 0$$

$$\oint_{l(S)} \vec{E}\cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{S} \vec{B}\cdot d\vec{S}$$

$$\oint_{l(S)} \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\iint_{S} \vec{E}\cdot d\vec{S}$$



B field surrounds electric field **E** (capacitor in the central part of the figure), although there is no "current" flowing here

Here the role of the added term by Maxwell is clearly visible (without it it would not be possible the explain the situation of **E** field acting in vacuum). The term $\varepsilon_0\, \mathrm{d}\Phi_E/\mathrm{d}t$ is sometimes called as displacement current.

Videos with an experiment: https://www.youtube.com/watch?v=NQeATJvuVPY

https://www.youtube.com/watch?v=uzbAv0Kbkg4

Excellent lecture from Walter Lewin: https://www.youtube.com/watch?v=8ZYFYUFRblM

Maxwell's equations – integral form

Name	Integral equations	Meaning
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega} \rho \,\mathrm{d}V$	The electric field leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
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Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0\iint_{\Sigma} \mathbf{J}\cdot \mathrm{d}\mathbf{S} + \mu_0\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{E}\cdot \mathrm{d}\mathbf{S}$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

We have this common form thanks to the work of Oliver Heaviside - he was able to rewrite Maxwell's original 20 equations into a mathematically equivalent 4 equation form.
Repetition: first two equations describe how the fields vary in space due to sources if any; electric fields emanating from electric charges in Gauss's law, and magnetic fields as closed field line in Gauss's law for magnetism.
The other two describe how the fields "circulate" around their respective sources; the magnetic field "circulates" around electric currents and time varying electric fields in Ampere's law with Maxwell's addition, while the electric field "circulates" around time varying magnetic fields in Faraday's law.

Maxwell’s equations – differential form

Name	Integral equations	Differential equations
Gauss's law	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_\Omega \rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oint\!\!\!\oint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \mu_0\iint_\Sigma \mathbf{J}\cdot\mathrm{d}\mathbf{S} + \mu_0\varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_\Sigma \mathbf{E}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial \mathbf{E}}{\partial t}\right)$

To transfer from the integral form to that – differential one is not easy, we have to know special properties of *div* and *rot* differential operators and so called Stokes theorem – we will not perform it in detail here...

There are several good web-sites and also videos about this, e.g.:
https://www.wikihow.com/Convert-Maxwell%27s-Equations-into-Differential-Form

Maxwell's equations – differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ Gauss law

$$\nabla \cdot \vec{B} = 0$$ Gauss law for magnetism

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \mathrm{t}}$$ Maxwell-Faraday equation

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial \mathrm{t}} \right)$$ Maxwell-Ampere equation

where: **E** is electrical field intensity vector, **B** is magnetic induction vector,
ρ is volume density of electrical charge,
J is density of electrical current (vector quantity),
ε_0 is electrical permitivity of vacuum, μ_0 is magnetic permeability of vacuum

Maxwell's equations – differential form (vacuum, empty space)

$$\nabla \cdot \mathbf{E} = 0 \text{ (Gauss)} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \text{ (Faraday)}$$

$$\nabla \cdot \mathbf{B} = 0 \text{ (no name)} \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \text{ (Ampère)}$$

$\rho = 0$ (electrical charge is not present),
$\mathbf{J} = 0$ (electrical current is not present).

Maxwell's equations – differential form

... these are so popular... (plotted on a wall close to our faculty)

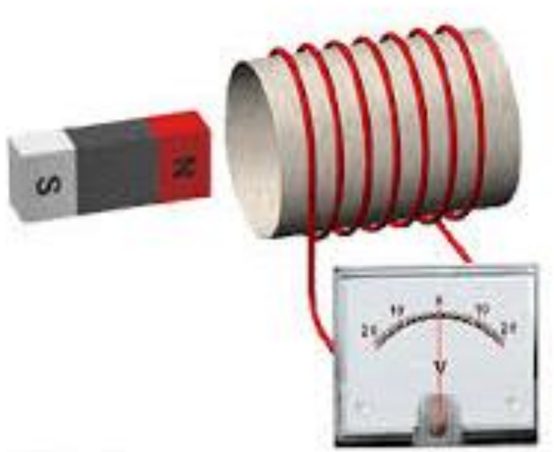
Maxwell's equations

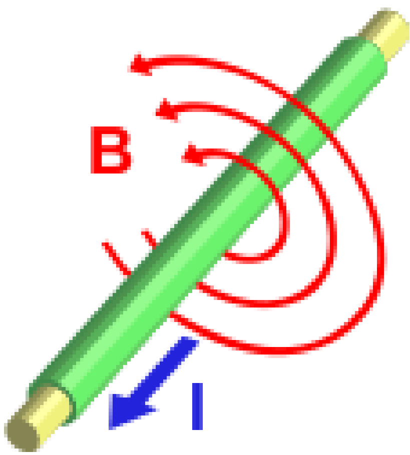
One very important contribution from Maxwell – symmetry (between electric and magnetic fields):

- a time varying magnetic field produces an electric field,

-a time varying electric field produces a magnetic field.

This symmetry is fully valid thanks to the additional term From Maxwell to the Ampere's equation.





EM waves

Electromagnetic waves (EM radiation) are synchronized oscillations of electric and magnetic fields that propagate at the speed of light through a vacuum.

Faraday's law: **dB/dt ⟶ electric field**

Maxwell's modification of Ampere's law

dE/dt ⟶ magnetic field

$$\oint_{l(S)} \vec{E}\cdot d\vec{l} = -\frac{\mathrm{d}}{\mathrm{dt}}\iint_S \vec{B}\cdot d\vec{S} \qquad \oint_{l(S)} \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{\mathrm{d}}{\mathrm{dt}}\iint_S \vec{E}\cdot d\vec{S}$$

These two equations can be solved simultaneously.

The result is:

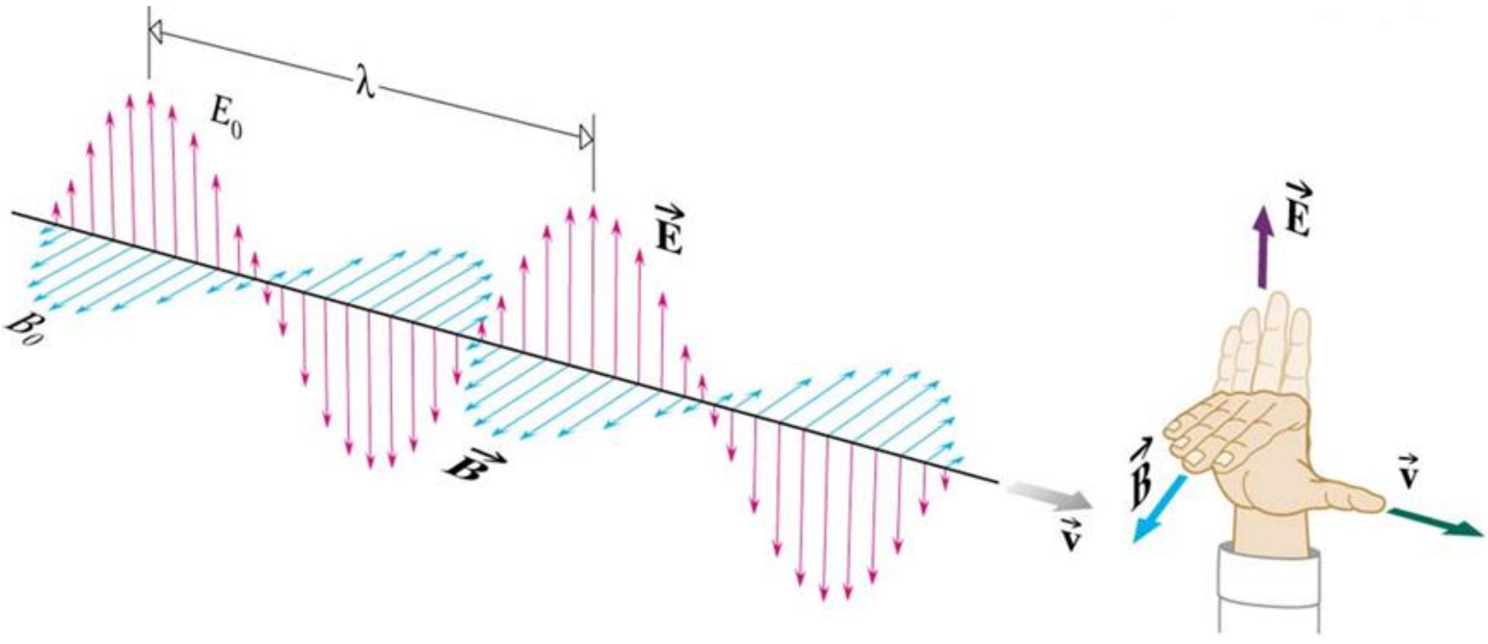
$$\mathbf{E}(x, t) = E_P \sin(kx-\omega t)\ \hat{\mathbf{j}}$$

$$\mathbf{B}(x, t) = B_P \sin(kx-\omega t)\ \hat{\mathbf{k}}$$

EM waves

One important consequence of the EM symmetry – origin of EM waves: changing electric and magnetic fields create a wave:

