Lecture 9: special and general relativity theory

Content:

- introduction, classical relativity
- two postulates in Einstein's relativity theory
- time dilatation, length contraction
- spacetime
- Lorentz transformations
- gravity is not a force
- black holes, gravitational lenses

Relativity theories from Albert Einstein

Special Theory of Relativity
(is about constant speed and time)

and

General Relativity
(is about accelerated motion and gravity)

Special Theory of Relativity

Einstein's two postulates (in special relativity)

1) **The principle of relativity**: The laws of physics are the same in all coordinate systems.

2) **The constancy of the speed of light**: Observers in all coordinate systems measure the same value for the speed of light in a vacuum.

Important comment:
Special relativity applies to specific situation – when the objects are moved by a constant speed (without acceleration).

Newtonian (Classical) Relativity

Question:

If you would in an isolated box on a ship (assume there is a smooth sea) and you are given a clock, simple pendulum, measuring scale, different objects (as spheres, cylinders, …) and weights of different masses, could you find out that the ship is at rest or moving with a constant velocity in a straight line?

Newtonian (Classical) Relativity

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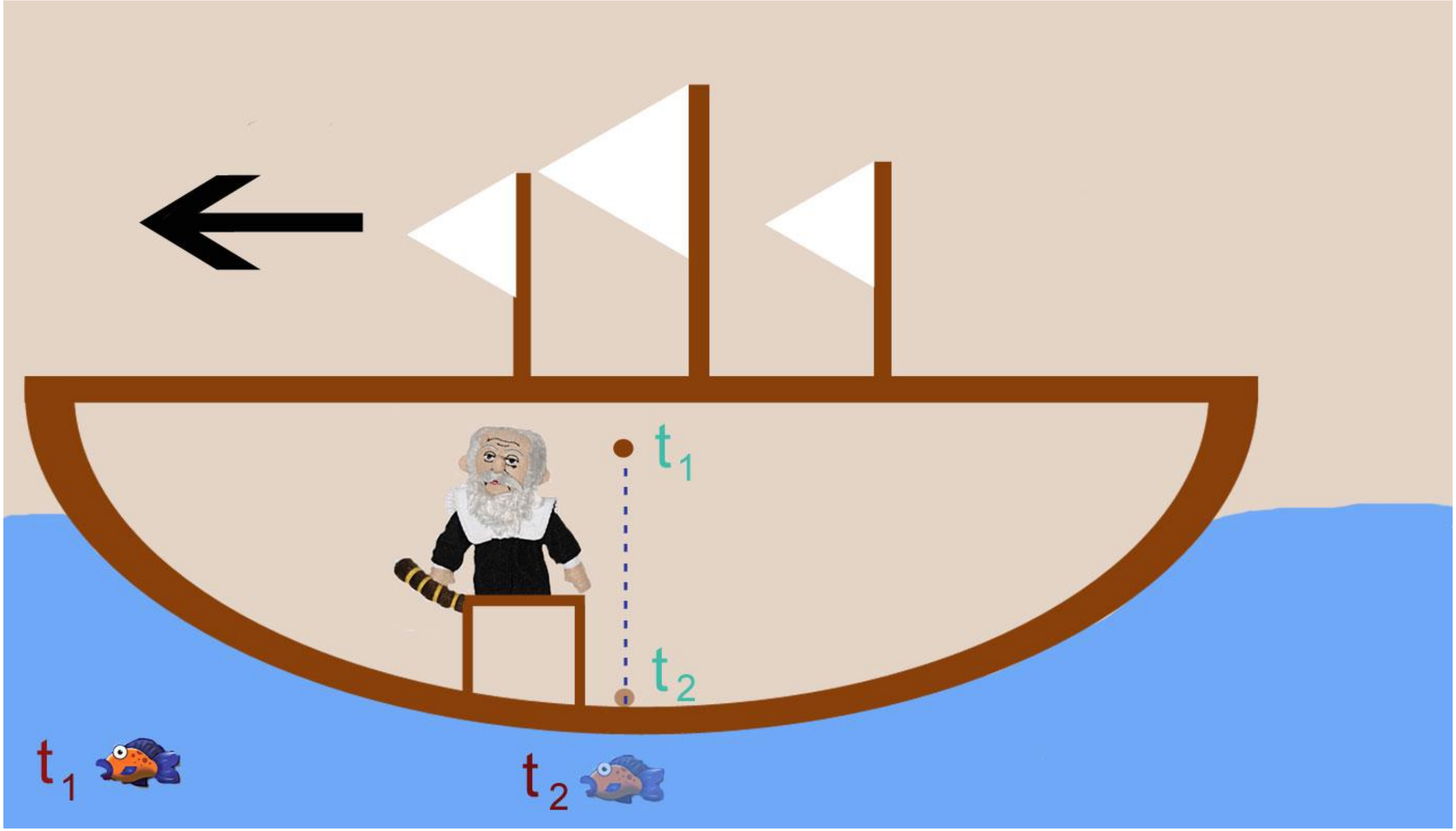
If you would in an isolated box on a ship (assume there is a smooth sea) and you are given a clock, simple pendulum, measuring scale, different objects (as spheres, cylinders, …) and weights of different masses, could you find out that the ship is at rest or moving with a constant velocity in a straight line?

Answer:

No, you could not. The most fundamental laws of physics have no (constant) velocity dependence, which means there is no physics experiment you could perform inside the box to tell whether the ship is at rest or moving in a straight line, without comparing the movement with the surroundings.

Newtonian (Classical) Relativity

this mental (thought) experiment came from Galileo…



Newtonian (Classical) Relativity

Principle of relativity

Any uniformly moving observer in an inertial frame cannot determine his "absolute" state of motion by a co-moving experimental arrangement.

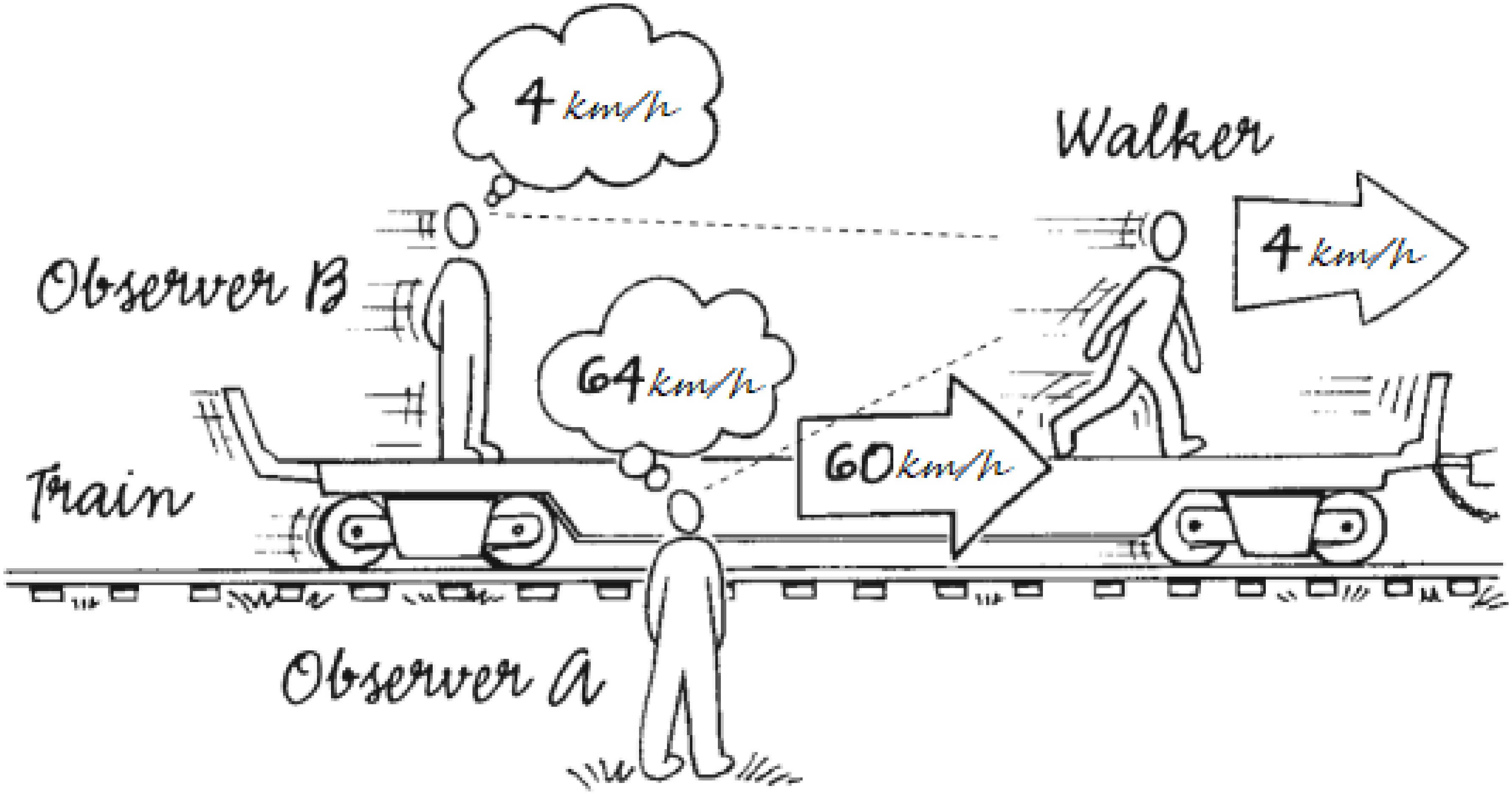
Comment 1:

In the Newtonian physics, movements (speeds) are relative – it all depends from the reference frames, where these movements are observed.

Comment 2:

The term “inertial frame” will be explained later on.

Speed of the second train is relative – to a person in the first train or to a person standing on a platform.



speed of the walker in the train is different for observer A (standing outside the train) and observer B (standing in the train).

Newtonian (Classical) Relativity

Assumption

- It is assumed that Newton’s laws of motion must be measured with respect to (relative to) some reference frame.

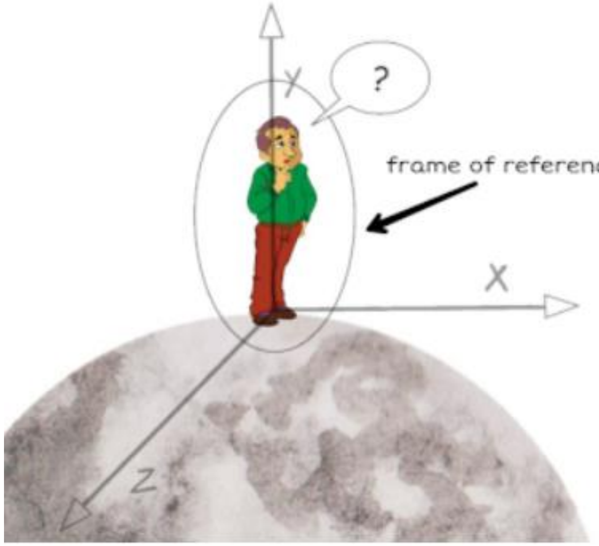
Comment:

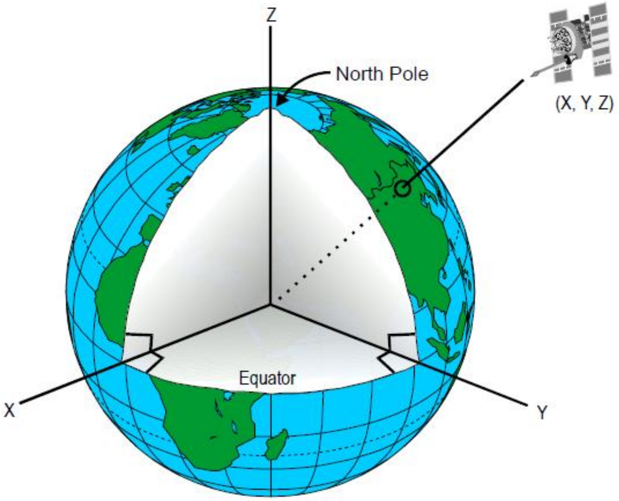
Reference frame (or frame of reference):

Is an abstract coordinate system whose origin, orientation, and scale are specified by a set of reference points.

There can exist several reference frames, “hosting” one situation.

example: local reference frame





example: geocentral reference frame

Newtonian (Classical) Relativity

There can exist several reference frames, “hosting” one situation.

Whether or not you are moving depends on your point-of-view.
(in the language of physics: depends on your reference frame)

In the Newtonian physics, movements (speeds) are relative – it depends from the reference frames, where these movements are observed.

