Mathematics for Biochemistry

LECTURE 4

Graphs

Content

- Linear functions
- Quadratic functions
- Rational functions
- Exponential and logarithmic functions
- Trigonometric and cyclometric functions

Linear functions

Example 1.

$$y = -2x + 3 \qquad \rightarrow \qquad D(f) = \mathbb{R}$$

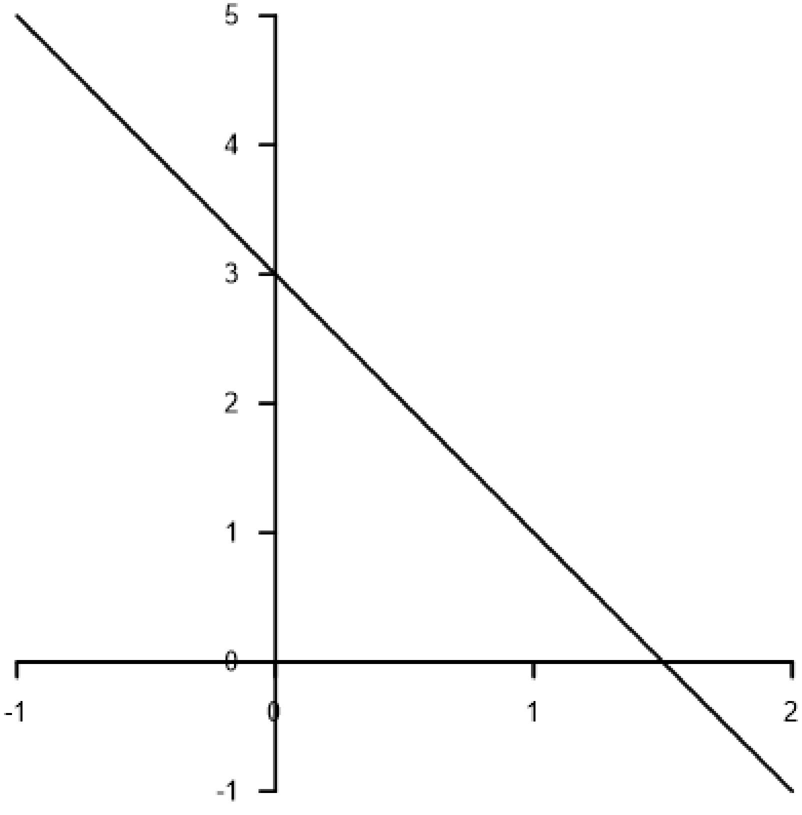
The linear term (slope) is negative number (-2) results in decreasing line, shifted to the number 3 (absolute term).

root $x_0 = \frac{3}{2}$

max, min, sup, inf $\rightarrow \nexists$

decreasing on D(f)

range $H(f) = \mathbb{R}$



Quadratic functions

Example 2.