- electric field creates a magnetic field
- magnetic field creates an electric field

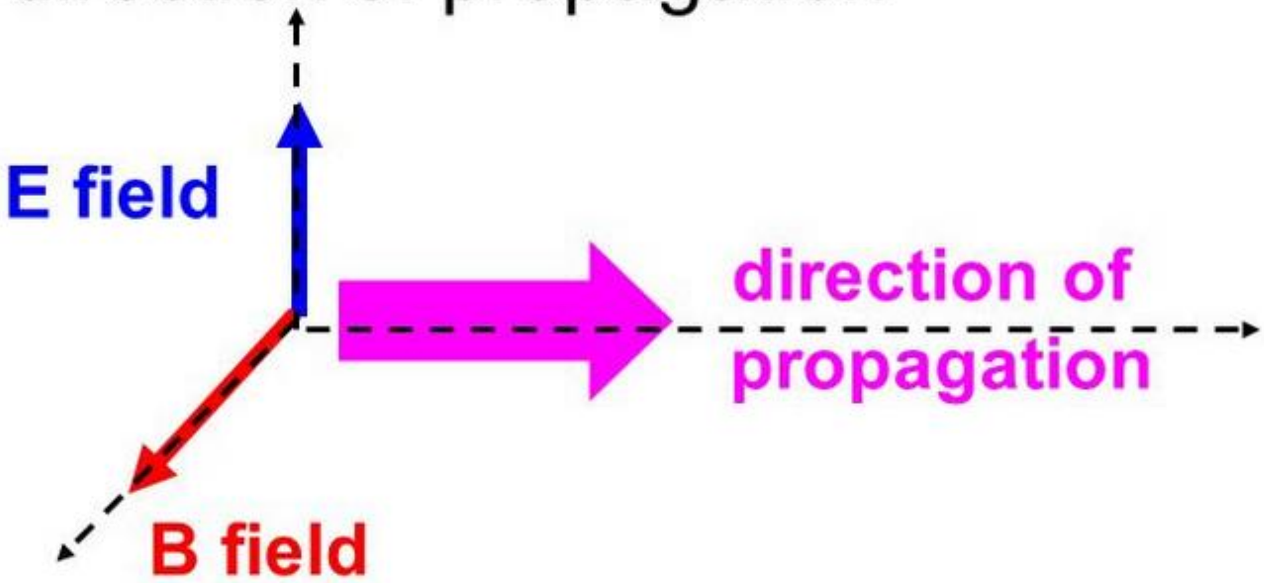


(a) (b)

Electromagnetic waves are produced whenever charged particles are accelerated, and these waves can subsequently interact with any charged particles. EM waves carry energy.

EM waves

- the electromagnetic wave is a *transverse wave*, the electric and magnetic fields oscillate in the direction perpendicular to the direction of propagation

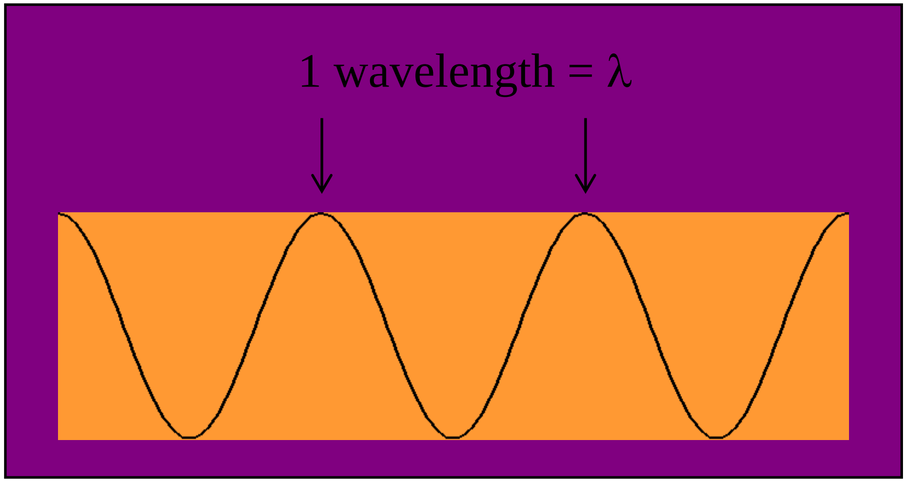


good visualisation:
https://en.wikipedia.org/wiki/Electromagnetic_radiation#/media/File:Electromagneticwave3D.gif

EM waves

- for a continuous wave the **speed** *v* is the **wavelength** compared to the **period** (reciprocal frequency):

$$v = \lambda / T = \lambda f$$



- for an electromagnetic wave the speed is based on the permittivity and permeability,

$$v = \sqrt{1/\mu\varepsilon}$$

- in the vacuum this is the speed of light $c = 2.99792 \cdot 10^8$ m/s.

$$c = \sqrt{1/\mu_0\varepsilon_0}$$

Task: check the value of *c* by entering values for electric permitivity of vacuum ε_0 and magnetic permeability of vacuum μ_0.

EM waves – derivation of velocity

Faraday law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right)$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B})$$

Ampére law

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times (\nabla \times \mathbf{B}) = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$$

$$\nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{E})$$

Now if you look carefully, you'll see that one term in each equation equals zero and the other can be replaced with a time derivative.

$$0 - \nabla^2 \mathbf{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \qquad 0 - \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \mathbf{B}}{\partial t}\right)$$

Let's clean it up a bit and see what we get.

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{E} \qquad \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \mathbf{B}$$

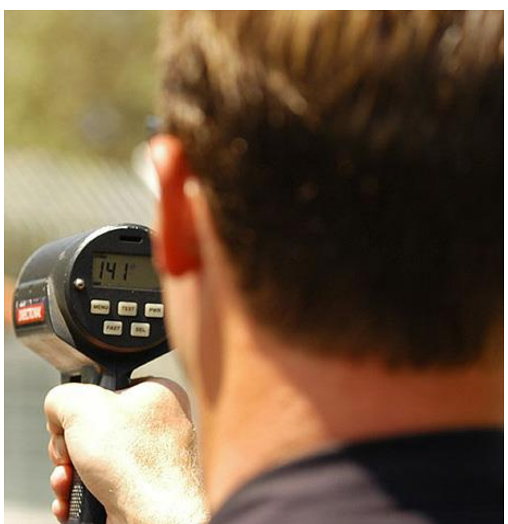
If you compare this equation to the mechanical wave equation: $\dfrac{\partial^2 D}{\partial x^2} = \dfrac{1}{v^2} \dfrac{\partial^2 D}{\partial t^2}$

Then it would be logical to define the speed of an electromagnetic wave to be

$$v_{EM-wave} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 300000 km/s = c.$$

EM waves

- All EM waves travel 300,000 km/sec in vacuum (speed of light-nature's limit!).
- EM waves usually travel slowest in solids and fastest in gases.



Material	Speed (km/s)
Vacuum	300,000
Air	<300,000
Water	226,000
Glass	200,000
Diamond	124,000
Soils	100,000