Inertial Reference Frame

- a reference frame is called an **inertial frame**, when Newton laws are valid in that frame

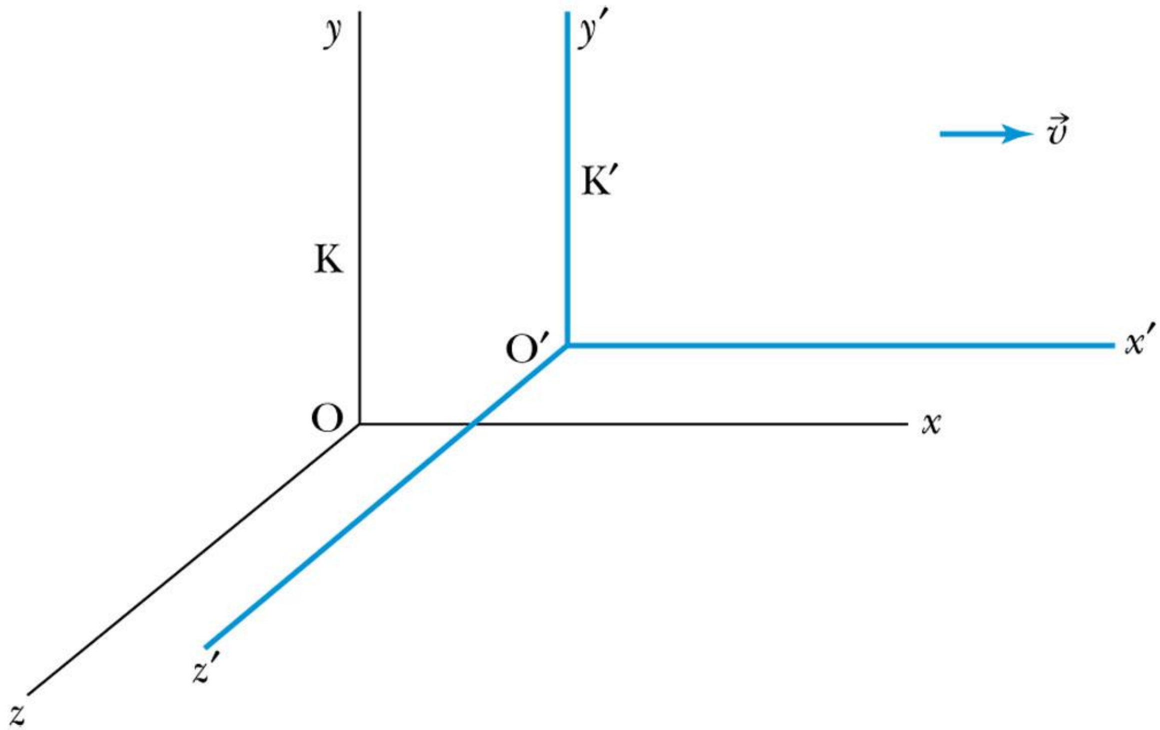
- such a frame is established when a body, not subjected to external forces, is observed to move in rectilinear motion (motion along a straight line) at **constant velocity**

Newtonian Principle of Relativity

- If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

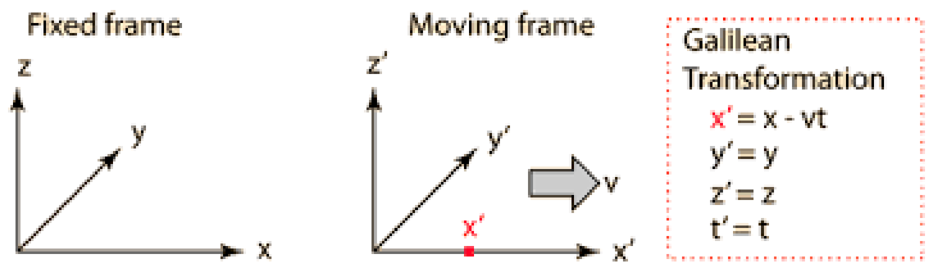
- This is referred to as the **Newtonian principle of relativity** or **Galilean invariance.**

Inertial Frames K and K'



- K is at rest and K' is moving with constant velocity $\vec{v}$
- Axes are parallel
- K and K' are *INERTIAL COORDINATE SYSTEMS*

Conditions of the Galilean Transformation



- Parallel axes
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

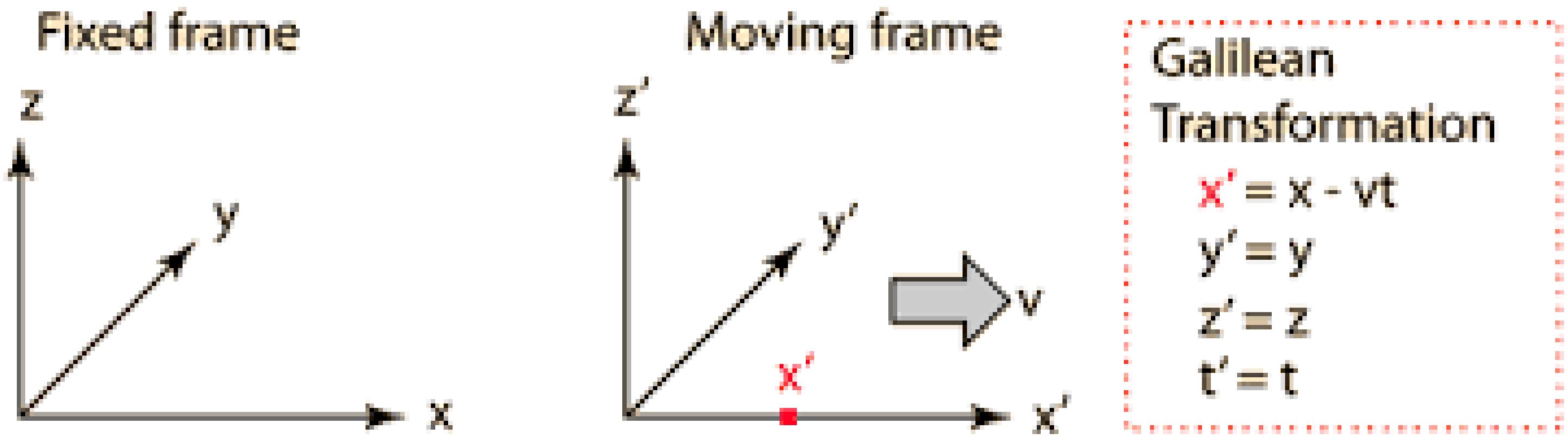
$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

(in Galileo's/Newton's physics)

Galilean Transformation



Important!

Galilean transformation is valid for Newtonian physics.

This transformation is applicable only when the bodies move at a speed much lower than that of the speeds of light.

The Galilean transformation relate the space and time coordinate of two systems that move at constant velocity.

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, <u>Maxwell's equations did not</u>

  (EM waves propagate in vacuum with

  the speed of light).

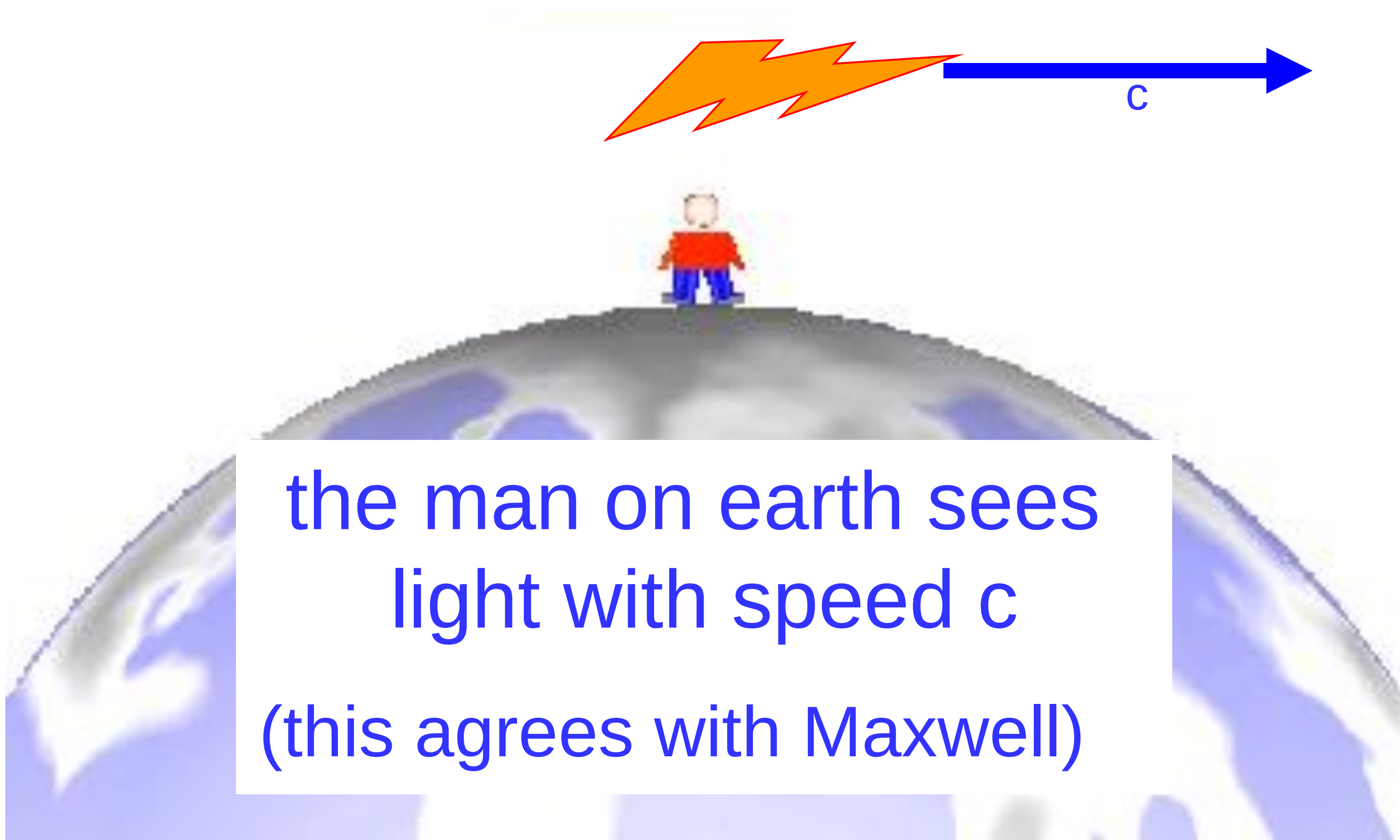
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

The Transition to Modern Relativity

Mental (thought) experiment:
A fast train with a light source in the front – what would be the speed of the light, when the train would travel with a very high speed (close to c)?

next mental (thought) experiment:

watching a light flash go by

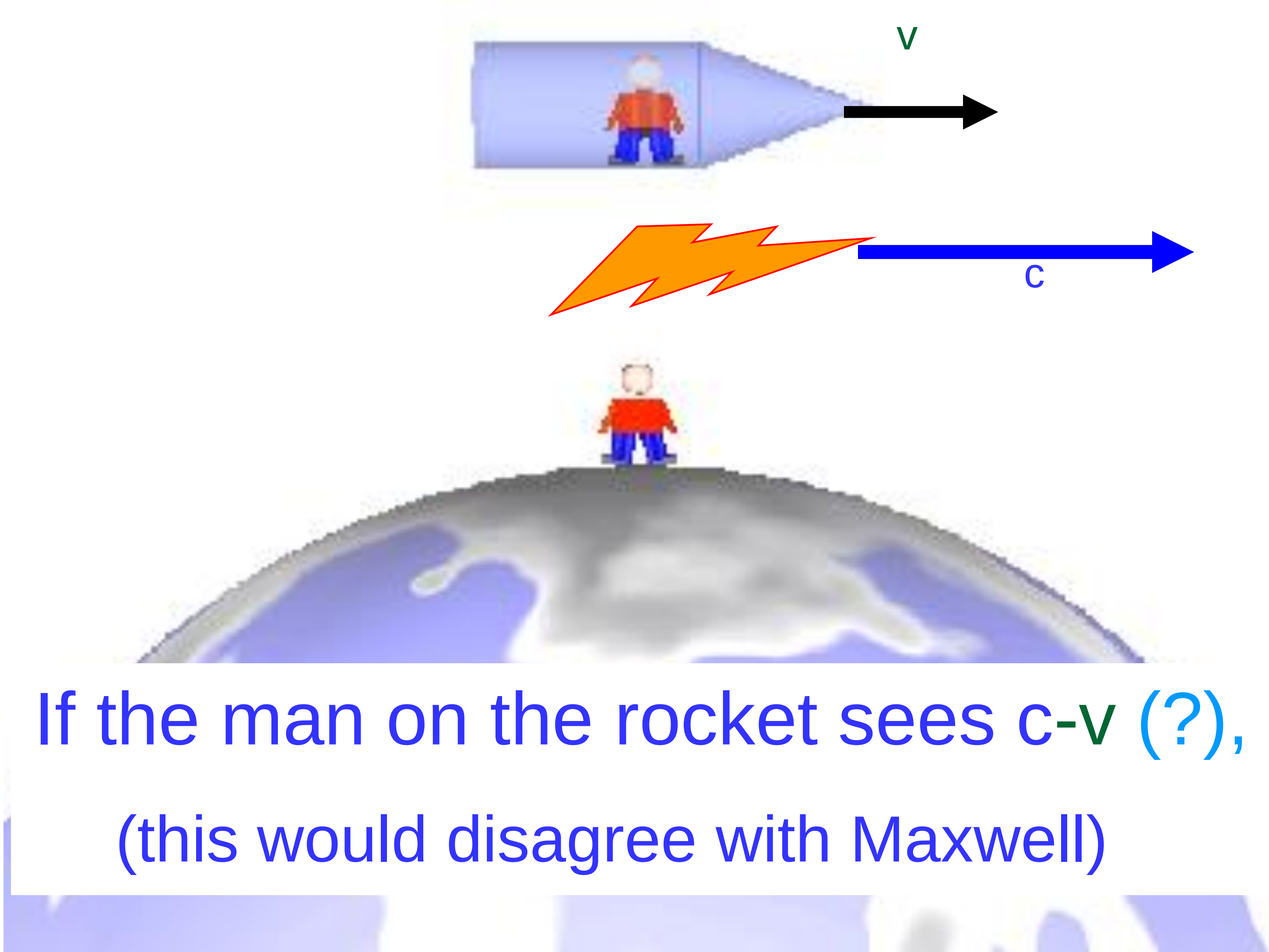


the man on earth sees
light with speed c

(this agrees with Maxwell)

next mental (thought) experiment:

watching a light flash go by



If the man on the rocket sees c-v (?),

(this would disagree with Maxwell)

The Transition to Modern Relativity

Special relativity was originally proposed by Albert Einstein in a paper published in 1905 titled "On the Electrodynamics of Moving Bodies".

Albert Einstein around 1905, the year his "*Annus Mirabilis* papers" were published. These included *Zur Elektrodynamik bewegter Körper*, the paper founding special relativity.

This study was motivated by the incompatibility of Newtonian mechanics with Maxwell's equations of electromagnetism and also experimentally (the so called Michelson–Morley experiment).