$$y = -x^2 + x - 3 \qquad \rightarrow \qquad D(f) = \mathbb{R}$$

The quadratic term is negative number (-1) results in concave shape of parabola

roots $x_1 = 1, x_2 = 3$

crossing on the vertical axis $y_0 = -3$

min, inf $\rightarrow \nexists$

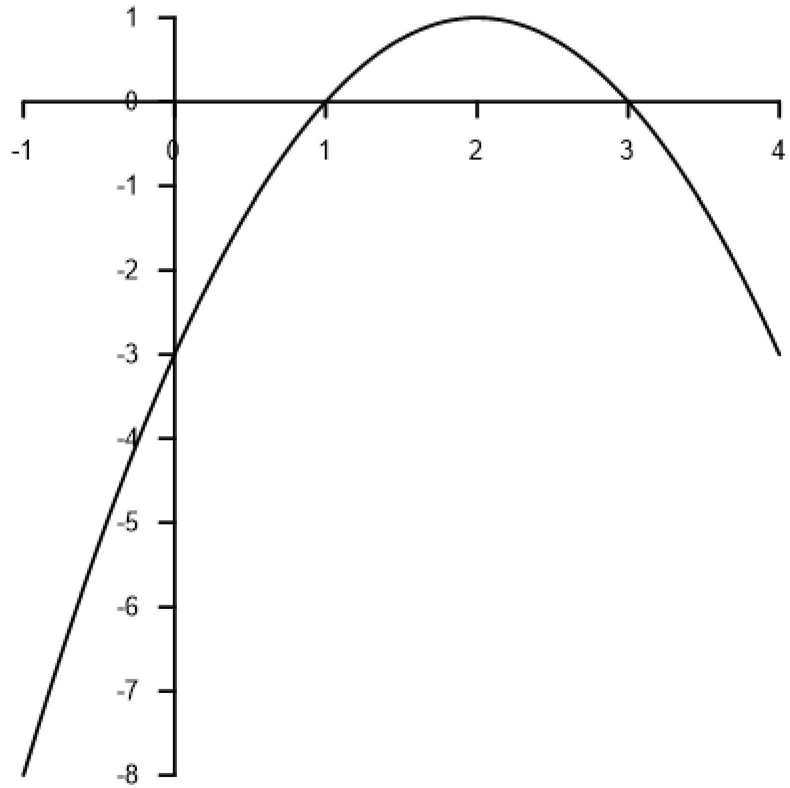
max $x_m = 2$

sup $y_s = 1$

increasing on $(-\infty, 2)$

decreasing on $(2, \infty)$

range $H(f) = \mathbb{R}$



Cubic functions

Example 3.

$$y = 2x^3 - 6 \qquad \rightarrow \qquad D(f) = \mathbb{R}$$

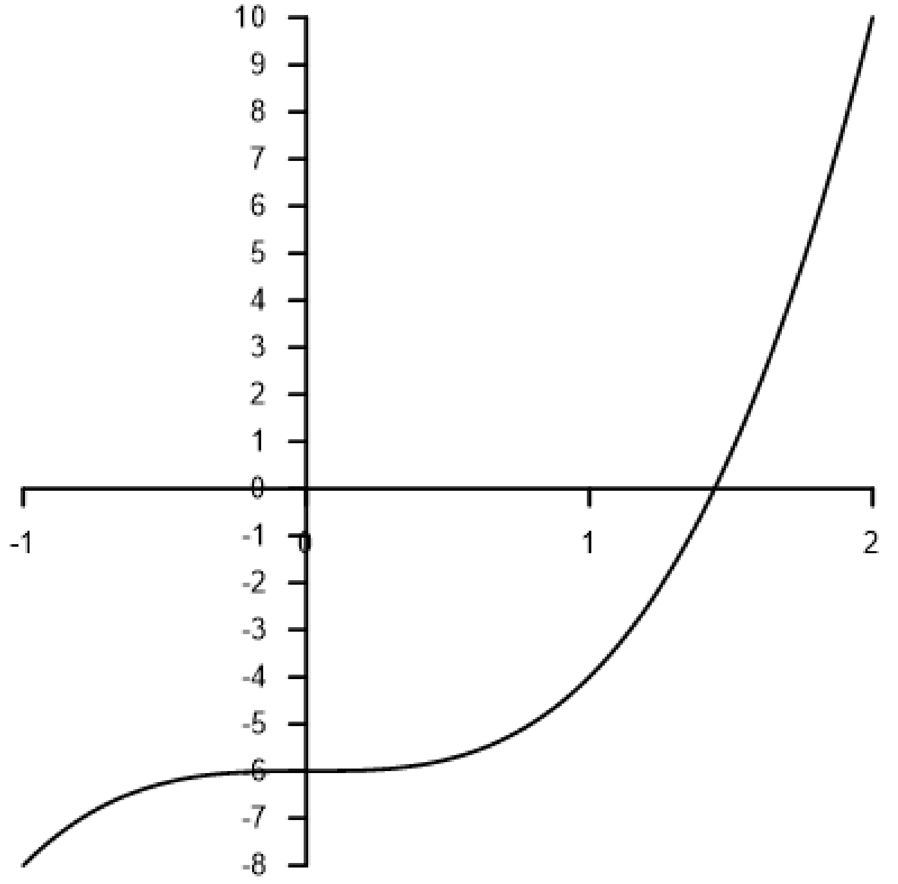
root $x_1 = \sqrt[3]{3}$

crossing on the vertical axis $y_0 = -6$

max, min, sup, inf $\rightarrow \nexists$

increasing on D(f)

range $H(f) = \mathbb{R}$



Rational functions

Example 4.