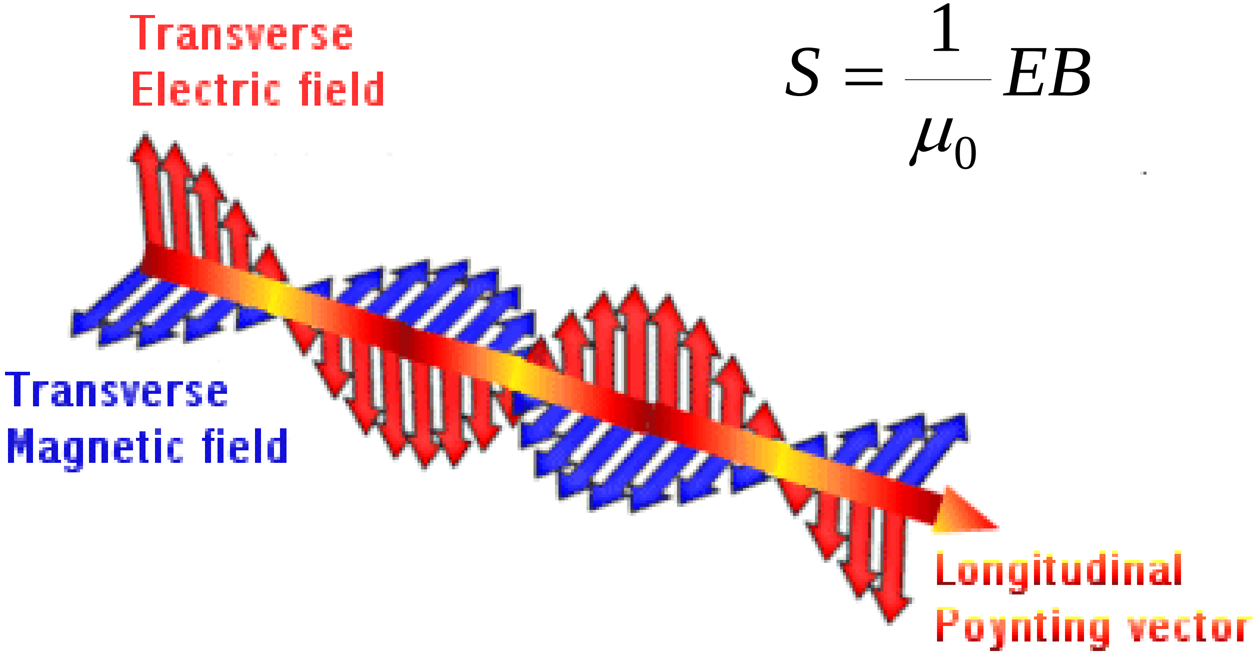
EM waves – Poynting vector

Poynting vector – has the direction of EM wave propagation and its size (amplitude) speaks about the rate of energy transport per unit area by the EM wave (unit: [W/m^2]):

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

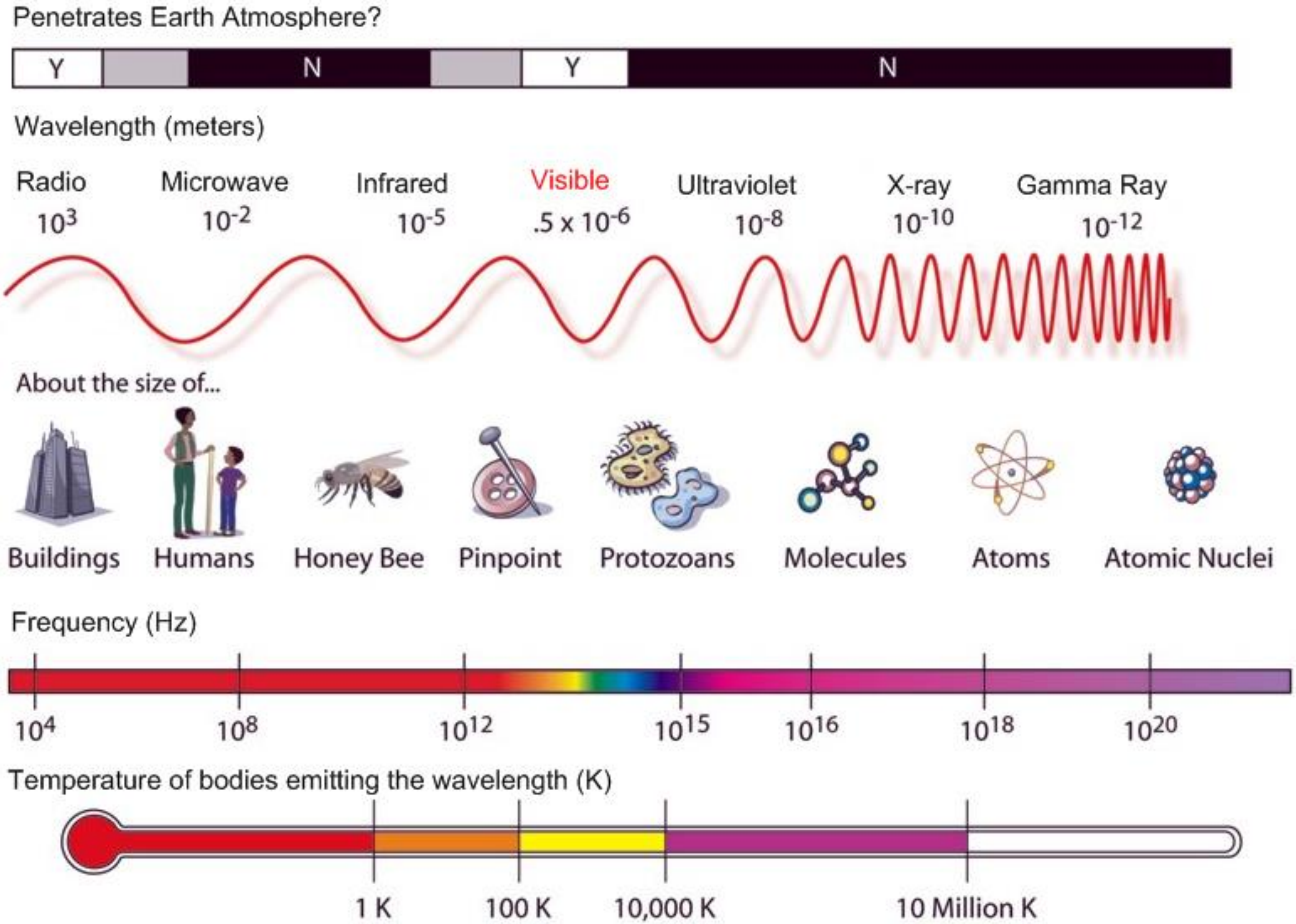
Due to the fact that vectors E and B are orthogonal (perpendicular), it is valid:

$$S = \frac{1}{\mu_0} EB$$



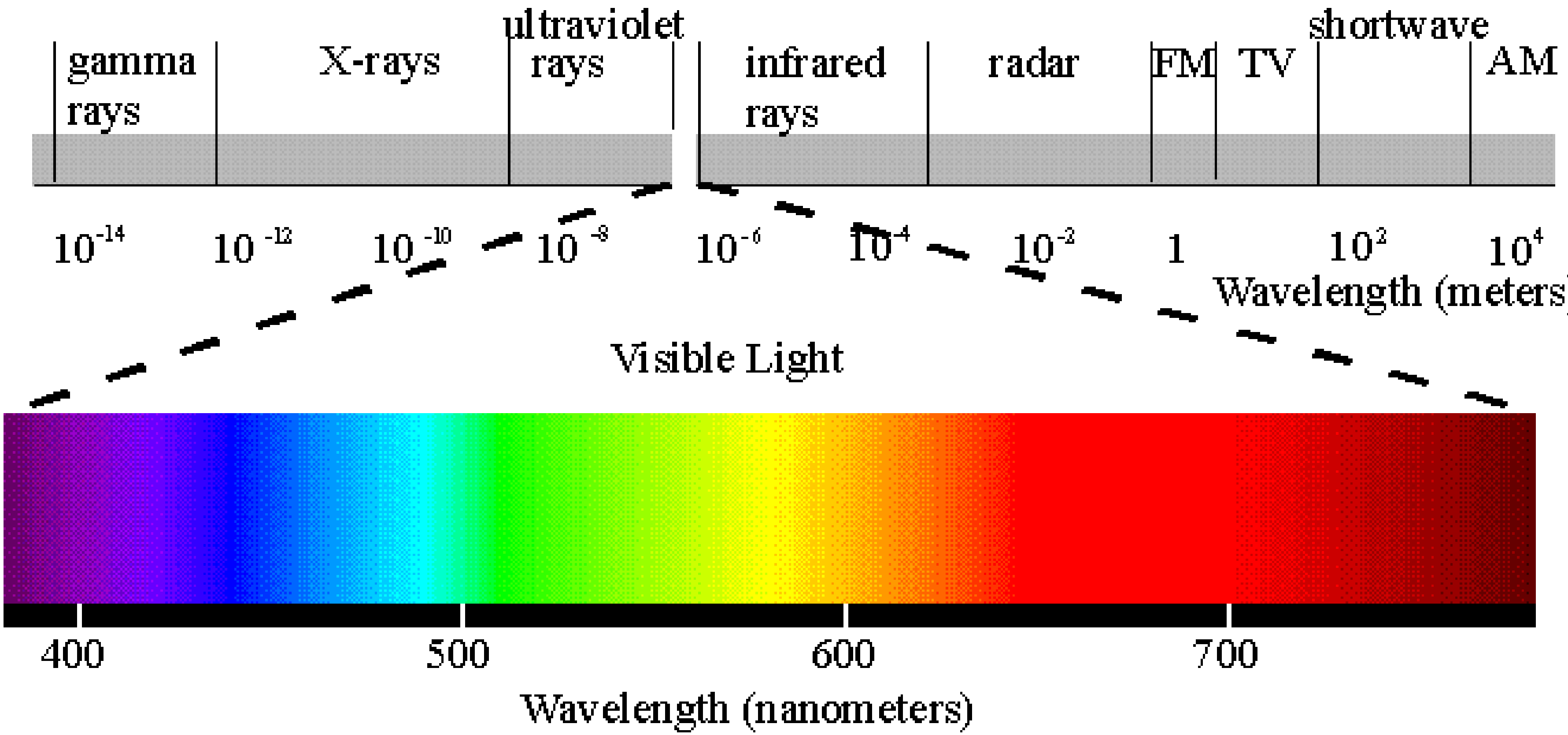
Named after its inventor John Henry Poynting.

