Mathematics for Biochemistry

LECTURE 1

Basic terms, Symbolism, Sets, Intervals

Basic info:

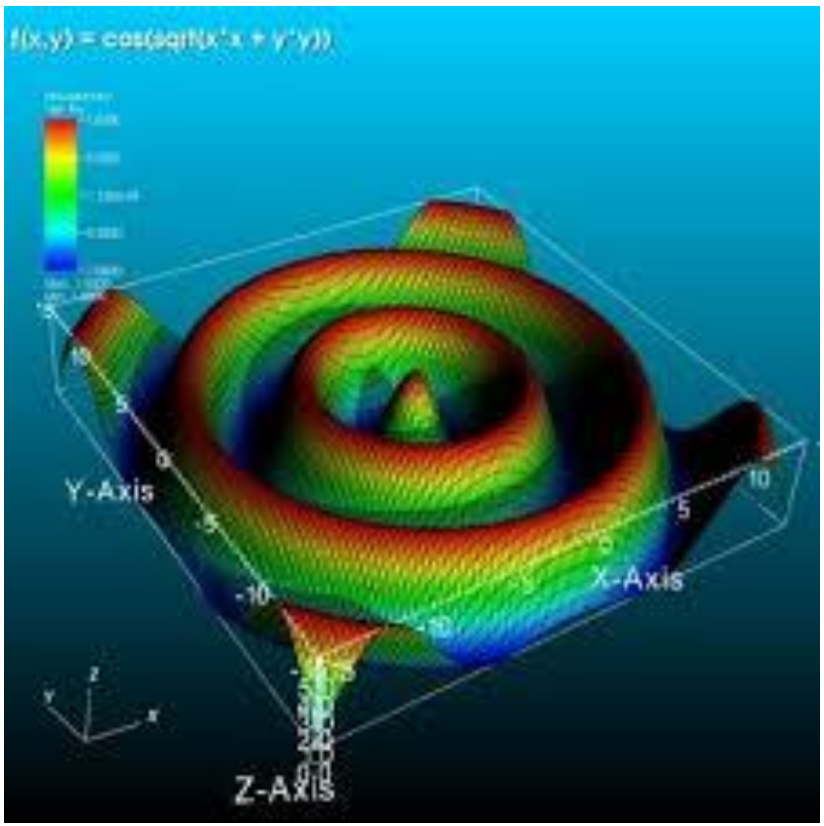
1. Organisation of the term (semester) and evaluation of the subject, all lectures on the website www.kaeg.sk, with each lecture also a small vocabulary will be given (in the end).
2. Evaluation of the subject – 100% final examination (few definitions and solution of exercises)

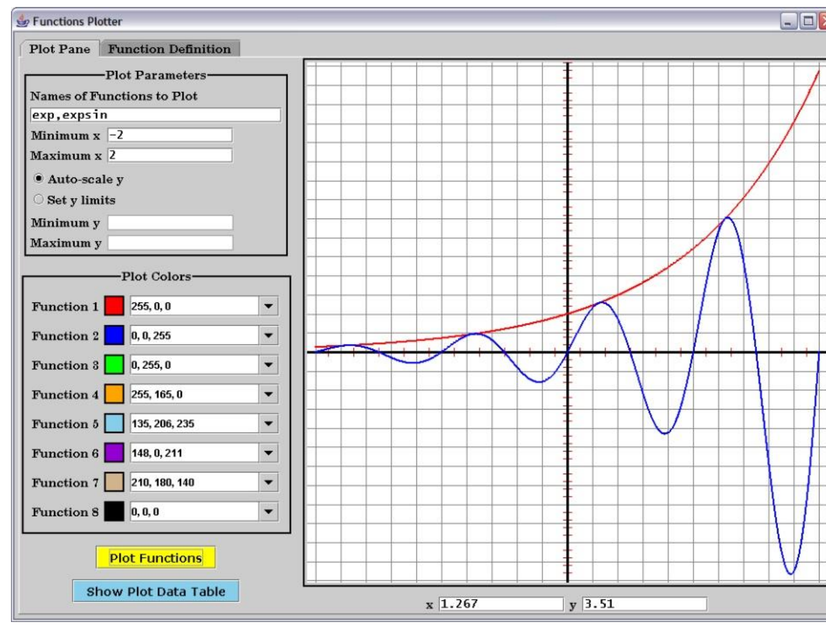
Content:

- Basic terms
- Symbolism
- Number systems
- Sets
- Intervals

Topic: Basic terms

Mathematics is a science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the objects.