$$y = \frac{1}{x+2} - 3 \quad \rightarrow \quad D(f) = \mathbb{R} - \{-2\}$$

root $x_1 = -\frac{5}{3}$

crossing on the vertical axis $y_0 = -\frac{5}{3}$

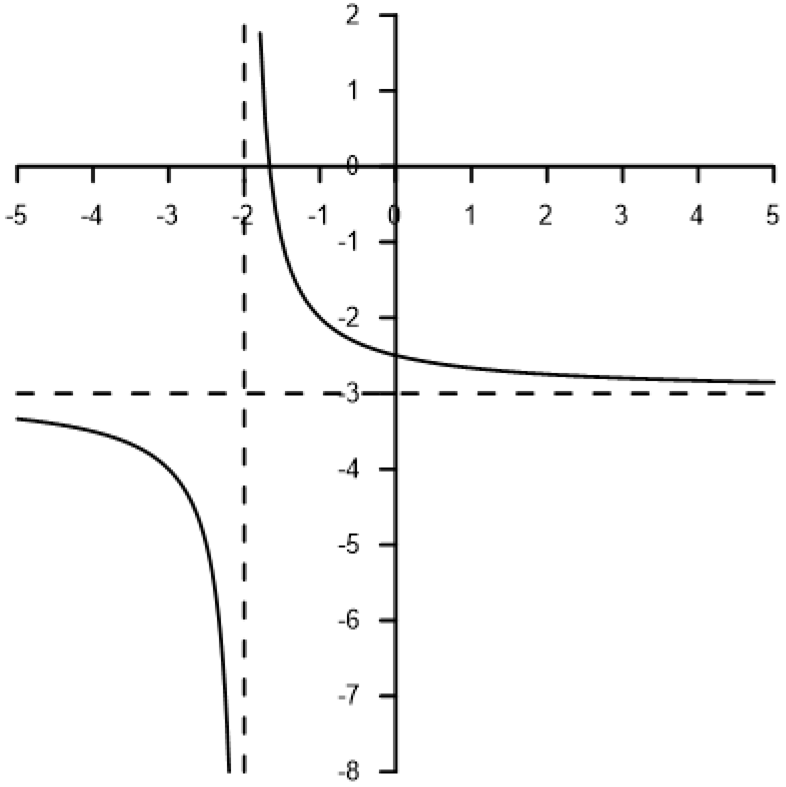
max, min, sup, inf $\rightarrow \nexists$

decreasing on D(f)

asymptote without slope $y = -2$

asymptote with slope $x = -3$

range $H(f) = \mathbb{R} - \{-3\}$



Exponential functions

Example 5.

$$y = 2\left(\frac{1}{2}\right)^{x} - 1 \qquad \rightarrow \qquad D(f) = \mathbb{R}$$