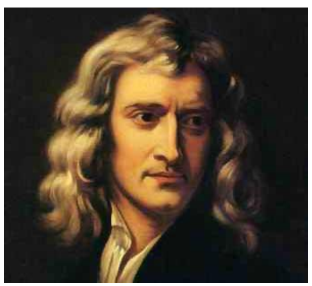
Special relativity has a wide range of consequences that have been experimentally verified. They include the relativity of simultaneity, length contraction, time dilation, the relativistic velocity addition formula, the relativistic Doppler effect, relativistic mass, a universal speed limit, mass–energy equivalence, the speed of causality and the Thomas precession.

3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter, so entsteht in der Umgebung des Magneten kein elektrisches Feld, dagegen im Leiter eine elektromotorische Kraft, welcher an sich keine Energie entspricht, die aber — Gleichheit der Relativbewegung bei den beiden ins Auge gefaßten Fällen vorausgesetzt — zu elektrischen Strömen von derselben Größe und demselben Verlaufe Veranlassung gibt, wie im ersten Falle die elektrischen Kräfte.

time is the 4th dimension

Einstein discovered that there is no "absolute" time, it too depends upon the state of motion of the observer.



Newton

space
&
time

completely different concepts

Einstein

space-time

2 different aspects of the same thing

The term space-time was not introduced by Albert Einstein, but by a German mathematician **Hermann Minkowski** (in German: Raumzeit).

Einstein's Two Postulates

With the belief that Maxwell's equations must be valid in all inertial frames, Einstein proposes the following postulates:

1) **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2) **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in a vacuum.

Comment: Letter "c" is used for the speed of light, because "*celeritas"* means in Latin "speed".

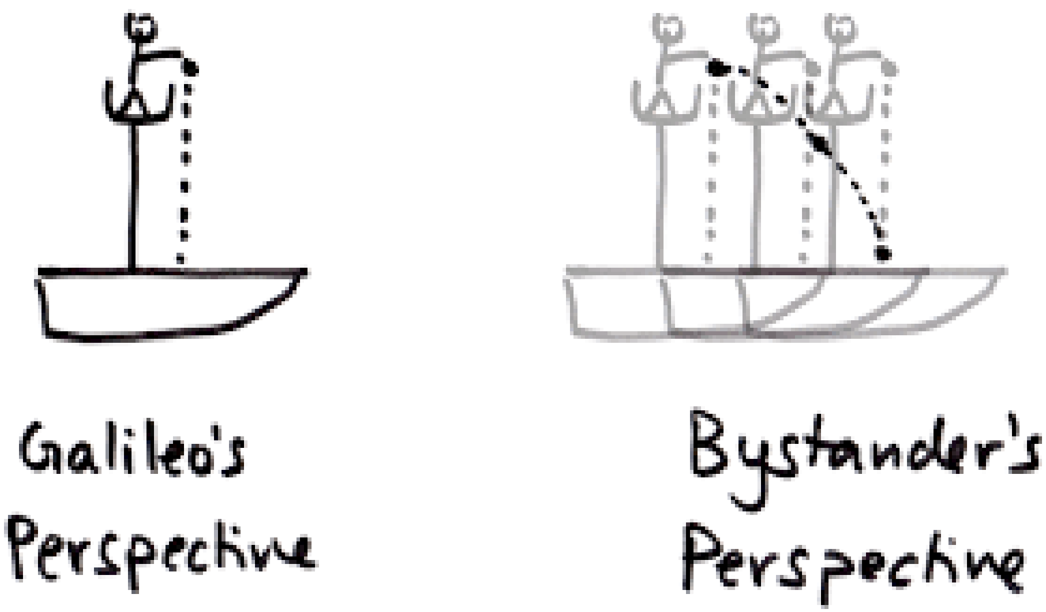
time is the 4^{th} dimension

Einstein discovered that there is no "absolute" time, it too depends upon the state of motion of the observer.

But how are the times seen by two different observers related?

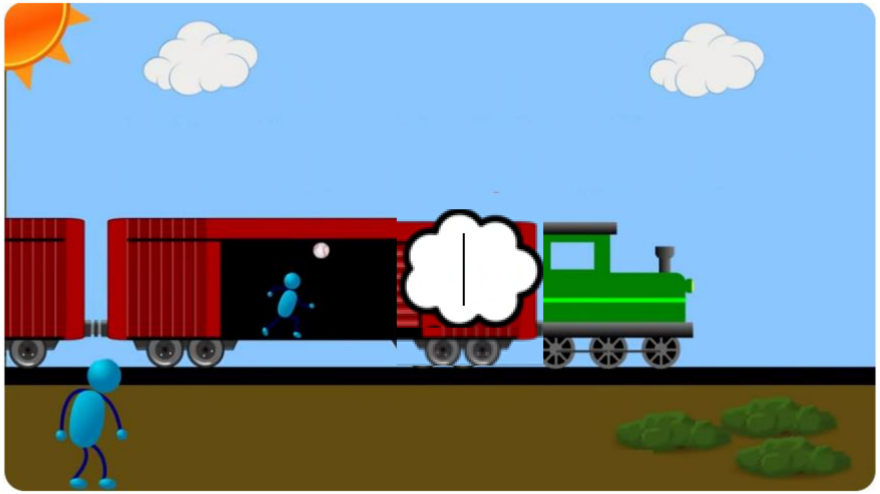
mental (thought) experiments

Galileo Galilei formulated in 1632
a thought experiment - of classical relativity:
A sailor drops a rock from the top par of the ship's mast – what will happen?
Where it will fall?
It will be different for different observers.

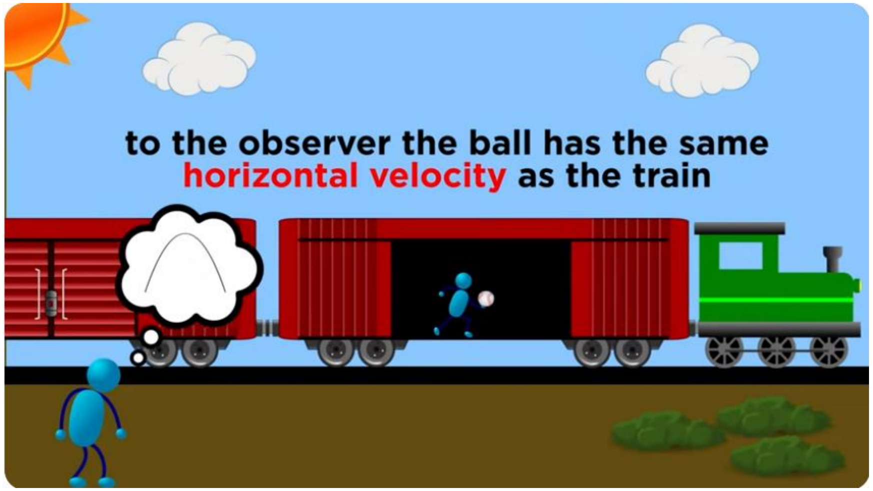


mental (thought) experiments

From the era of Albert Einstein we have various mental experiments with a train:
A person in the waggon is throwing a ball in vertical direction…



for the person in the waggon – a vertical trajectory



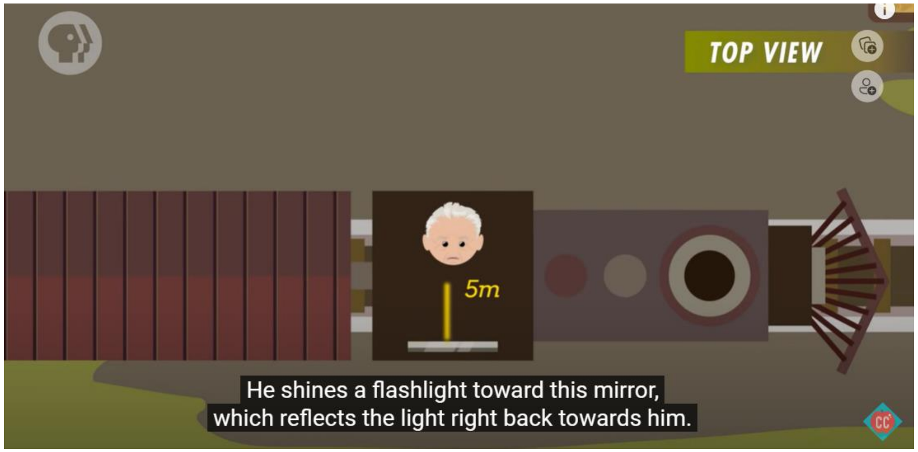
for the person outside – a parabolic trajectory

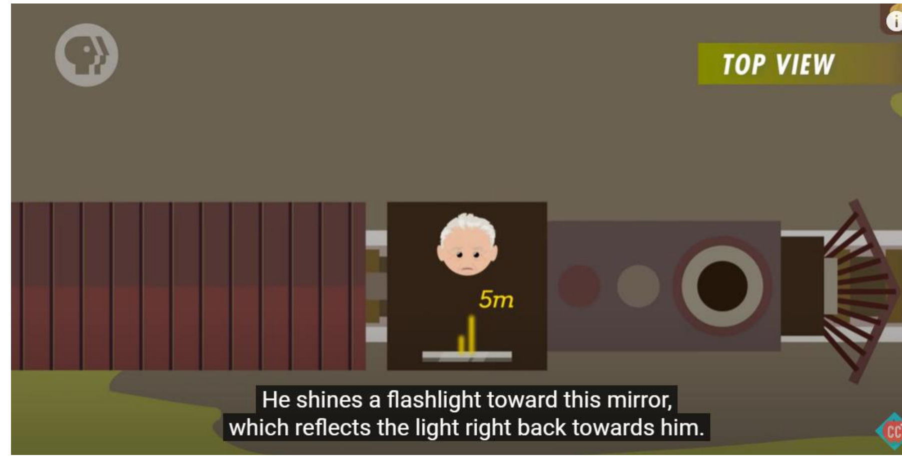
Will be there a difference in the measured times of the throw?

video:
https://www.google.at/search?q=falling+ball+in+a+fast+train+for+external+observer&source=lmns&tbm=vid&bih=722&biw=1536&hl=de&sa=X&ved=2ahUKEwjswrPm0dD9AhXz47sIHTmCCsAQ_AUoAnoECAEQAg#fpstate=ive&vld=cid:30164e6a,vid:wD7C4V9smG4

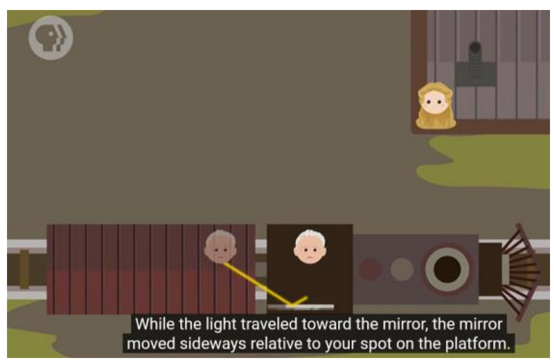
mental (thought) experiments

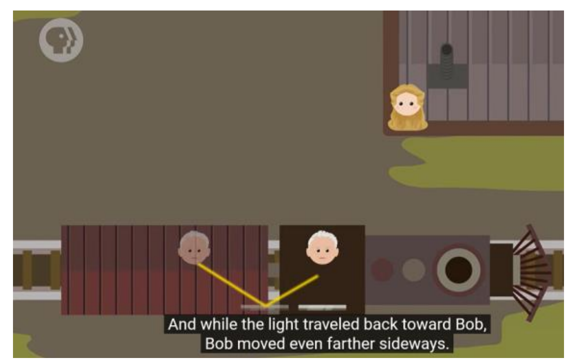
The same property (influence of the velocity on measured time) is valid for seeing things – different for a person in the train and outside of it:





for the person in the waggon – looking into a mirror to the personal image…



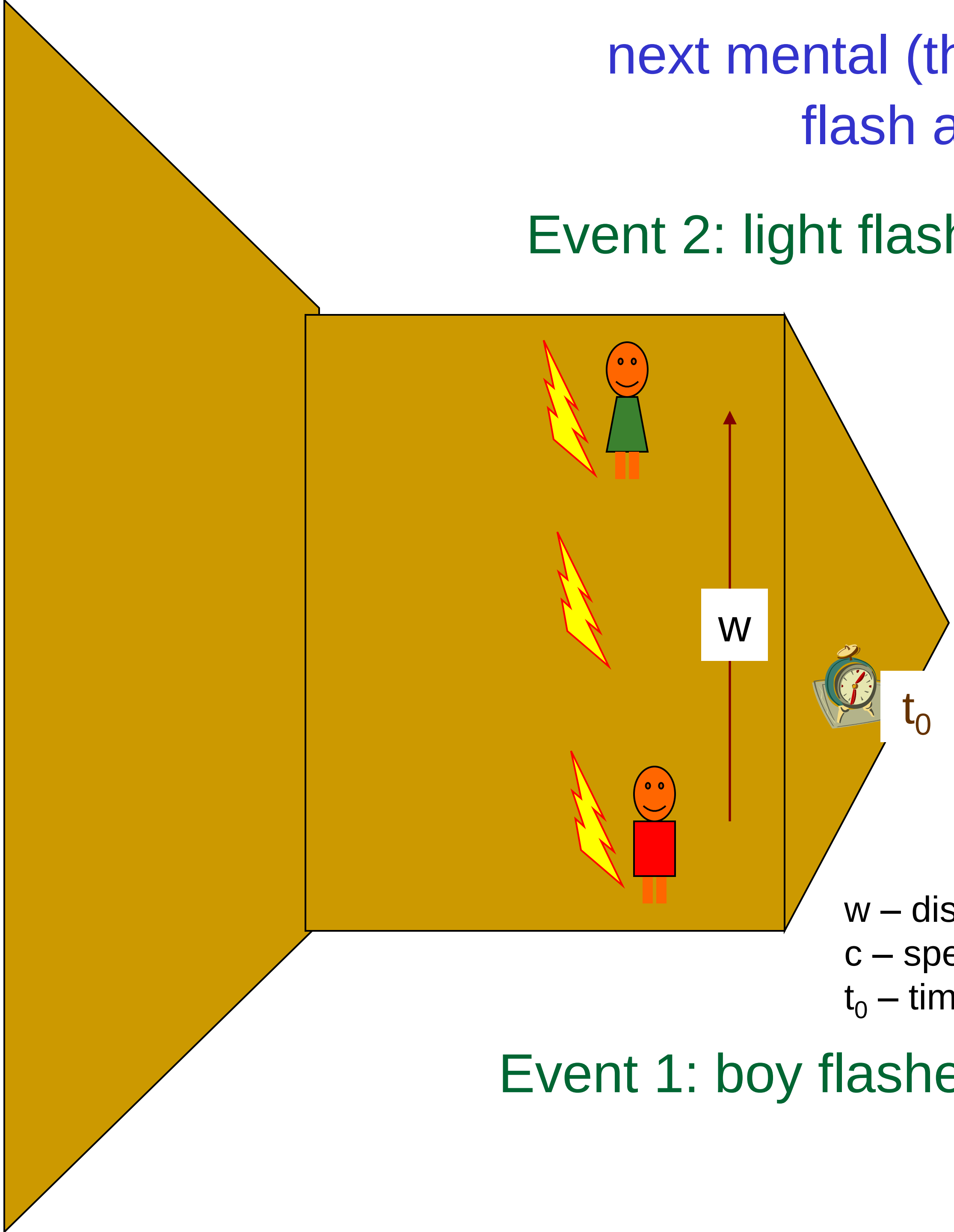


the light ray will take longer time for the person outside the waggon

video: https://www.youtube.com/watch?v=AInCqm5nCzw

next mental (thought) experiment:
flash a light a rocket ship

Event 2: light flash reaches the girl



$$c = \frac{w}{t_0}$$

$$w = ct_0$$

w – distance between boy and girl
c – speed of light
t_0 – time (clock on the moving rocket)