The Electromagnetic Spectrum



A good video from NASA: https://www.youtube.com/watch?v=cfXzwh3KadE

EM waves spectrum

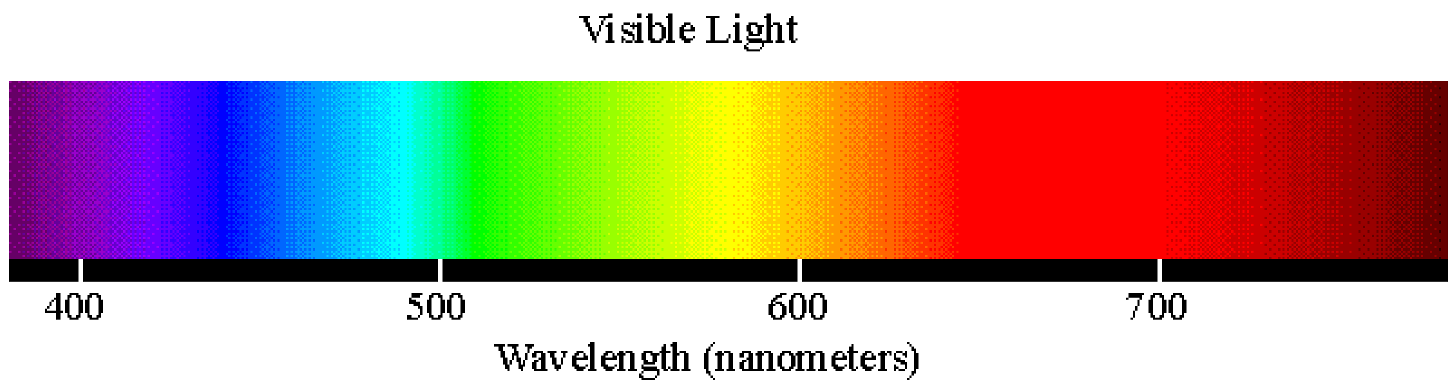


A typical human eye will respond to wavelengths from about 400 to 700 nm.
In terms of frequency, this corresponds to a band in the vicinity of 430–770 THz.

Seeing things

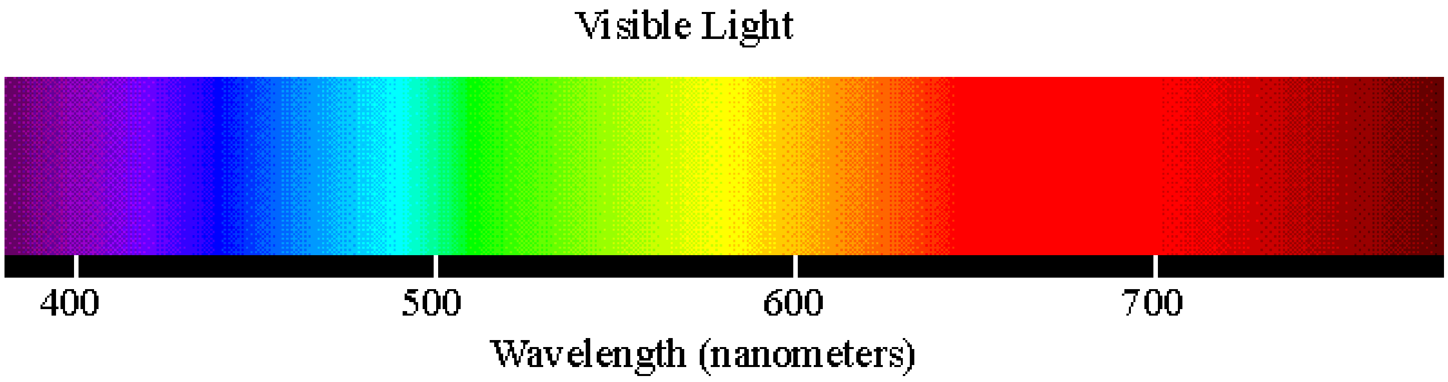
Not everything does emit radiation. How do we see things?

- when waves run into a boundary they are partially transmitted (refracted) and partially reflected (light too),
- therefore, an object does not need to emit waves (photons) itself to be seen, it just has to reflect light back to our eyes where we can detect it,
- objects that do not allow light to pass through them are called opaque,
- objects that allow light to pass through them are considered transparent,
- objects in between are called translucent.

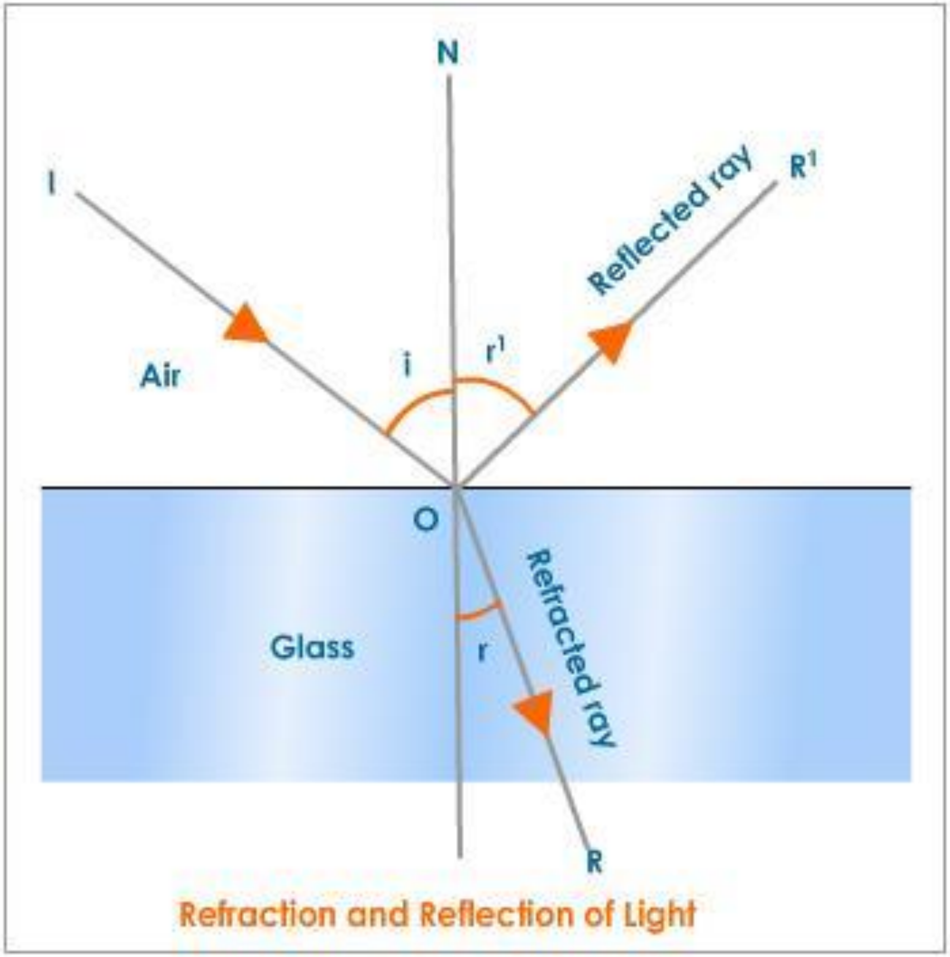


Seeing things

- the light we see is know as visible or white light,
- the light is not really white, the white we see is a combination of all the colors of the rainbow,
- when all of these light waves are combined we see white light (all wavelengths are reflected back to our eyes),
- if we see something as RED or BLUE?
  - it reflected only the RED or only the BLUE wavelengths, the others were absorbed,
- and if we see something as black?
  - it did not reflect back any of the light.