Topic: Basic terms

Basic objects in mathematics:

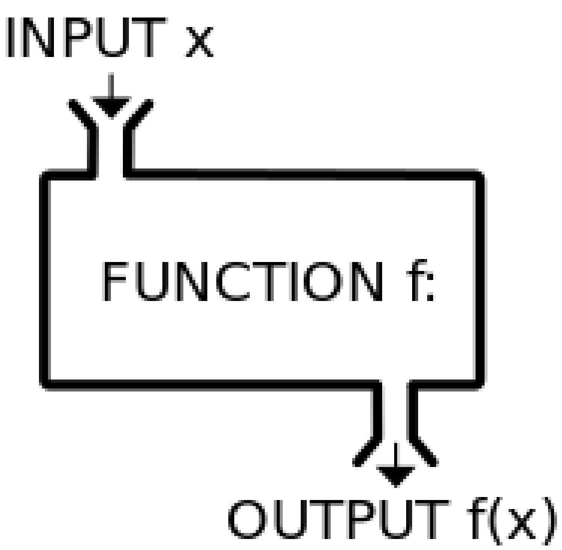
- numbers, variables, functions, functionals.

A **number** is a mathematical object used to count, measure and label. Numbers can be classified into sets, called number systems, such as the natural numbers and the real numbers (more details will come in a moment).

$$1, 2, 3, 4, \cdots$$

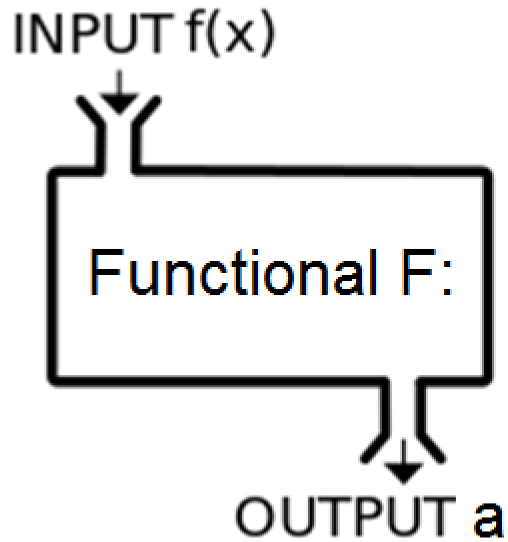
A **variable** is an alphabetic character representing a number (e.g. x, a).