Event 1: boy flashes the light

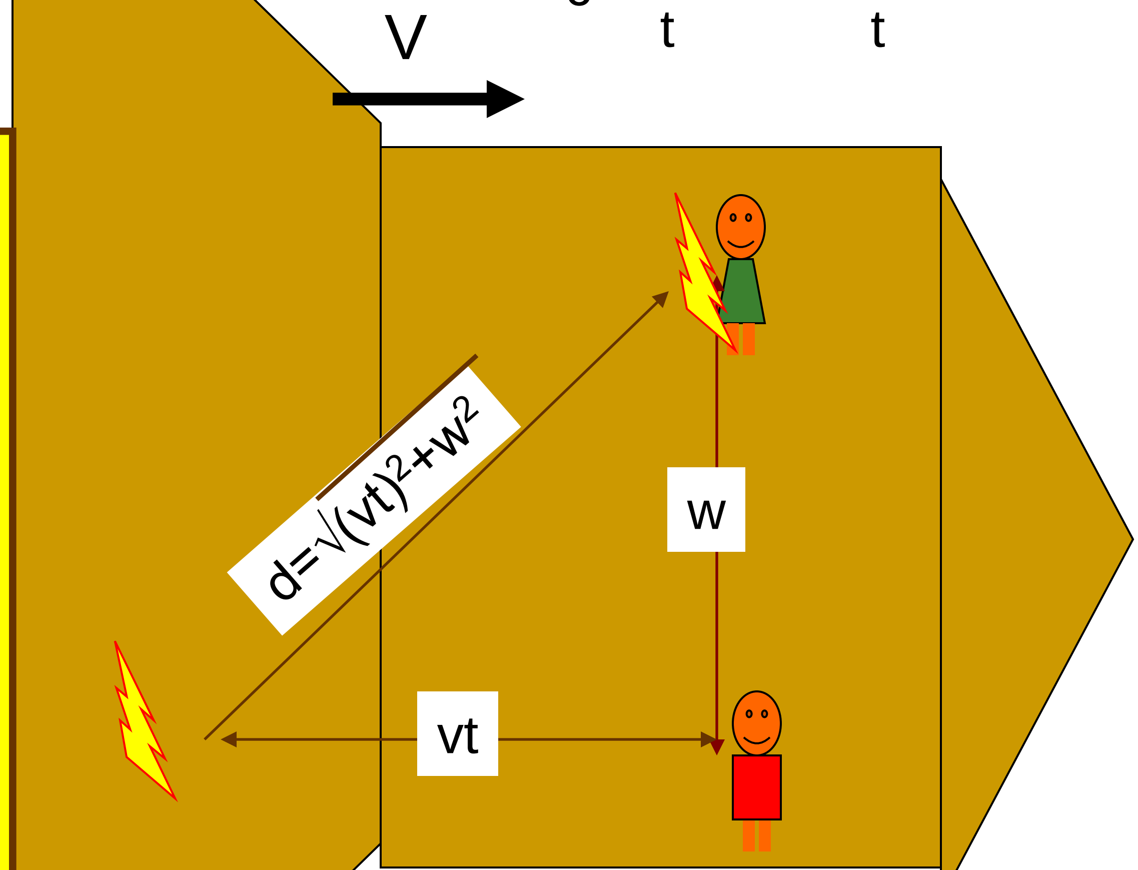
seen from the Earth

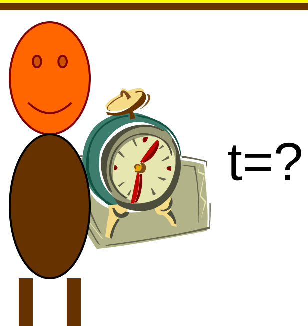
$$c = \frac{d}{t} = \frac{\sqrt{(vt)^2 + w^2}}{t}$$

speed of light c has to be the same

distance d is longer

time t must be longer (time dilation)





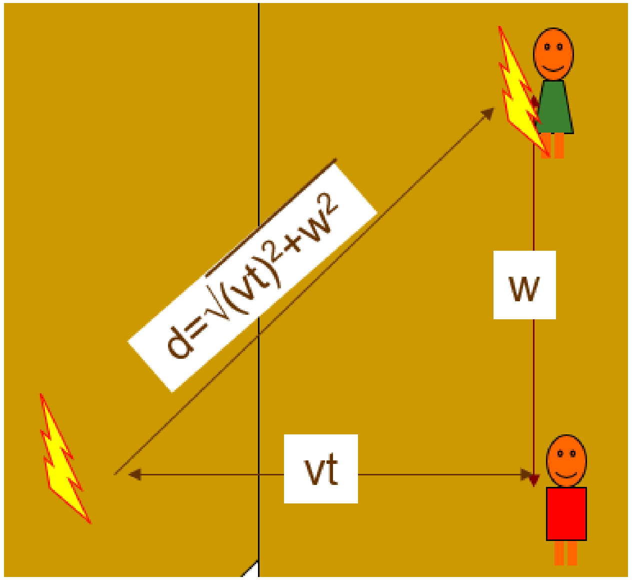
w – distance b. boy and girl
d – distance between girl and observer
c – speed of light
v – speed of the rocket ship
t_0 – time, on moving clock (rocket clock)
t – time, on the Earth clock

derivation of the formula for the time (t) calculation (1/3)

From previous slides we know:

$$c = \frac{w}{t_0}$$

$$c = \frac{d}{t} = \frac{\sqrt{(vt)^2 + w^2}}{t}$$



w – distance b. boy and girl
d – distance between girl and observer
c – speed of light
v – speed of the rocket ship
t_0 – time, on moving clock (rocket clock)
t – time, on the Earth clock

$$c = \frac{w}{t_0} \quad \Rightarrow \quad w = ct_0 \qquad (1)$$

$$c = \frac{\sqrt{(vt)^2 + w^2}}{t} \qquad / \cdot t$$

$$ct = \sqrt{(vt)^2 + w^2} \qquad /^2$$

$$(ct)^2 = (vt)^2 + w^2$$

here we enter the expression for w (from Eq. 1) $w=ct_0$

$$(ct)^2 = (vt)^2 + (ct_0)^2 \qquad /-(vt)^2$$

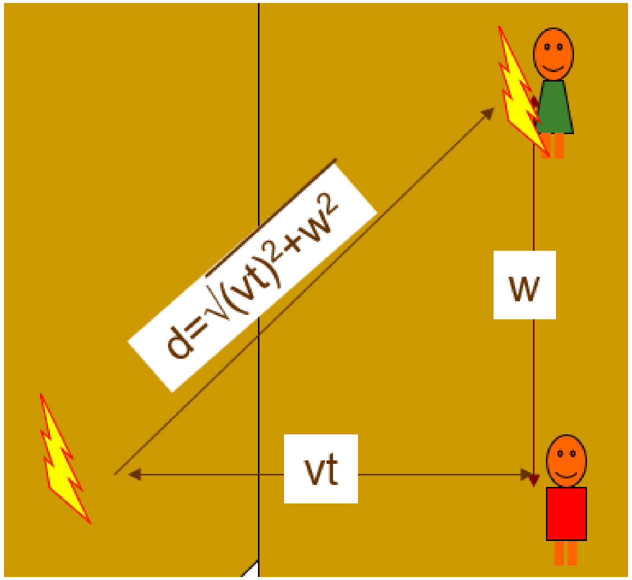
$$(ct)^2 - (vt)^2 = (ct_0)^2$$

derivation of the formula for the time (t) calculation (2/3)

From previous slides we know:

$$c = \frac{w}{t_0}$$

$$c = \frac{d}{t} = \frac{\sqrt{(vt)^2 + w^2}}{t}$$



w – distance b. boy and girl
d – distance between girl and observer
c – speed of light
v – speed of the rocket ship
t_0 – time, on moving clock (rocket clock)
t – time, on the Earth clock

$$(ct)^2 - (vt)^2 = (ct_0)^2$$

$$t^2\left(c^2 - v^2\right) = c^2 t_0^{\,2}$$

$$t^2 = \frac{c^2 t_0^{\,2}}{c^2 - v^2} \qquad / \cdot \frac{c^2}{c^2}$$

$$t^2 = \frac{1}{1 - v^2/c^2} t_0^{\,2} \qquad / \sqrt{}$$

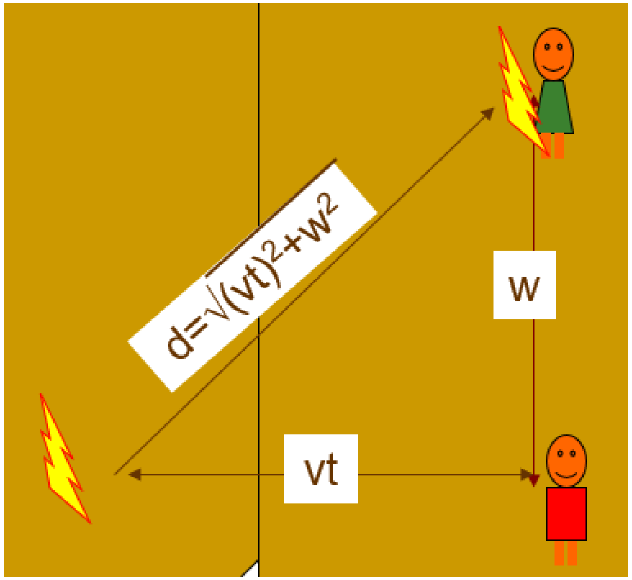
$$t = \frac{1}{\sqrt{1 - v^2/c^2}} t_0$$

derivation of the formula for the time (t) calculation (3/3)

From previous slides we know:

$$c = \frac{w}{t_0}$$

$$c = \frac{d}{t} = \frac{\sqrt{(vt)^2 + w^2}}{t}$$



w – distance b. boy and girl
d – distance between girl and observer
c – speed of light
v – speed of the rocket ship
t_0 – time, on moving clock (rocket clock)
t – time, on the Earth clock

$$t = \frac{1}{\sqrt{1 - v^2/c^2}} t_0$$

$$t = \gamma t_0$$

Here we substitute a new variable γ (so called **Lorentz factor or relativistic factor**):

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

This phenomenon is called relativistic time dilation.

Lorentz factor (called also relativistic factor)

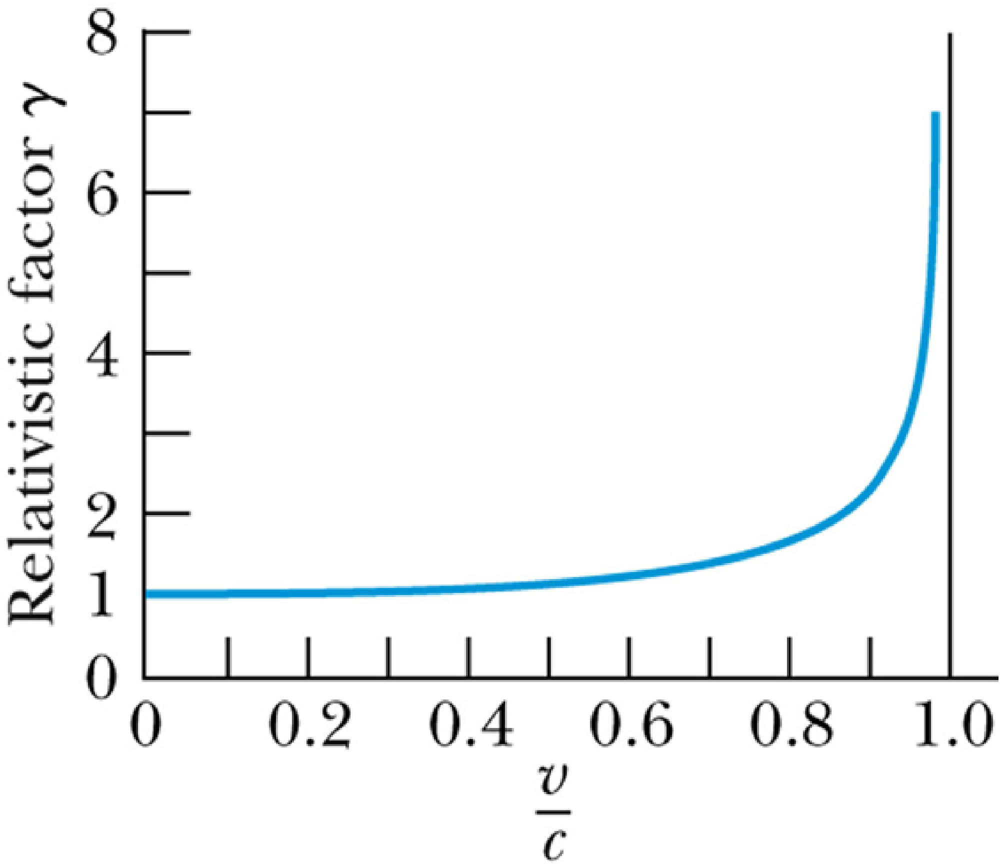
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

v	γ
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67
0.9c	2.29
0.99c	7.07
1.00c	∞
larger than c	imaginary number

Lorentz factor (called also relativistic factor)

When speed v is much smaller than c, the Lorentz factor is negligibly different from 1, but as v approaches c, γ grows without bound. The value of v must be smaller than c for the transformation to make sense.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



Time dilation equation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Δt = *r*elative time (time measured in frame moving relative to actual event),

Δt_o= *p*roper time (time measured in the same frame as the event itself),

v = *r*elative velocity of two frames.