root $x_1 = 1$

crossing on the vertical axis $y_0 = 1$

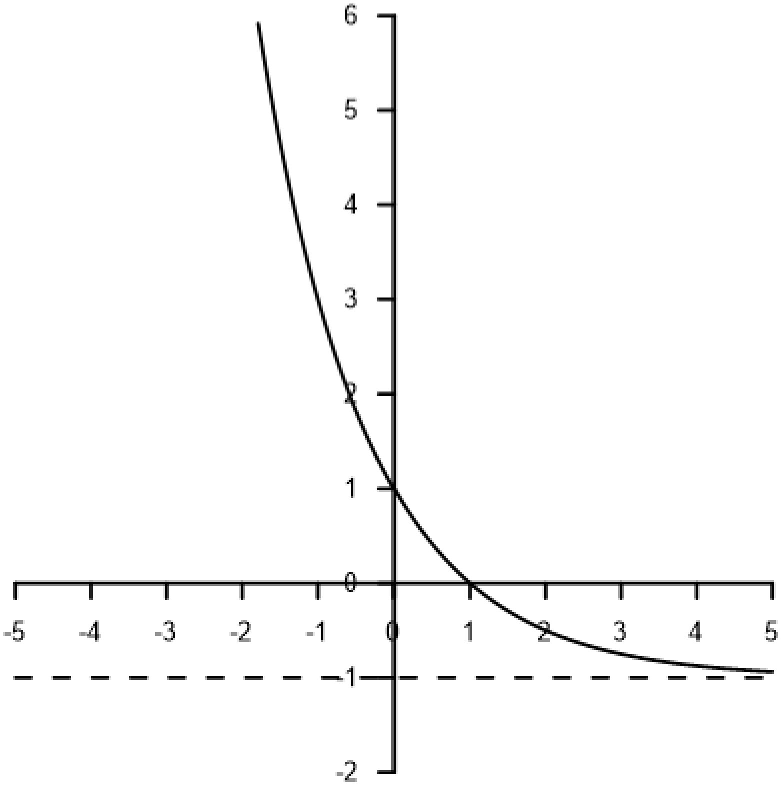
max, min, sup $\rightarrow \nexists$

inf $y_i = -1$

decreasing on D(f)

asymptote with slope $x = -1$

range $H(f) = (-1, \infty)$



Logarithmic functions

Example 6.

$$y = \frac{1}{2}\ln(2x-5)+3 \quad \rightarrow \quad D(f) = \left(\frac{5}{2}, \infty\right)$$

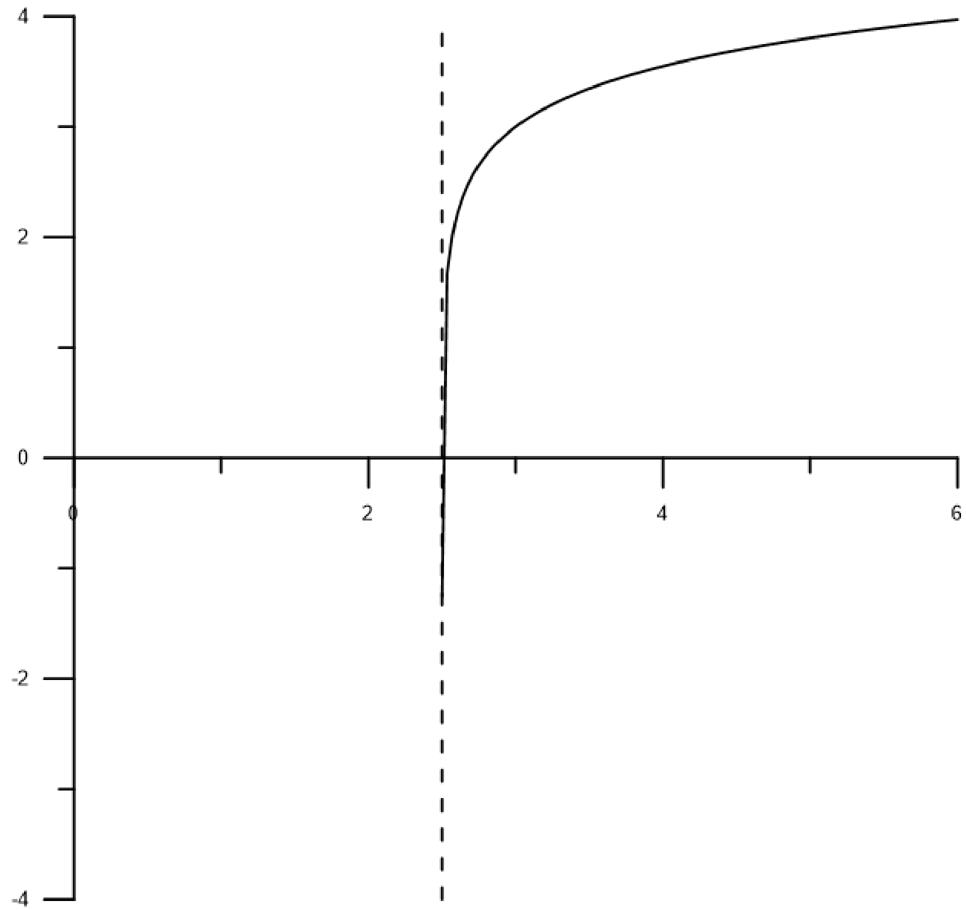
root $x_1 = \dfrac{e^{-6}+5}{2}$

max, min, sup, inf $\rightarrow \nexists$

increasing on D(f)

asymptote without slope $x = \dfrac{5}{2}$

range $H(f) = \mathbb{R}$



Logarithmic functions

Example 7.