refraction vs reflection

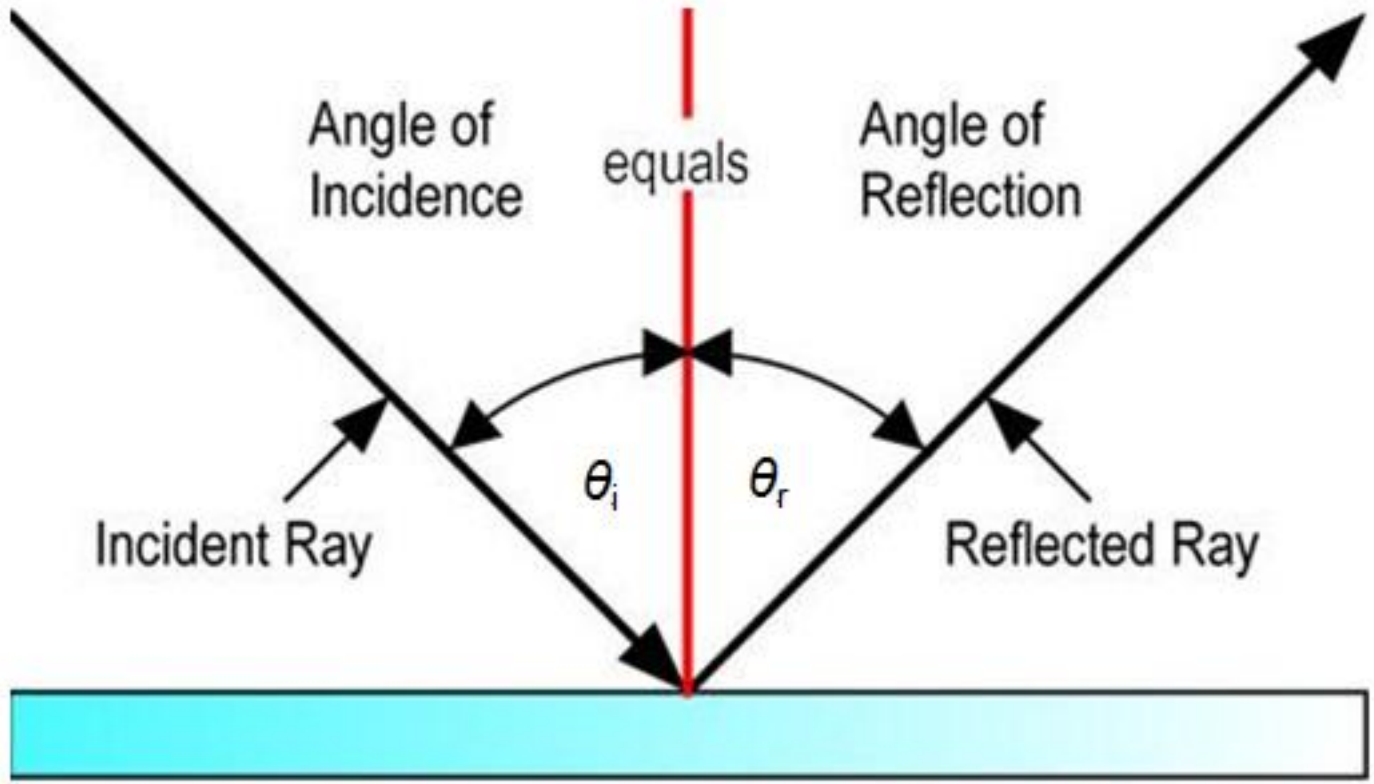


Refraction is the change in direction of propagation of a wave due to a change in its transmission medium.

Reflection is the change in direction of a wavefront at an interface between two different media so that the wavefront returns into the medium from which it originated.

law of reflection

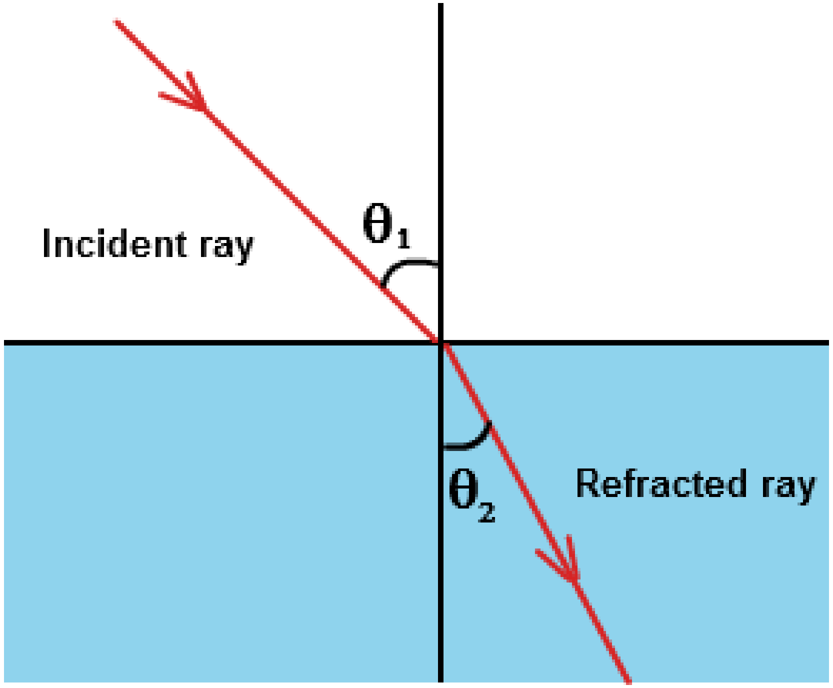
The law of reflection states that $\theta_i = \theta_r$, or in other words, the angle of incidence equals the angle of reflection.



Snell's law of refraction

Snell's law states that the ratio of the sines of the angle of incidence θ_1 and angle of refraction θ_2 is equivalent to the ratio of phase velocities (v_1/v_2) in the two media, or equivalently, to the opposite ratio of the indices of refraction (n_2/n_1):

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$



Index of refraction n of a material is a dimensionless number that describes how light propagates through that medium.
It is defined as:

$$n = \frac{c}{v}$$

where c is the speed of light in vacuum and v is the phase velocity of light in the medium.

Snell's law of refraction

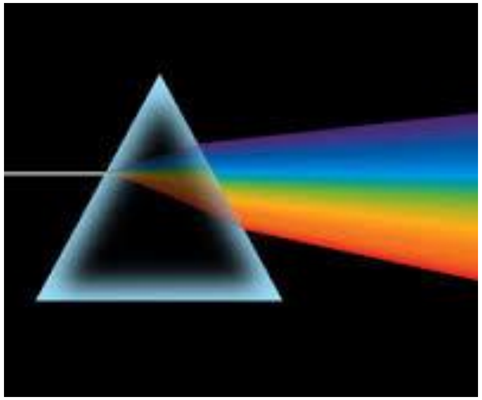
Index of Refraction for various media

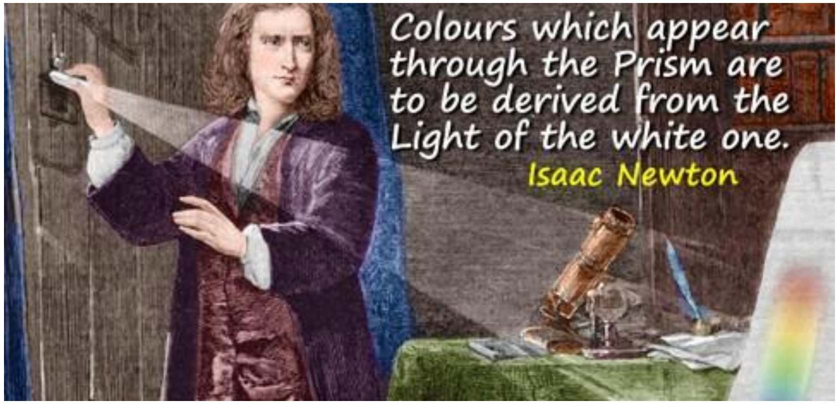
Media	Index of Refraction
Vacuum	1.00
Air	1.0003
Carbon dioxide gas	1.0005
Ice	1.31
Pure water	1.33
Ethyl alcohol	1.36
Quartz	1.46
Vegetable oil	1.47
Olive oil	1.48
Acrylic	1.49
Table salt	1.51
Glass	1.52
Sapphire	1.77
Zircon	1.92
Cubic zirconia	2.16
Diamond	2.42
Gallium phosphide	3.50

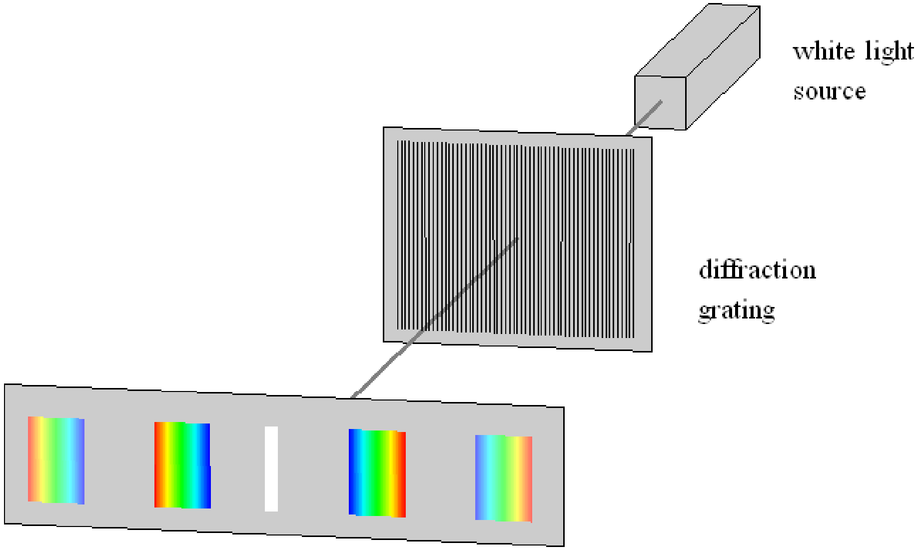
decomposition of white light

White light can be decomposed by a prims or diffraction grating.