Re-evaluation of Time

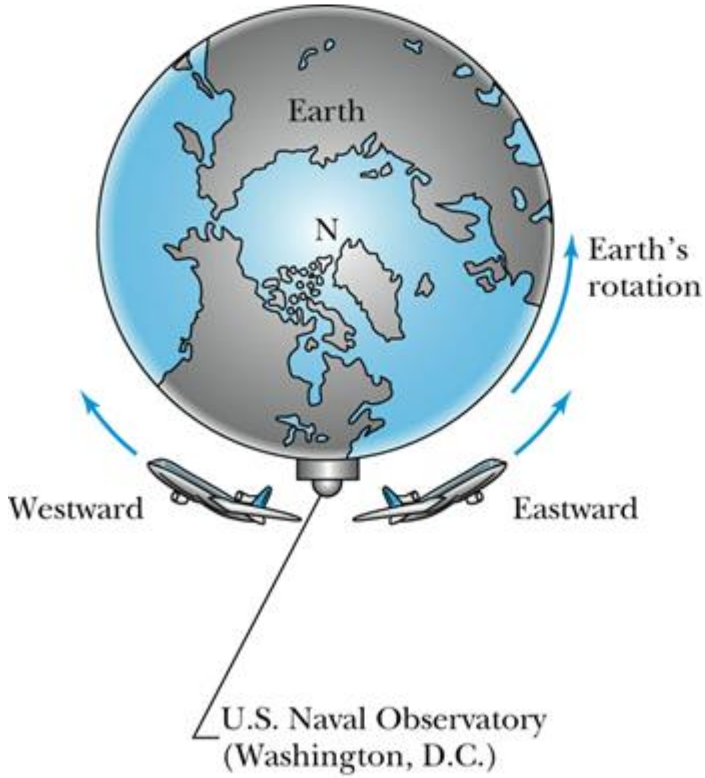
- In Newtonian physics we previously assumed that $t = t'$
  - *Thus with "synchronized" clocks, events in K and K' can be considered simultaneous*
- Einstein realized that each system must have its own observers with their own clocks and meter sticks
  - Thus events considered simultaneous in K may not be in K'

We thus observe…

- *Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K’) moving with respect to the first frame.*

Atomic Clock measurement (1972)

so called Hafele–Keating experiment



Cesium atomic clock accuracy
is on the level of 10^{-15} s

They flew twice around the world, first eastward, then westward, and compared clocks against other that remained at the United States Naval Observatory. When reunited, the 3 sets of clocks were found to disagree with 1 another, and their differences were consistent with the predictions of special and general relativity. **Differences were on the level of tens of nanoseconds.**

Result: moving clocks in the airplanes ran slower.

the so called Twin Paradox

Let us consider the case of two hypothetical twins, Abigail (**Abby**) and Gabrielle (**Gabby**). **Abby** is someone who likes to stay **at home** while **Gabby is an adventurous astronaut**.

When both of them are 30 years old, **Gabby** decides to travel to the star Tau Ceti, which is 12 light years away. She travels at a speed of 99.9% the speed of light in her ultra-fast spaceship.

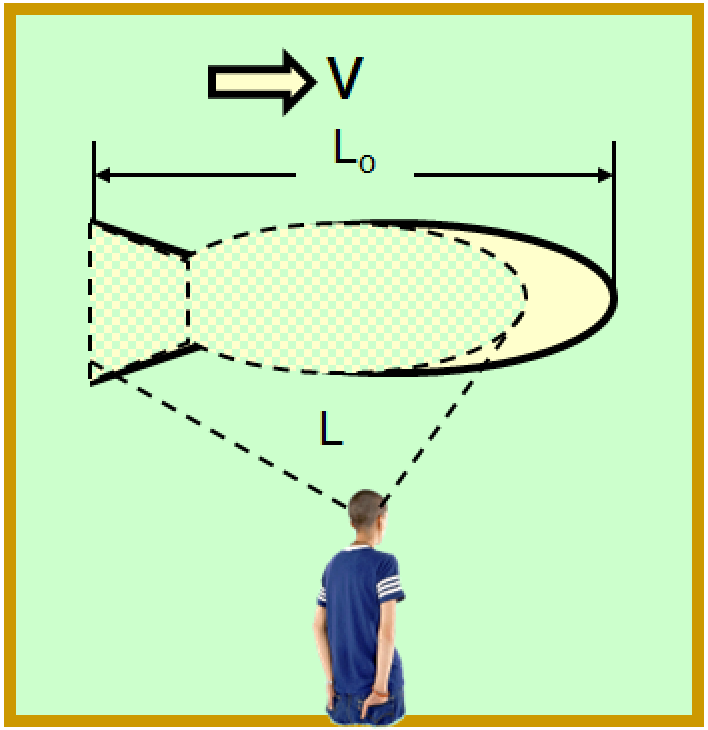
The travelling twin returns significantly younger than her sibling who stayed at home. Thus, **Gabby** would have experienced just about 13 months of time and be 31 years old. However, **Abby** who lived on Earth would have waited for 24 years to see her twin and would be 54 years old when **Gabby** returned.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{13}{\sqrt{1 - \left(\frac{0.999c}{c}\right)^2}} = \frac{13}{\sqrt{0.00199}} \approx 13 \cdot 22.366 \approx 290 \text{ months}$$

Length contraction equation

The phenomenon time dilation causes length contraction. When the velocity v is constant (v = L/t), and time t is prolongated → length L must be shortened (contracted).

$$L = L_0 \sqrt{1 - v^2/c^2}$$

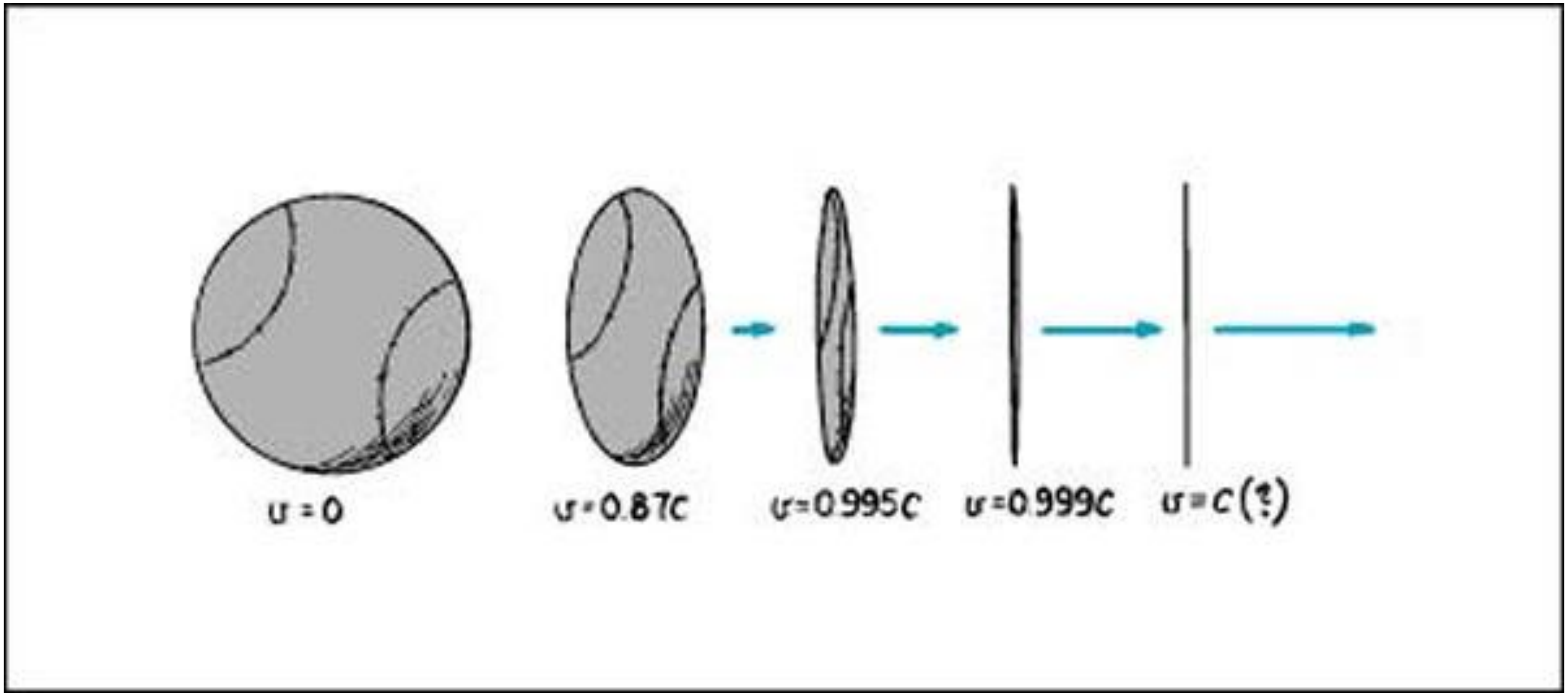


L_o is proper length,

L is relative length.

Is negligible for small velocities (v<<c).

Length contraction equation



$$L = L_0\sqrt{1 - v^2/c^2}$$

L_o is proper length,

L is relative length.

relativistic mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Note that as an object is accelerated by a resultant force, its mass increases, which requires even more force.

relativistic momentum

The basic conservation laws for momentum and energy can not be violated due to relativity.

Newton's equation for momentum (*mv*) must be changed as follows to account for relativity:

relativistic momentum: $$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

m_o is the proper mass, often called the rest mass.
Note that for small values of v, this equation reduces to Newton's equation.

Mass and Energy

Prior to the theory of relativity, scientists considered mass and energy as separate quantities, each of which must be conserved.
Now mass and energy must be considered as the same quantity. We may express the mass of a baseball in joules or its energy in kilograms! The motion adds to the mass-energy.

$$E_{rest} = mc^2$$

Total kinetic energy includes rest energy and energy of motion.

$$E_{kinetic} = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)$$

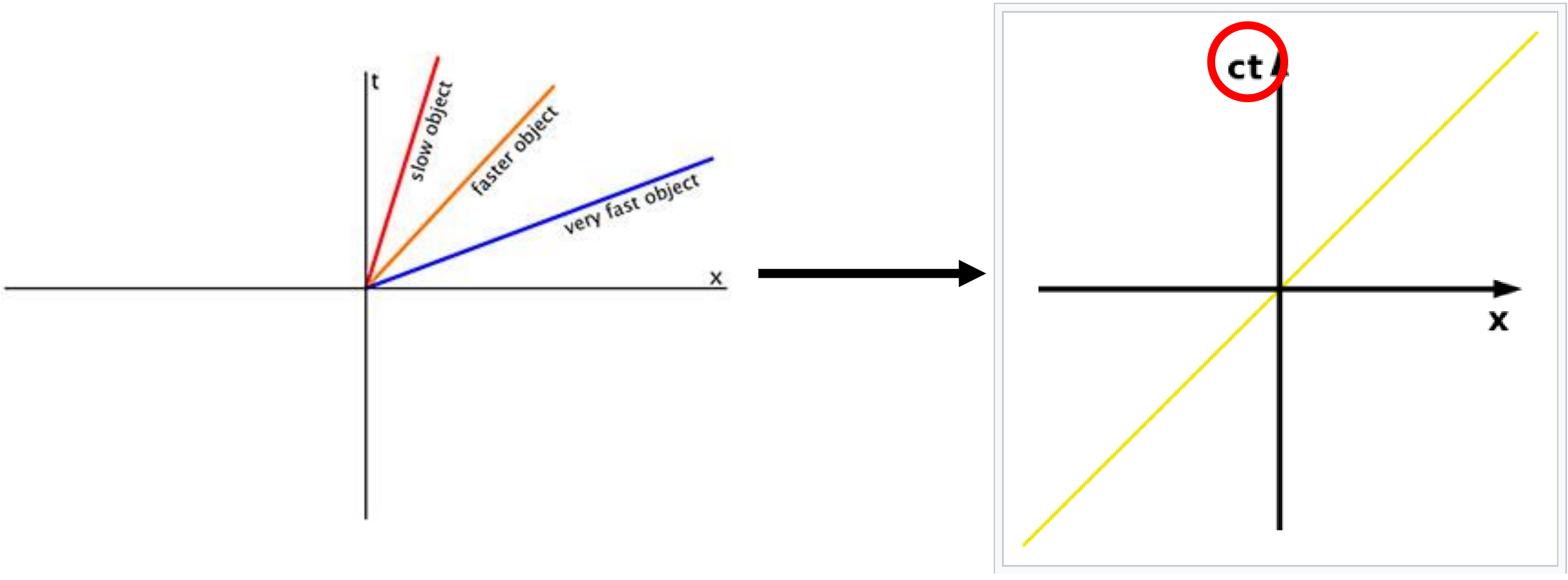
This relativistic kinetic energy equation shows that the energy of an object approaches infinity as the velocity approaches the speed of light.