$$y = 2\log_{\frac{1}{2}}(x+1)+1 \qquad \rightarrow \qquad D(f) = (-1,\infty)$$

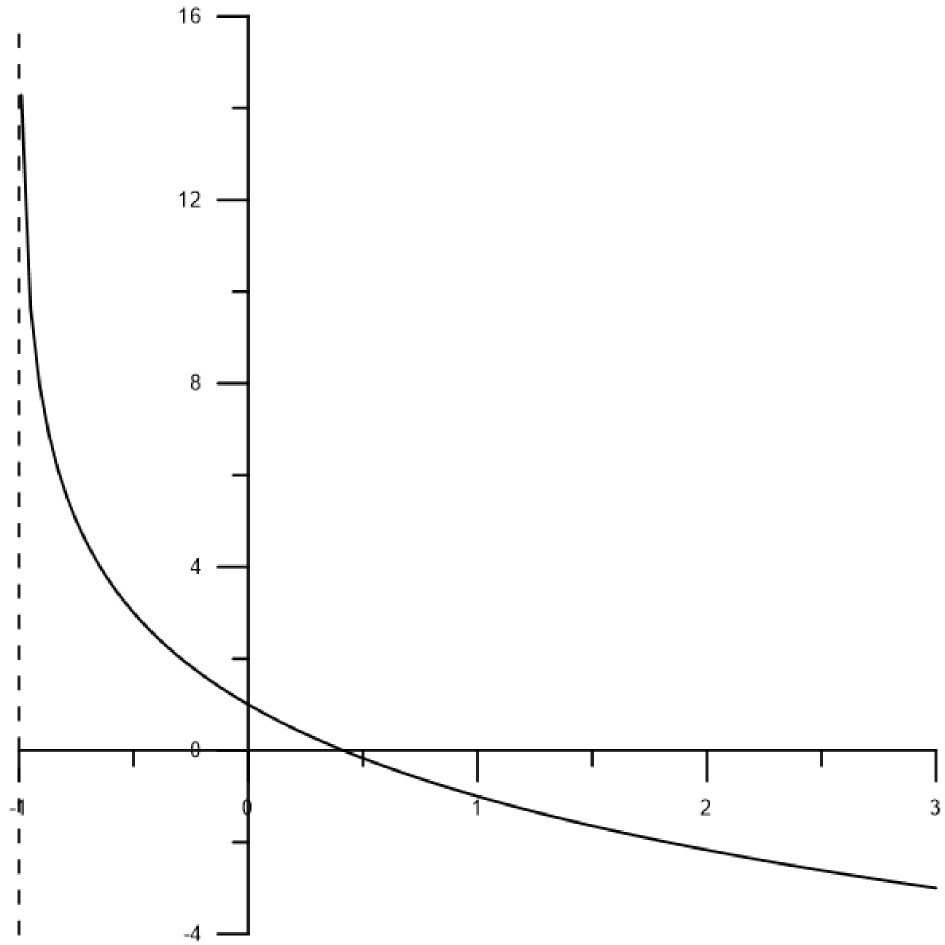
root $x_1 = \sqrt{2} - 1$

max, min, sup, inf $\rightarrow \nexists$

decreasing on D(f)