A **function** is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

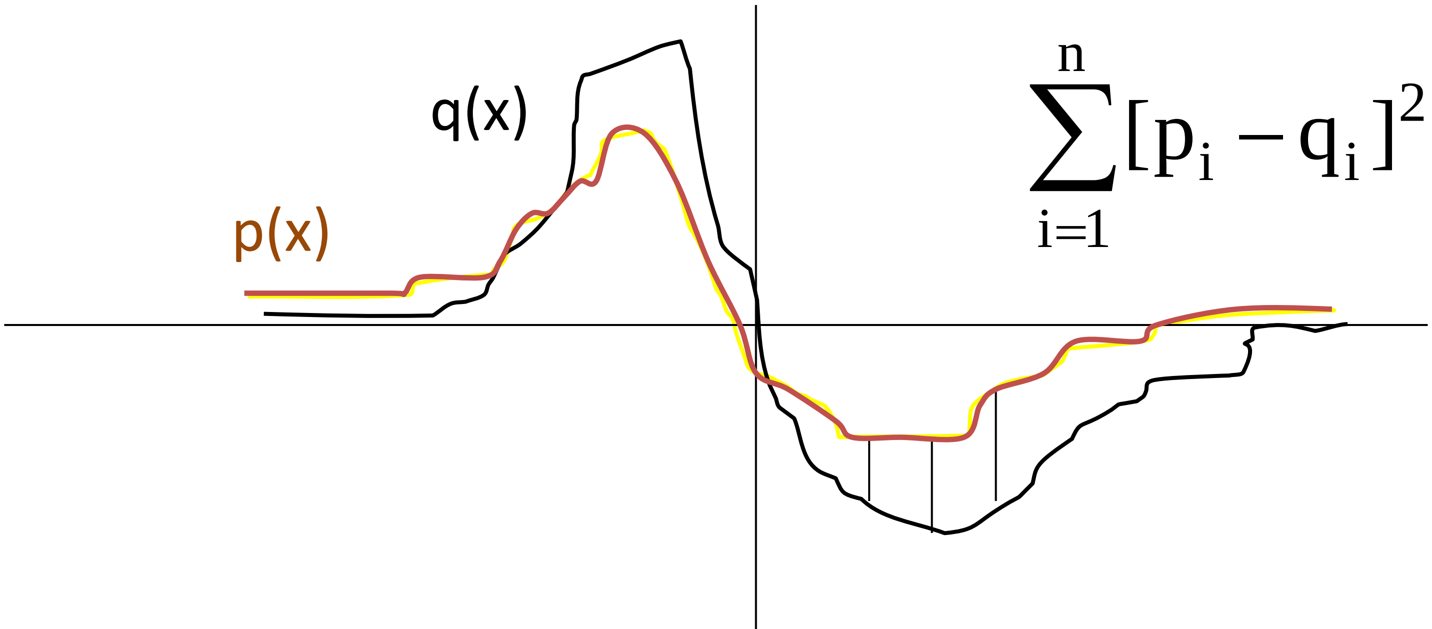


Topic: Basic terms

A **functional** is a mathematical object (operator), which has in the input a function (even more functions) and in the output a number (variable).



Example: so called Least Squares (LSQ) functional



$$\sum_{i=1}^{n}[p_i - q_i]^2 = a$$

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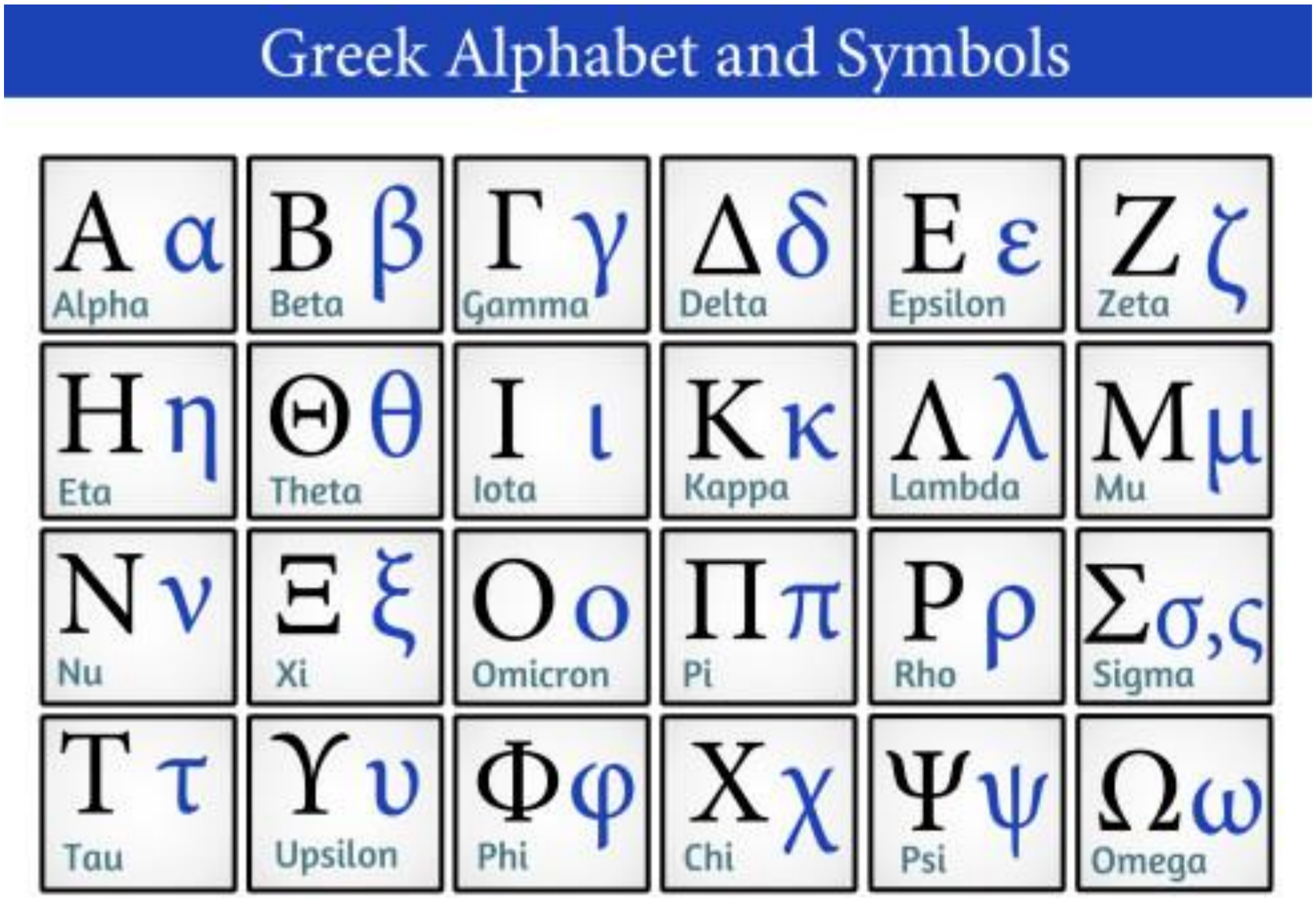
Topic: Symbolism