One possible derivation:
https://www.google.at/search?q=derivation+of+e%3Dmc2&sxsrf=AJOqlzWhqyWE1V-zaulP4po3ERgVDZSvEg%3A1679126580187&source=hp&ei=NHAVZLiSCJTfkwXmuo34Cw&iflsig=AK50M_UAAAAAZBV-RMbJR3eT7SaRjfh4u0H_t0xrBmyE&oq=derivation+of+the+E%3Dmc&gs_lcp=Cgdnd3Mtd2l6EAEYADIGCAAQFhAeOgQIIxAnOgYIIxAnEBM6BQgAEIAEOgsILhCABBDHARDRAzoLCC4QgAQQxwEQrwE6BAgAEEM6BQguEIAEOg4ILhCABBDHARDRAxDUAjoGCAAQChBDOggIABCABBDLAToKCAAQgAQQChDLAToICAAQFhAeEA9QAFjZP2C8VGgBcAB4AIAB7wKIAdUbkgEIMi4xNy4yLjKYAQCgAQE&sclient=gws-wiz#fpstate=ive&vld=cid:586cfc12,vid:1yF0PO6lidg

Spacetime

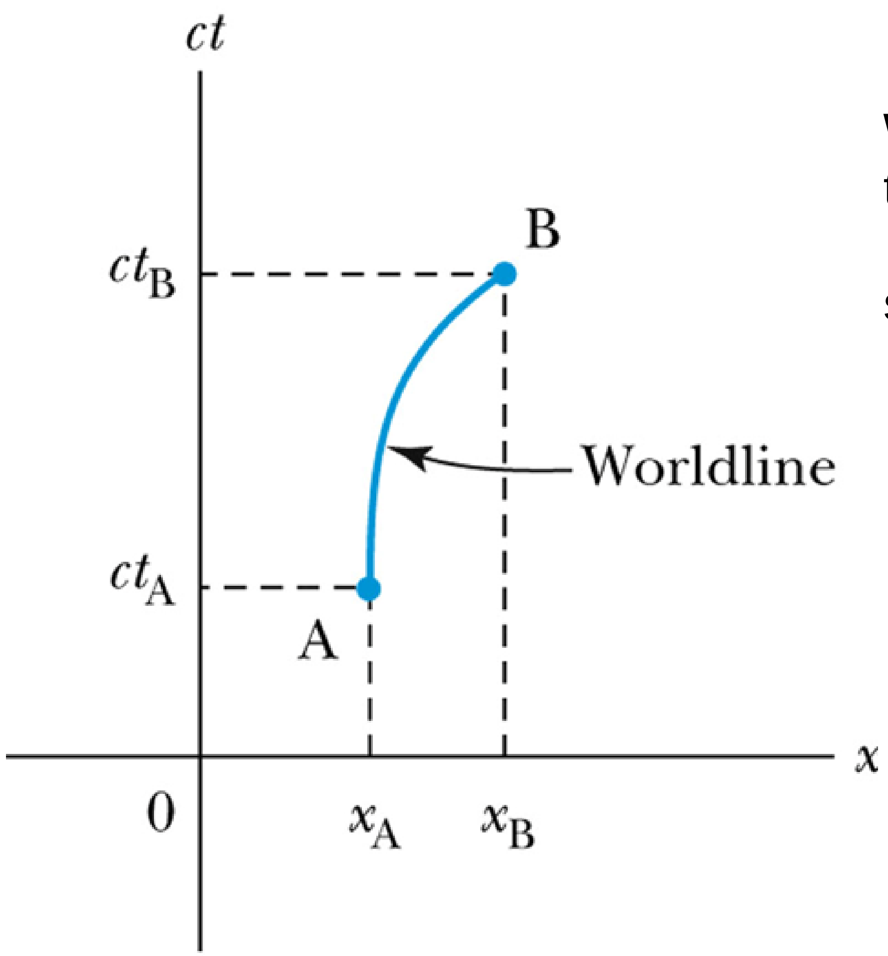
Spacetime is a mathematical model that combines the three dimensions of space and one dimension of time into a single four-dimensional manifold.

- When describing events in relativity, it is convenient to represent events on a **spacetime diagram**.
- In this diagram one spatial coordinate *x,* to specify position, is used and instead of time *t*, *ct* is used as the other coordinate so that both coordinates will have dimensions of length.



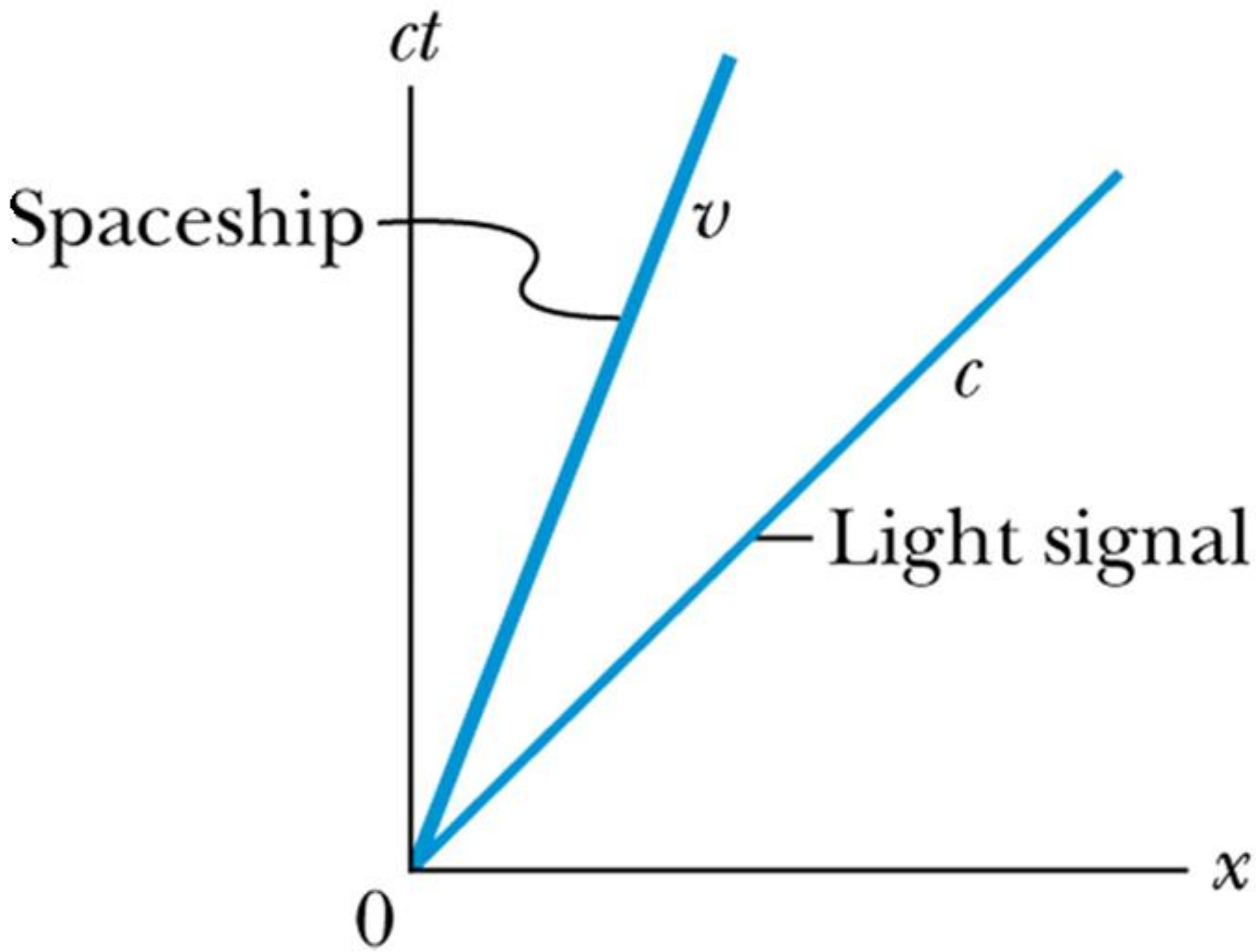
Spacetime Diagram

A **spacetime diagram** is a graphical illustration of the properties of space and time in the special theory of relativity.
It allows a qualitative understanding of the corresponding phenomena, connected with the special relativity theory – e.g. so called **worldline**.



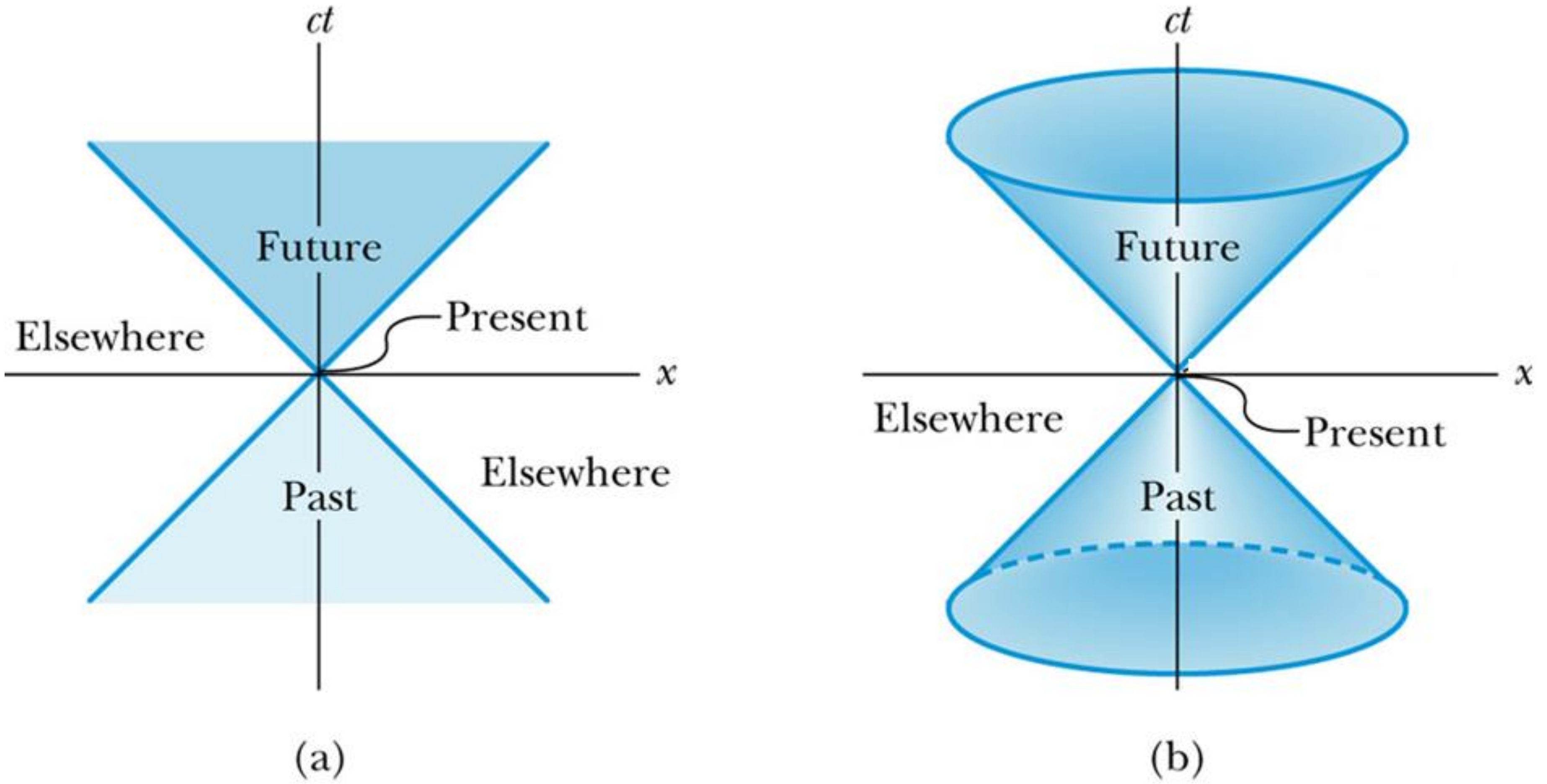
Worldline of an object is the path that an object traces in 4-dimensional spacetime.

Spacetime Diagram



particular worldlines

The Light Cone



Lorentz Transformations

Lorentz transformation is the relationship between two different coordinate frames that move at a constant velocity and are relative to each other (incorporating relativity).

It is a special set of linear transformations that:

1) preserve the constancy of the speed of light between inertial observers;

and,

2) account for the problem of simultaneity between these observers.

These equations handle also time dilation and length contraction.

Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz Transformation Equations

A more simple form:

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \gamma\left(x - \beta ct\right)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{\beta x}{c}\right)$$

if $v << c$, i.e., $\beta \approx 0$ and ≈ 1, we see these equations reduce to the Galilean transformation

Lorentz transformation - remarks

1) If $v << c$, i.e., $\beta \approx 0$ and $\gamma \approx 1$, we see these equations reduce to the Galilean transformation.

2) Space and time are now not separated.

3) For non-imaginary transformations, the frame velocity cannot exceed c.

Time Dilation and Length Contraction

Consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in K' run slow with respect to stationary clocks in K.

- **Length Contraction**:

  Lengths in K' are contracted with respect to the same lengths stationary in K.

General Relativity

General Relativity

Tenets of General Relativity

- General relativity is the extension of special relativity. It includes the effects of accelerating objects and their mass on spacetime.

- As a result, the theory is an explanation of gravity.