asymptote without slope $x = -1$

range $H(f) = \mathbb{R}$



Trigonometric functions

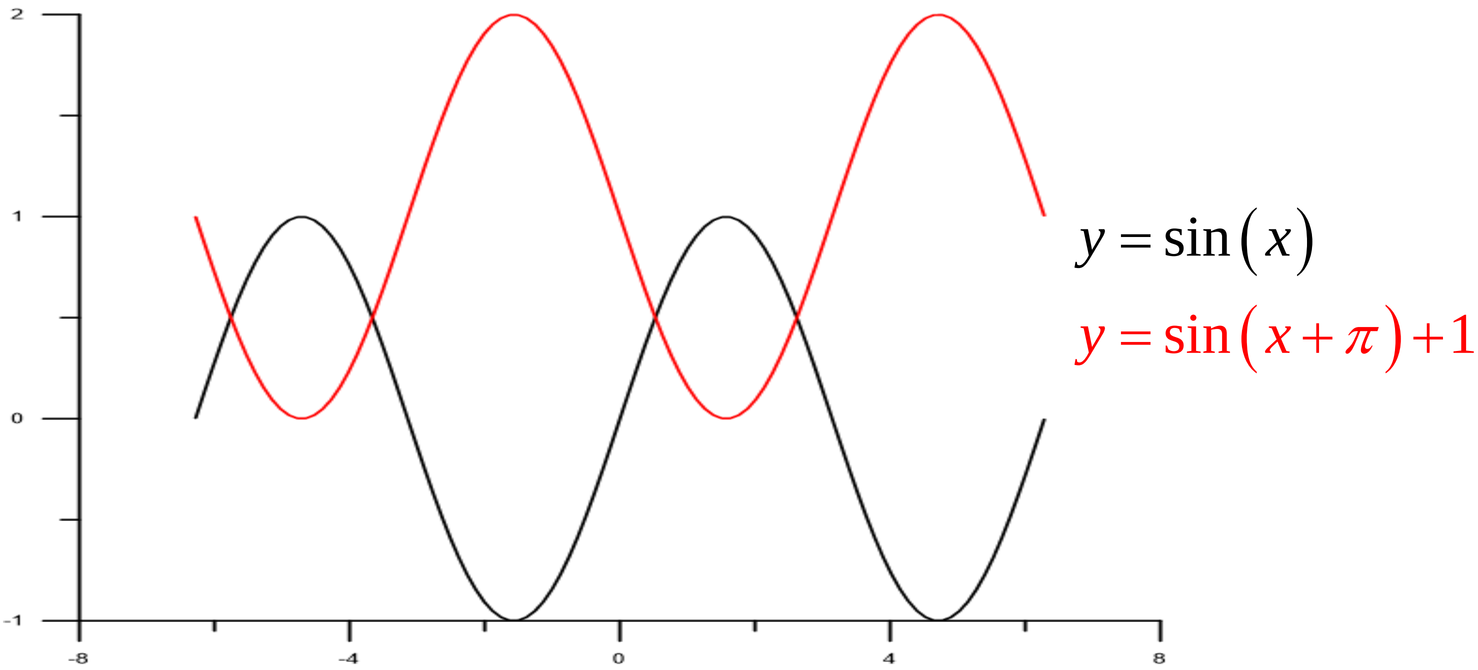
Example 8.

$$y=\sin\left(x+\pi\right)+1 \qquad \rightarrow \qquad D\left(f\right)=\mathbb{R}$$

period $T=2\pi$ roots $x=-\frac{3}{2}\pi+2k\pi;\quad k\in\mathbb{Z}$

max $x_{\max}=-\frac{\pi}{2}+2k\pi;\quad k\in\mathbb{Z}$ min $x_{\min}=\frac{\pi}{2}+2k\pi;\quad k\in\mathbb{Z}$ sup $y_s=2$ inf $y_i=0$

range $H\left(f\right)=\left\langle 0,2\right\rangle$



Trigonometric functions

Example 9.

$$y = \tan\left(x + \frac{3\pi}{2}\right) \qquad \rightarrow \qquad D(f) = \mathbb{R} - \{k\pi\}, k \in \mathbb{Z}$$

period $T = \pi$

range $H(f) = \mathbb{R}$

roots $x = -\frac{3}{2}\pi + k\pi; \quad k \in \mathbb{Z}$

max, min, sup, inf $\rightarrow \nexists$