- A prism is a block made of glass or another transparent material having a triangular base.
- A diffraction grating consists of fine parallel striated slits, equally spaced.

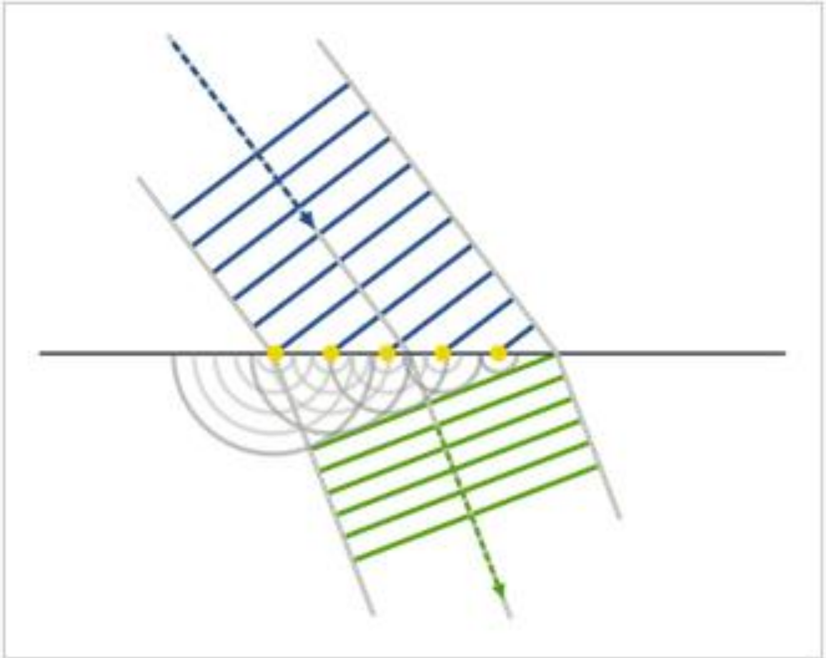




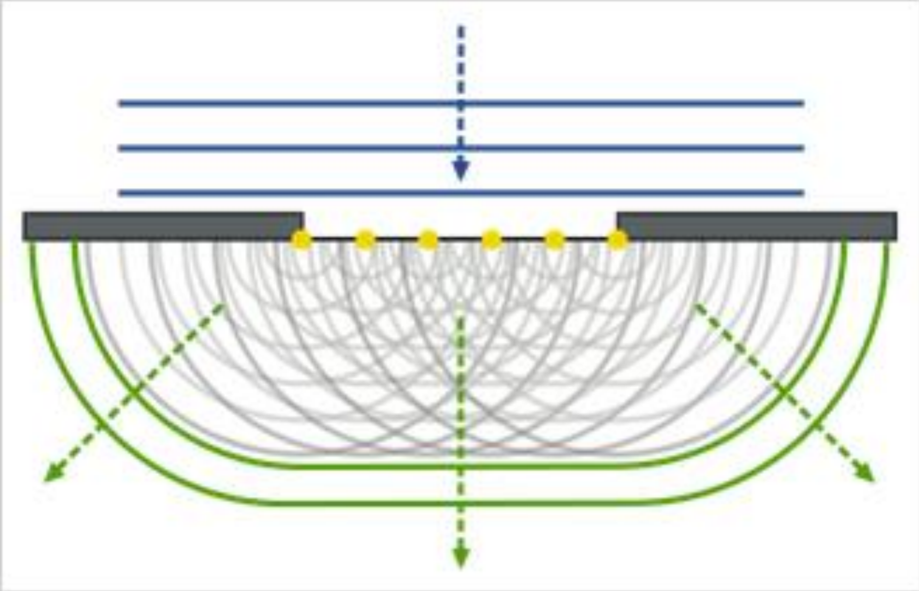


Huygens–Fresnel principle

The Huygens-Fresnel principle as modified by Fresnel states that every unobstructed point on a wavefront acts, at a given instant, as an individual source of outgoing secondary spherical waves.
The resulting net light amplitude at any position in the scattered light field is the vector sum of the amplitudes of all the individual waves.



Wave refraction in the manner of Huygens

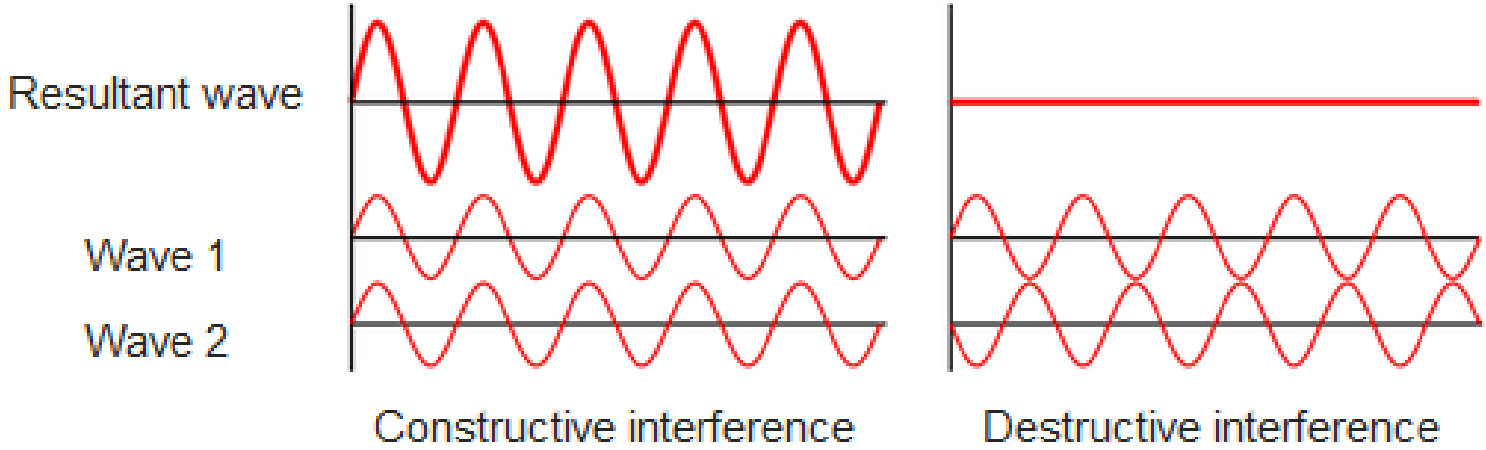


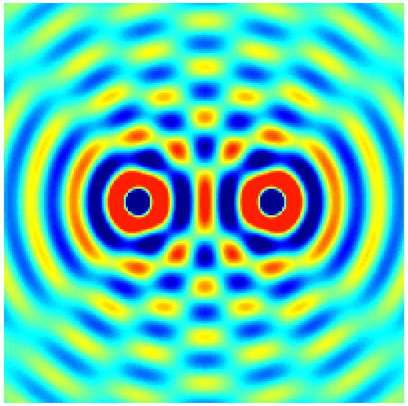
Wave diffraction in the manner of Huygens and Fresnel

Huygens–Fresnel principle

interference

Interference is a phenomenon in which two waves superpose
to form a resultant wave of greater, lower, or the same amplitude.
Interference usually refers to the interaction of waves that are correlated
with each other, either because they come from the same source
or because they have the same or nearly the same frequency.