- It is based on two concepts:

  (1) the principle of equivalence, which is an extension of Einstein’s first postulate of special relativity and

  (2) the curvature of spacetime due to gravitation.

Principle of Equivalence

A uniform gravitational field in some direction is indistinguishable from a uniform acceleration in the opposite direction.

Comment:

Keep in mind that an accelerating frame introduces pseudo-forces in the direction opposite to the true acceleration of the frame (e.g. inside a car when brakes are applied).

Inertial Mass and Gravitational Mass

- Recall from Newton's 2nd law that an object accelerates in reaction to a force according to its inertial mass:

$$\vec{F} = m_I \vec{a}$$

- Inertial mass measures how strongly an object resists a change in its motion.

- Gravitational mass measures how strongly it attracts other objects.

$$\vec{F} = m_G \vec{g}$$

- For the same force, we get a ratio of masses: $\vec{a} = \left(\frac{m_G}{m_I} \right) \vec{g}$

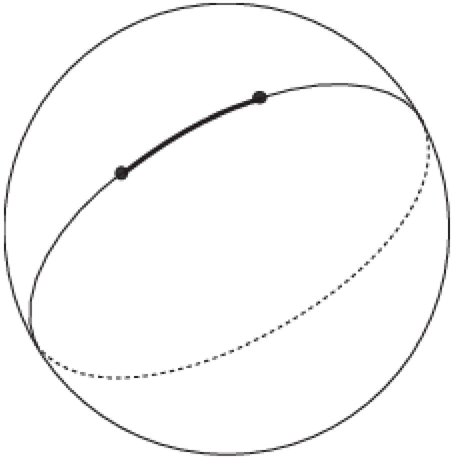
- According to the principle of equivalence, the inertial and gravitational masses are equal.

Spacetime – and its curvature

- Einstein recognized that we need to expand our definition of a *straight line.*

- The shortest distance between two points on a flat surface appears different than the same distance between points on a sphere. The path on the sphere appears curved. We shall expand our definition of a *straight line* to include any minimized distance between two points.

- Thus if the spacetime near the Earth is not flat, then the straight line path of light near the Earth will appear curved.



Spacetime – and its curvature

- Einstein mandated that the mass of the Earth creates a dimple on the spacetime surface. In other words, the mass changes the geometry of the spacetime.
- The geometry of the spacetime then tells matter how to move.
- Einstein's famous field equations sum up this relationship as:

  * *mass-energy tells spacetime how to curve*

  * *spacetime curvature tells matter how to move*

- The result is that a standard unit of length such as a meter stick increases in the vicinity of a mass.

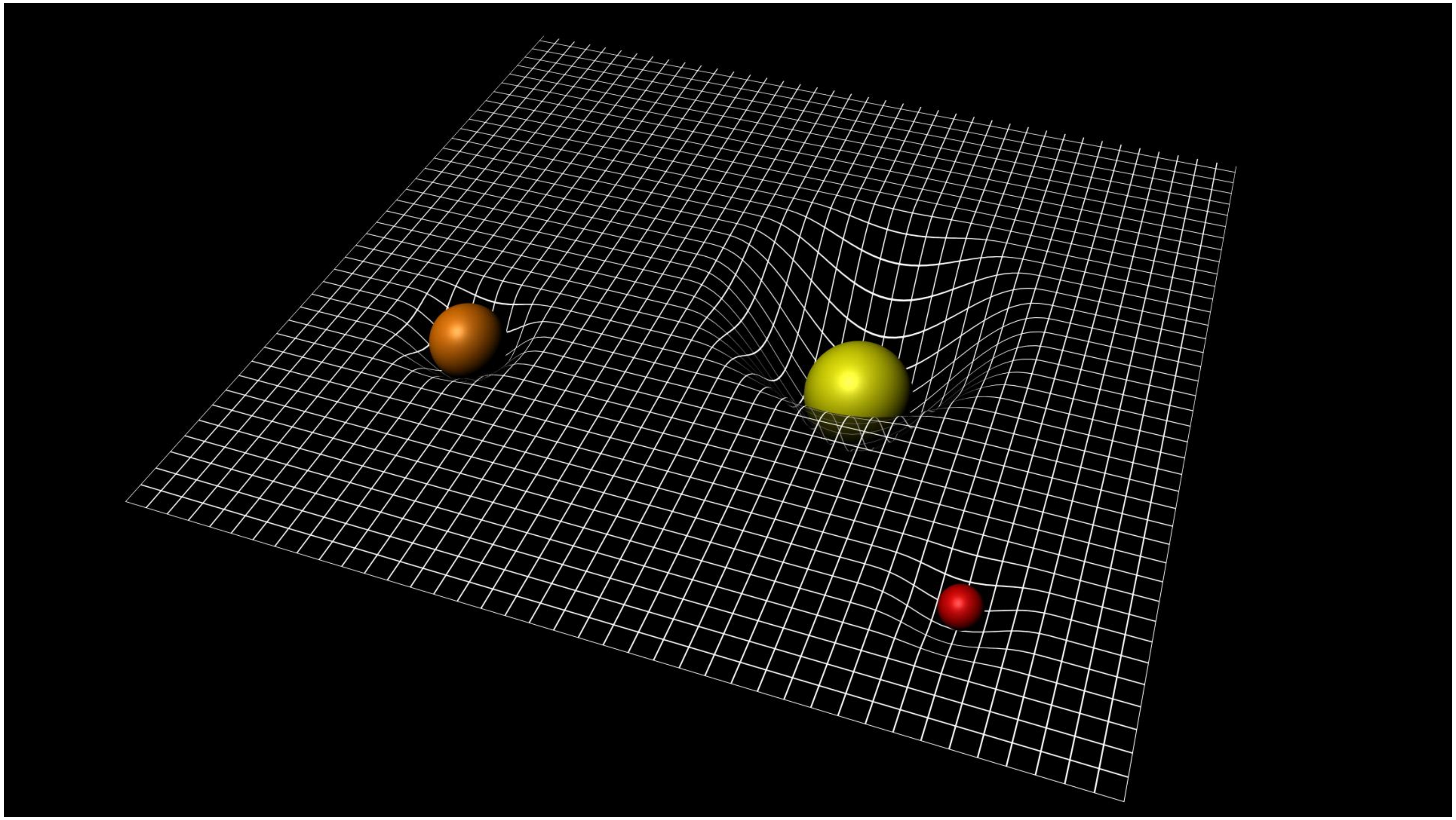
Spacetime – and its curvature

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, G is Newton's gravitational constant, c is the speed of light in vacuum, and $T_{\mu\nu}$ is the stress–energy tensor.

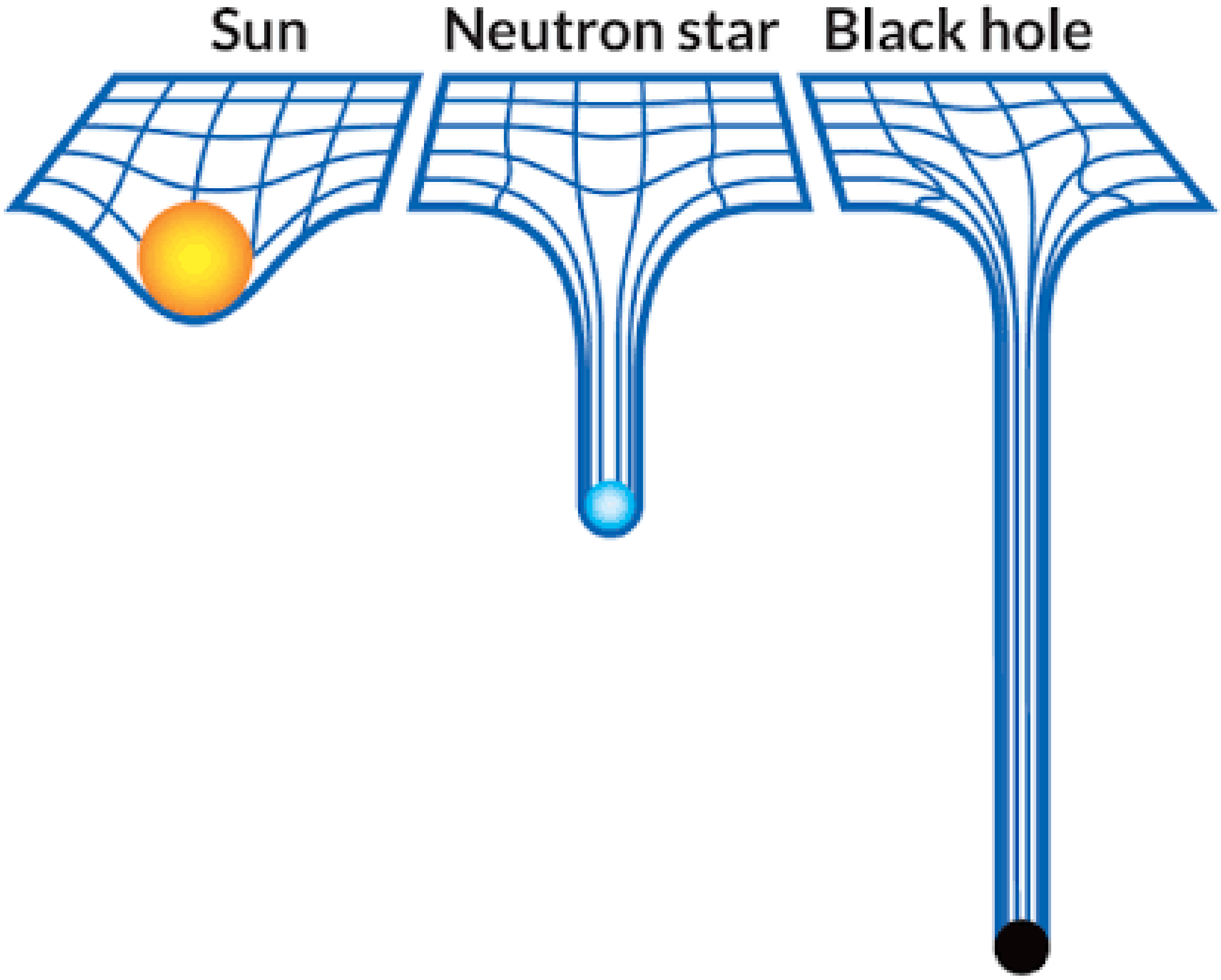
Despite the simple appearance of the equations they are actually quite complicated. Given a specified distribution of matter and energy in the form of a stress–energy tensor, the EFE (Einstein Field Equations) are understood to be equations for the metric tensor $g_{\mu\nu}$, as both the Ricci tensor and scalar curvature depend on the metric in a complicated nonlinear manner. In fact, when fully written out, the EFE are a system of 10 coupled, nonlinear, hyperbolic-elliptic partial differential equations.

Spacetime – and its curvature



very often used visualization
(spacetime is the net and heavy object cause its deformation - curvature)