- <u>Comment:</u> Different symbolism in math
- Mathematics use very **strict rules** in the formalism.
- Even a difference in a used style of writing (e.g. normal or **bold** or *italics*) can express important differences in the used meaning (e.g. between matrices and usual variables or in physics we distinguish between scalars and vectors).
- Of course <u>that there exists differences</u> in the variety of text-books, but some rules are valid in general.

Topic: Symbolism

Greek alphabet is must!



Topic: Symbolism

$\forall$- for each, for all

$\exists$ - exist

! - exactly one, one and only one

$\wedge$ - and

$\vee$ - or

$\Longrightarrow$- implies

$\Leftrightarrow$ - is equivalent to

… and plenty of others symbol which will be discussed later throughout the lessons

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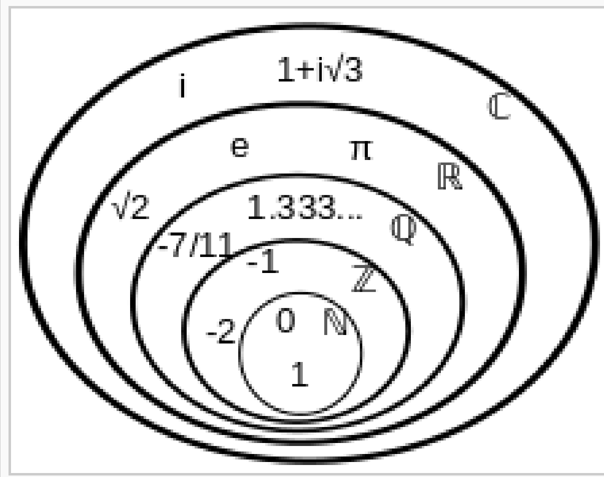
Basic objects in mathematics: **numbers, sets**

Different types of numbers have many different uses.
Numbers can be classified into sets, called **number systems**.



Subsets of the complex numbers.

Main number systems

$\mathbb{N}$	**Natural**	0, 1, 2, 3, 4, ... **or** 1, 2, 3, 4, ...
$\mathbb{Z}$	**Integer**	..., −5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ...
$\mathbb{Q}$	**Rational**	$\frac{a}{b}$ where *a* and *b* are integers and *b* is not 0
$\mathbb{R}$	**Real**	The limit of a convergent sequence of rational numbers
$\mathbb{C}$	**Complex**	*a* + *bi* where *a* and *b* are real numbers and *i* is the square root of −1

$$1.5 = \frac{3}{2}$$ Ratio

Rational

$$\pi = 3.14159... = \frac{?}{?}$$ (No Ratio)

Irrational

Basic objects in mathematics: **numbers, sets**

Definition: A **set** is a collection of things (members, elements, objects)

{1, 2, 3} - a set whose objects are positive numbers 1, 2, 3.

If a set consists of a finite (reasonable) number of objects, we may denote the set by listing them. If a set consists of an infinitely many objects, we denote the set:

a) by naming a property common to all objects of the set
e.g. {x | x is a positive number} (the bar "|" is read "such that")

b) in some conventional way (**N**, **Z**, **Q**, **R**, $\mathbf{R}^+$...).

Basic objects in mathematics: **numbers, sets**

If p is an object of a set **A**, we write: p $\in$ **A**
(“p is an element of **A**”, “p belongs to **A**”).