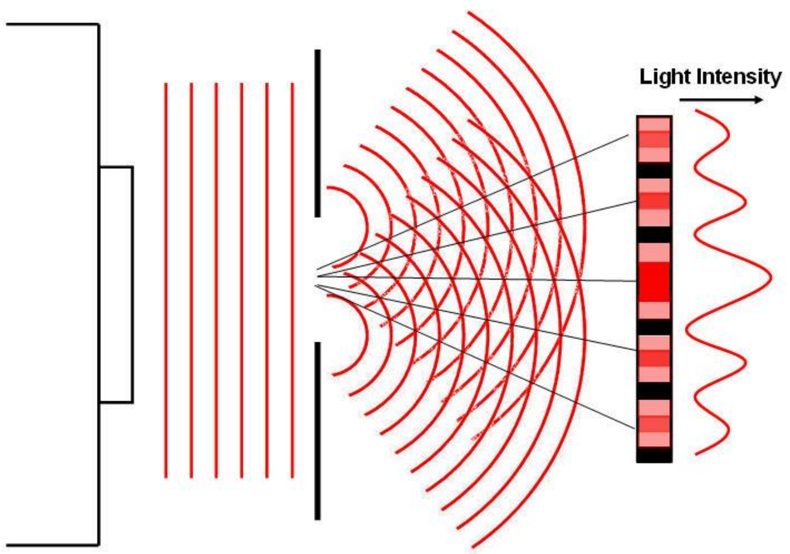


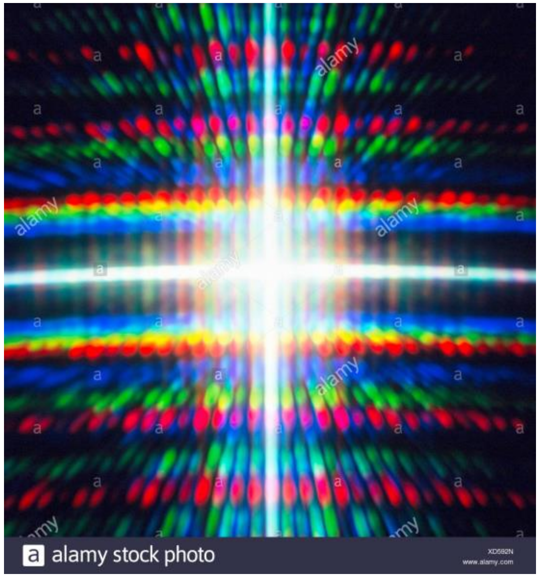
https://www.google.sk/search?q=interference&biw=1525&bih=734&source=lnms&tbm=isch&sa=X&ved=0ahUKEwib4qeE-4_MAhXHC5oKHTc_D5oQ_AUIBigB&dpr=0.9#imgrc=QNRJ5EAyC-1D7M%3A

light - diffraction

Diffraction refers to various phenomena which occur when a wave encounters an obstacle or a slit. It is defined as the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle.
In classical physics, the diffraction phenomenon is described as the interference of waves according to the Huygens–Fresnel principle.

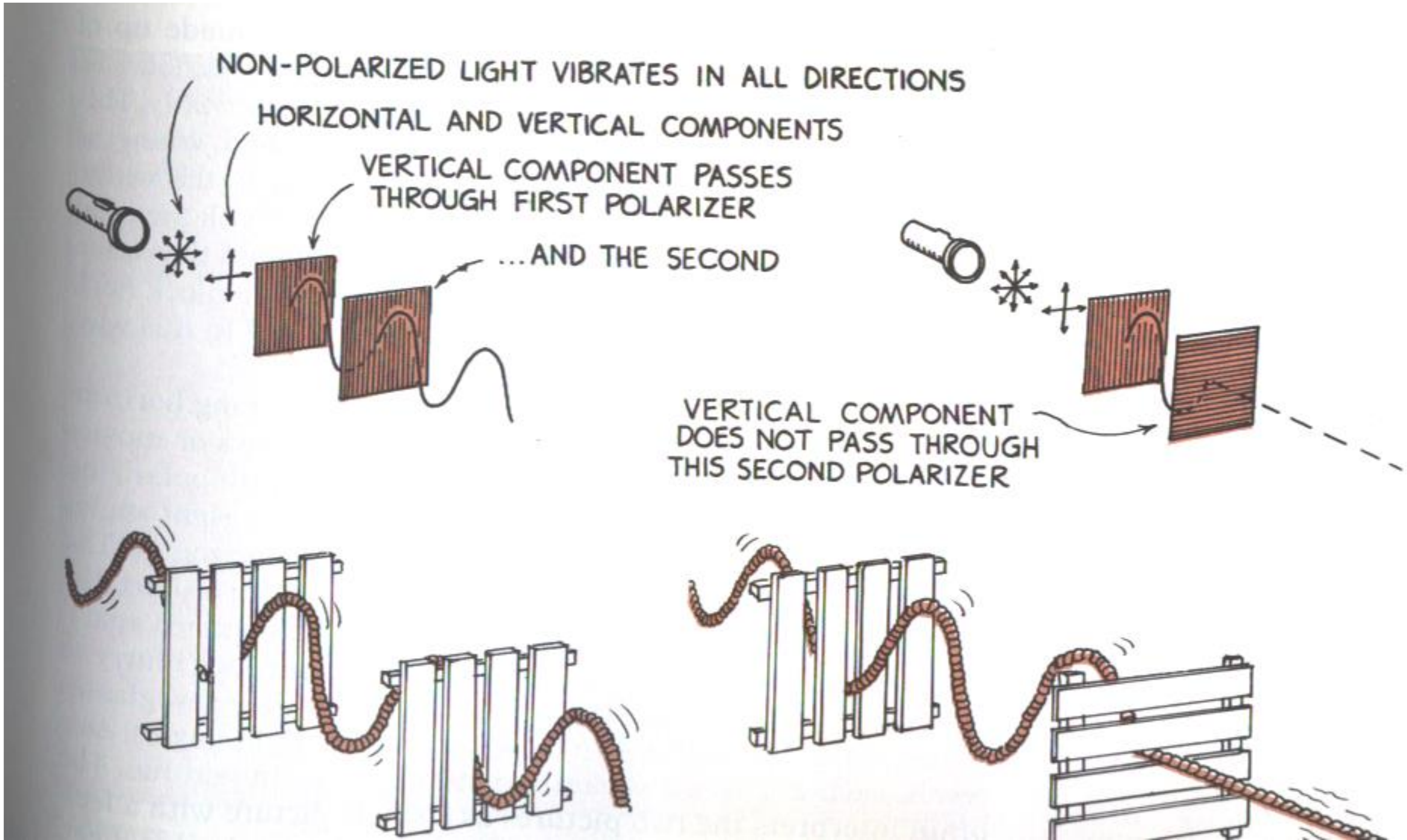
Single Slit Diffraction showing lines of maximum intensity