If p is not an element of the set **A**, we write: p $\notin$ **A** .
(“p is not an element of **A**”, “p does not belong to **A**”).

$$\pi \in \mathbb{R} \text{ but } \pi \notin \mathbb{Z}$$

The union of sets **A** and **B** is the set:

$$\mathbf{A} \cup \mathbf{B} = \left\{ x \middle| x \in \mathbf{A} \vee x \in \mathbf{B} \right\}$$

$$\mathbf{D} = \bigcup_{\mathrm{i}=1}^{\mathrm{n}} \mathbf{A}_{\mathrm{i}}$$

The intersection of sets **A** and **B** is the set:

$$\mathbf{A} \cap \mathbf{B} = \left\{ x \middle| x \in \mathbf{A} \wedge x \in \mathbf{B} \right\}$$

$$\mathbf{E} = \bigcap_{\mathrm{i}=1}^{\mathrm{n}} \mathbf{A}_{\mathrm{i}}$$

The difference of sets **A** and **B** is the set:

$$\mathbf{A} \setminus \mathbf{B} = \left\{ x \middle| x \in \mathbf{A} \wedge x \notin \mathbf{B} \right\}$$

The subset:

$$\mathbf{A} \subseteq \mathbf{B} \rightarrow \forall x \in \mathbf{A} : x \in \mathbf{B}$$

The complement of set **A**:

$$\mathbf{A}^{C} = \left\{ x \in \mathbf{U} : x \notin \mathbf{A} \right\}$$

The empty set is the set with no element, it is denoted $\varnothing$ or { }.

The power set of set **A** is the set of all subsets of **A** (denoted $\mathbf{P}_{\mathbf{A}}$)

Basic objects in mathematics: **numbers, sets**

<u>Example:</u>

Let
A = {1, 2, 3, 4},
B = {1, 3, 5, 7, 9}.

Compute the following sets:

a) $\mathbf{A \cup B}$

b) $\mathbf{A \cap B}$

c) $\mathbf{A \setminus B}$

d) **find the power set of set A**

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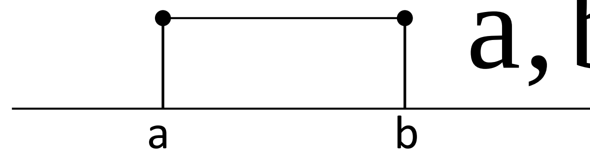
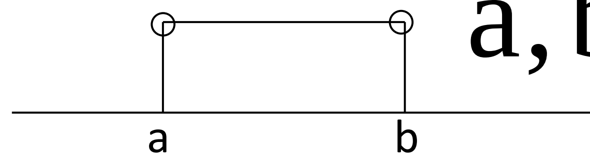
Topic: Intervals

closed interval: $\left\langle a, b \right\rangle$



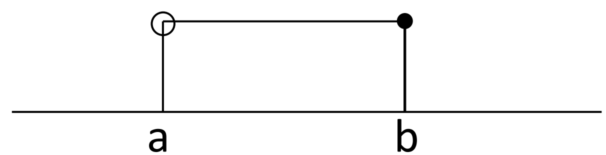
$a, b \in I$

open interval: $\left(a, b \right)$



$a, b \notin I$

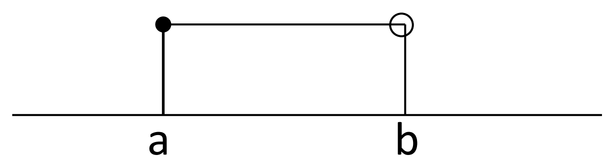
left-open interval: $\left(a, b \right\rangle$



$a \notin I$

$b \in I$

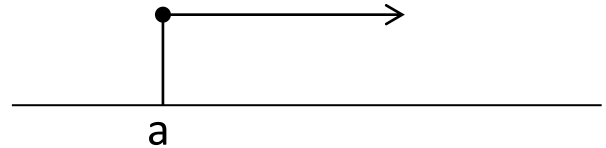
right-open interval: $\left\langle a, b \right)$



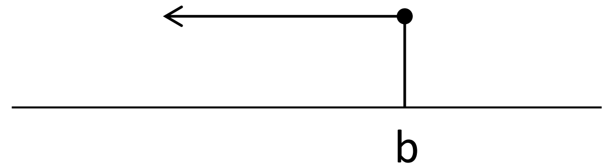
$a \in I$

$b \notin I$

if $a \vee b \to \infty$ $\quad \left\langle a, \infty \right)$

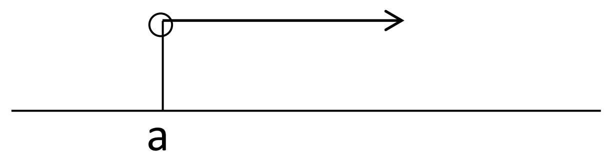


$$\left(-\infty, b \right\rangle$$



or

$$\left(a, \infty \right)$$



$$\left(-\infty, b \right)$$