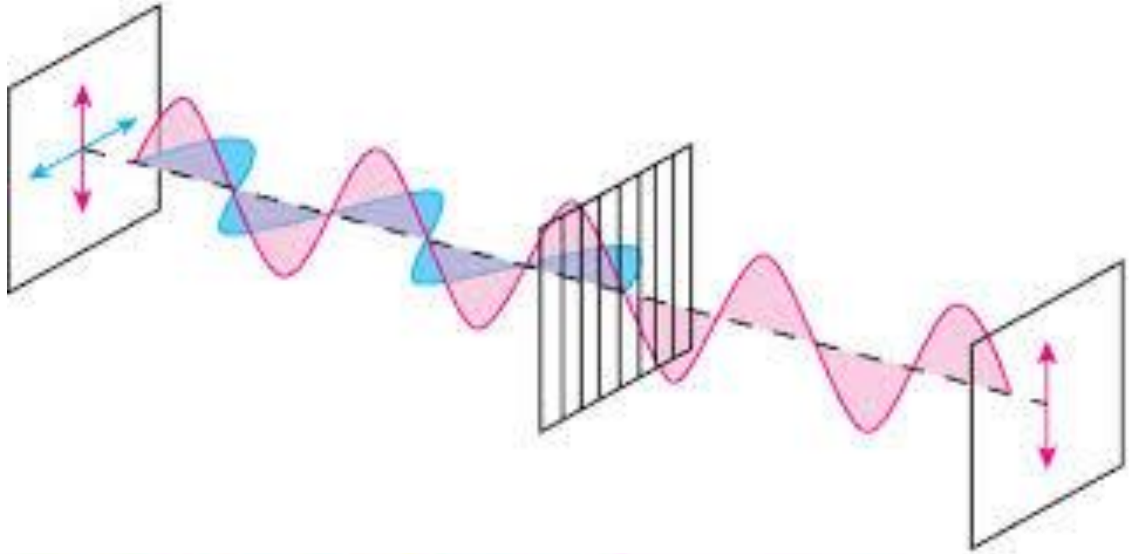
polarisation

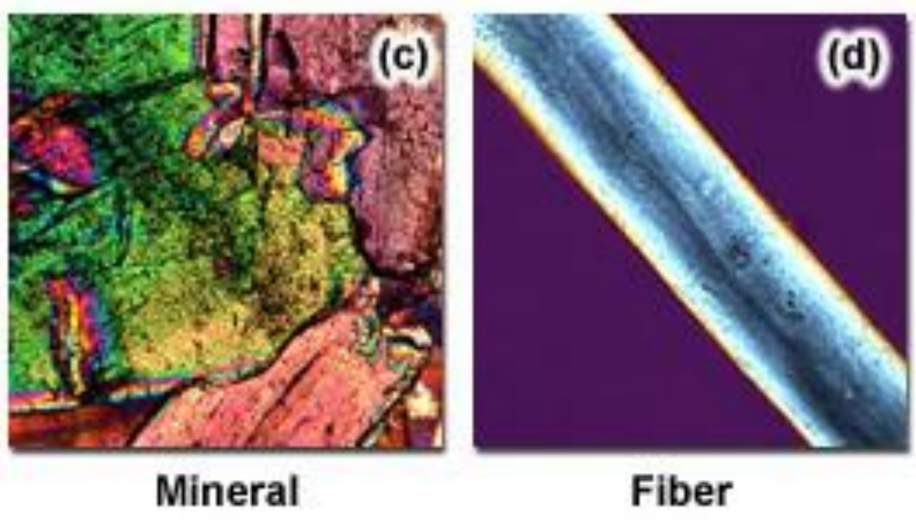
- light vibrates (oscillates) in all directions,
- a polarizing filter acts like a picket fence. It only lets certain direction vibrations pass through it,
- therefore, if you pass light through two of them you can completely block the light from passing through.



polarisation

This is used in science (so called polarising microscope) and in sun-glasses and 3-D movies.

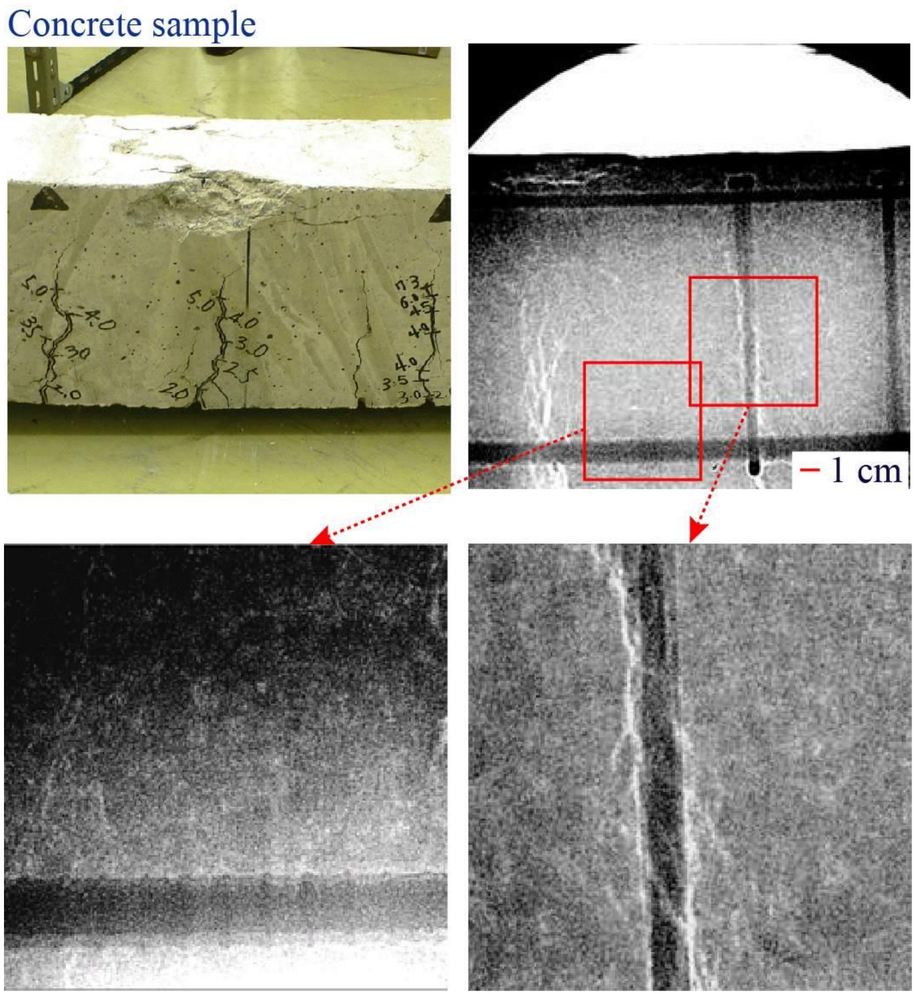




Inclined Monocular Polarising Microscope

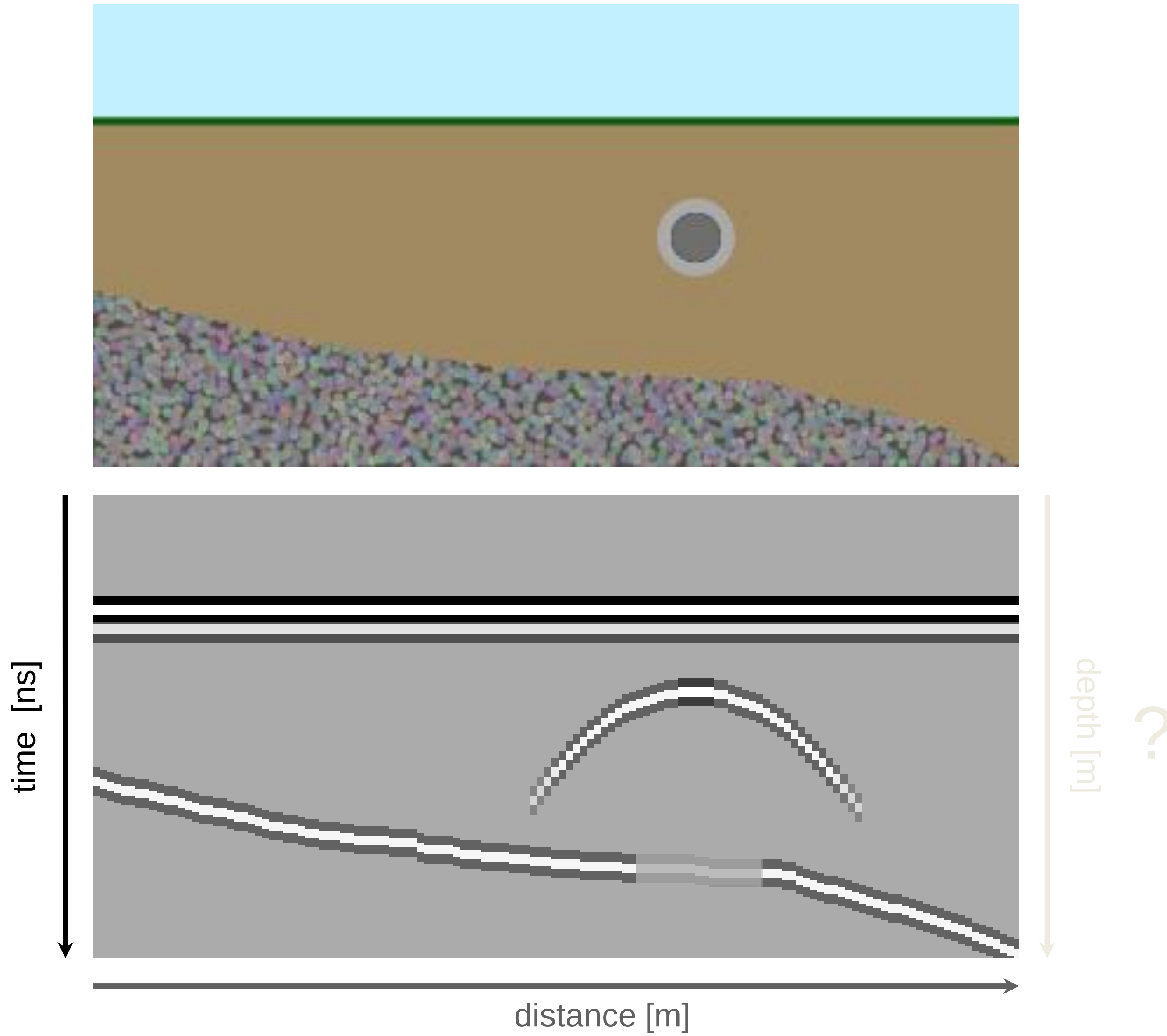
use of EM waves

in communication, science, medicine, engineering …



X-rays in materials inspection

use of EM waves



GPR= Ground Penetrating Radar