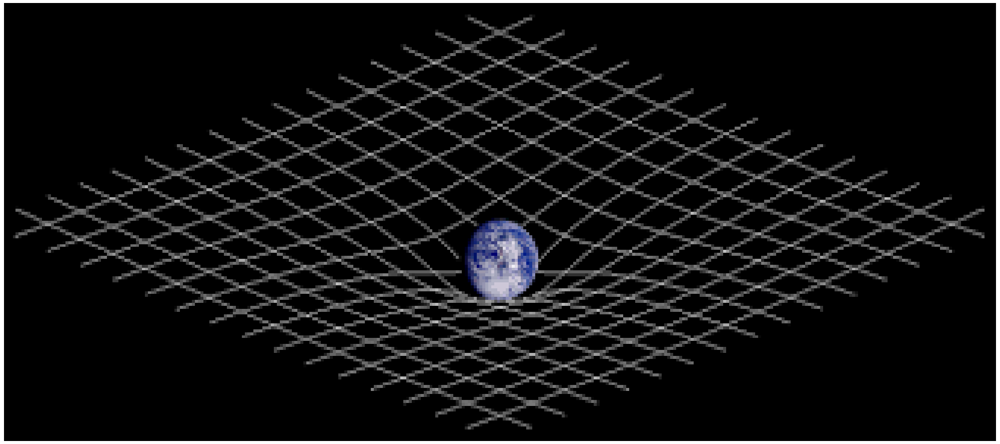
Spacetime – and its curvature

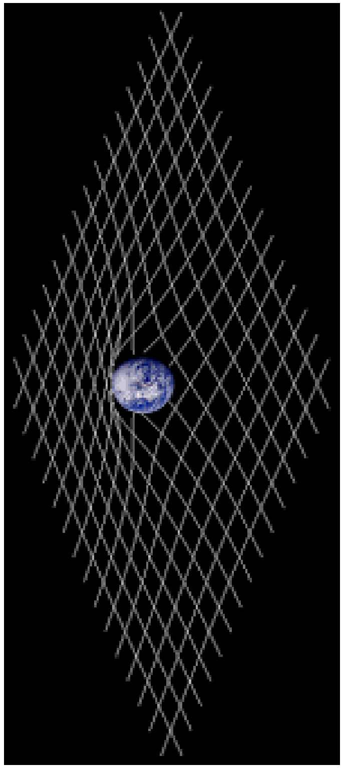


Spacetime – and its curvature

https://www.youtube.com/watch?v=MTY1Kje0yLg

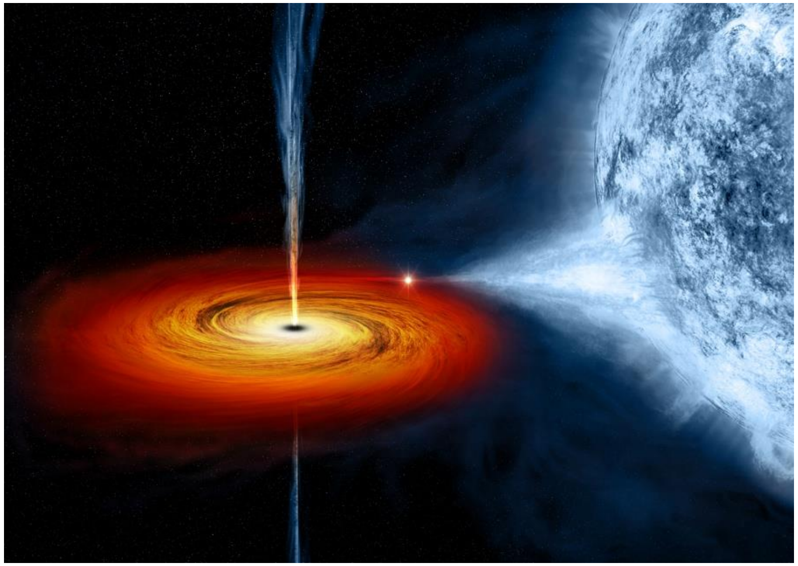
Spacetime – and its curvature

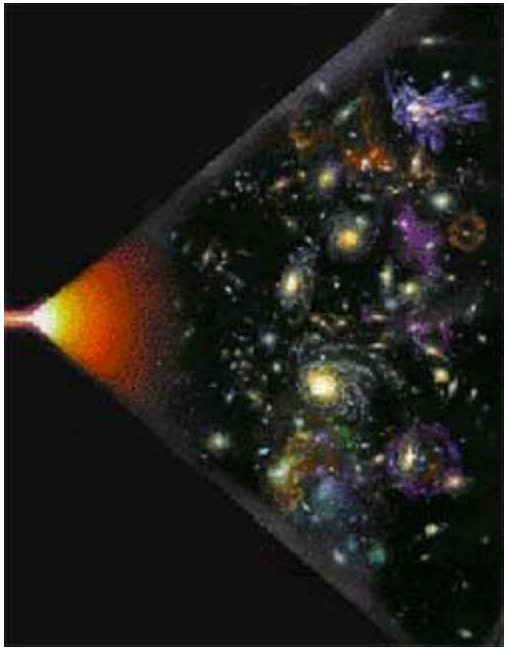


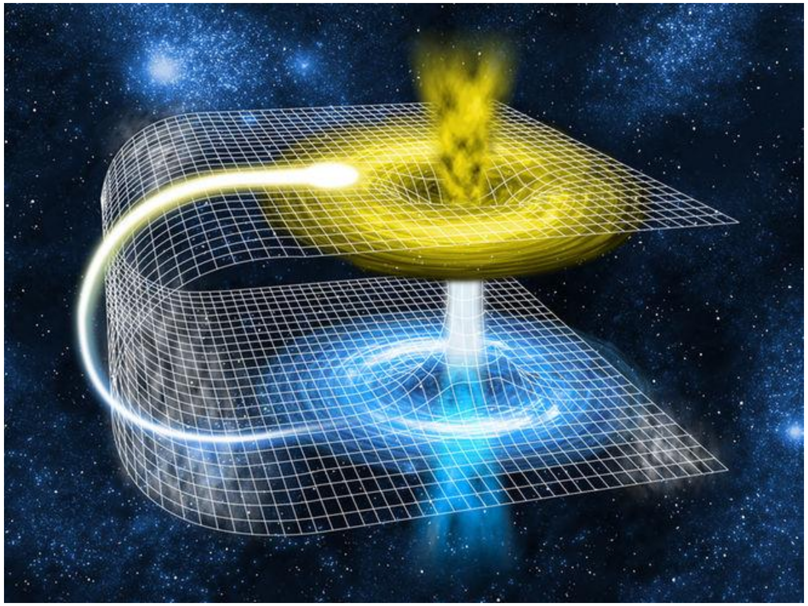


some further predictions of General Relativity

- bending of light
- expanding universe
- black holes
- wormholes



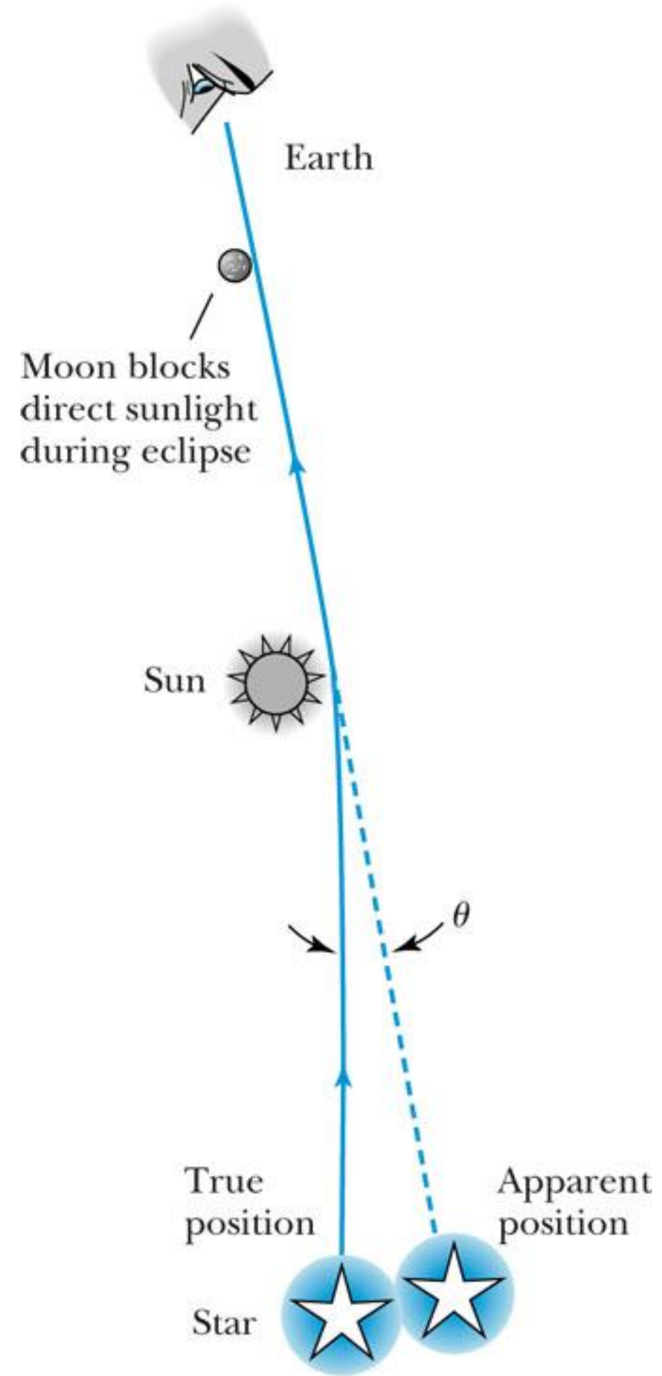




Tests of General Relativity

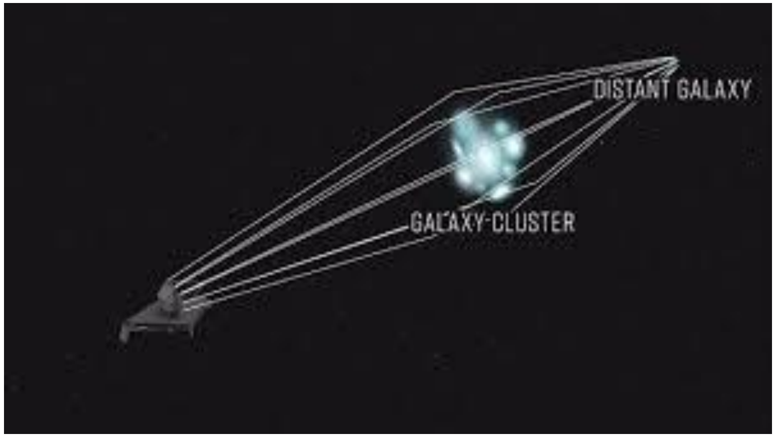
Bending of Light

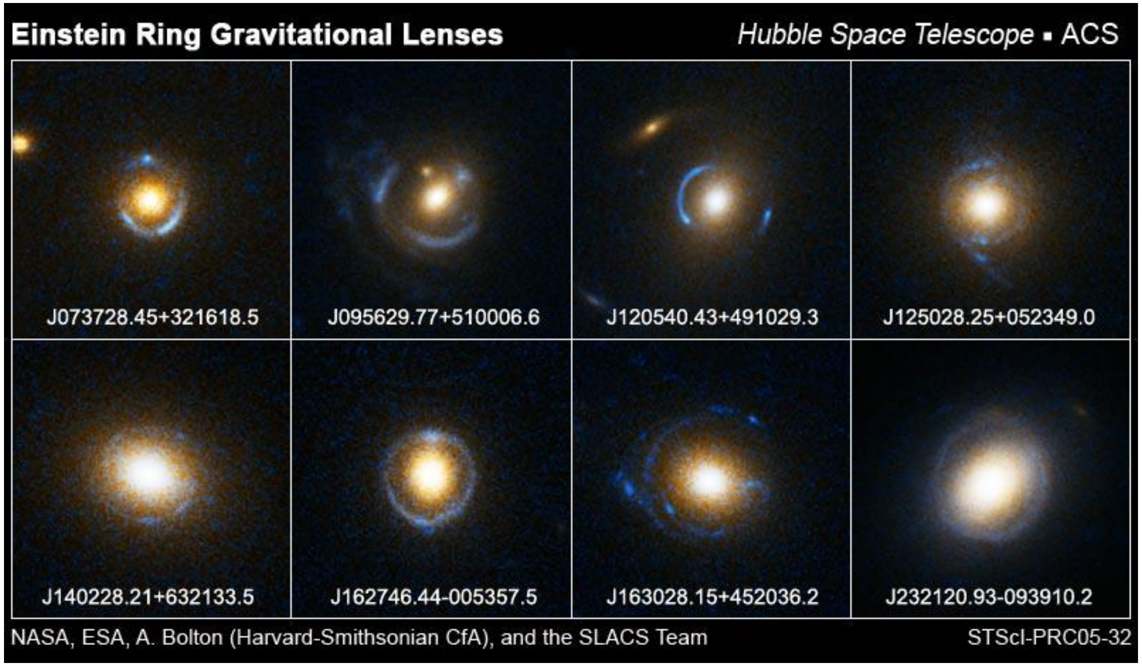
- during a solar eclipse of the sun by the moon, most of the sun's light is blocked on Earth, which afforded the opportunity to view starlight passing close to the sun in 1919. The starlight was bent as it passed near the sun which caused the star to appear displaced.
- Einstein's general theory predicted a deflection of 1.75 seconds of arc, and the two measurements found 1.98 ± 0.16 and 1.61 ± 0.40 arc seconds.
- measured by Eddington, Dyson and Davidson during two British expeditions
- since the eclipse of 1919, many experiments, using both starlight and radio waves from quasars, have confirmed Einstein's predictions about the bending of light with increasingly good accuracy.



Gravitational Lensing

When light from a distant object like a quasar passes by a nearby galaxy on its way to us on Earth, the light can be bent (even multiple times as it passes in different directions around the galaxy)

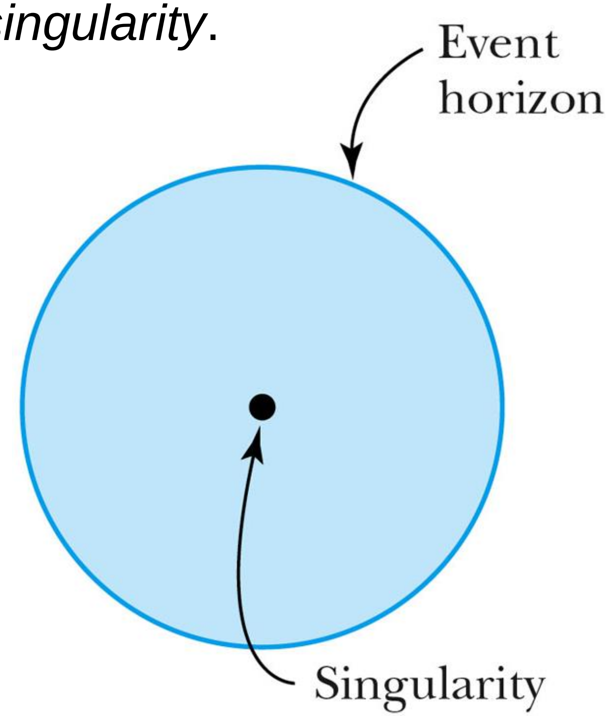




Black Holes

- While a star is burning, the heat produced by the thermonuclear reactions pushes out the star's matter and balances the force of gravity. When the star's fuel is depleted, no heat is left to counteract the force of gravity, which becomes dominant. The star's mass collapses into an incredibly dense ball that could wrap spacetime enough to not allow light to escape. The point at the center is called a *singularity*.
- A collapsing star greater than 3 solar masses will distort spacetime in this way to create a black hole.
- Karl Schwarzschild determined the radius of a black hole known as the *event horizon*.

$$r_S = \frac{2GM}{c^2}$$



Black Holes

- Since light can't escape, they must be detected indirectly:
- Severe redshifting of light.
- Hawking radiation results from particle-antiparticle pairs created near the event horizon. One member slips into the singularity as the other escapes. Antiparticles that escape radiate as they combine with matter. Energy expended to pair production at the event horizon decreases the total mass-energy of the black hole.
- Hawking calculated the blackbody temperature of the black hole to be:

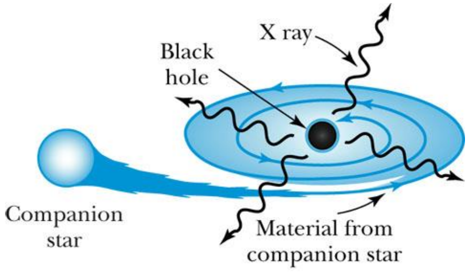
$$T = \frac{\hbar c^3}{8\pi kGM}$$

- The power radiated is: $P(T) = 4\pi\sigma r_S^{\,2}\left(\frac{\hbar c^3}{8\pi kGM}\right)^4$

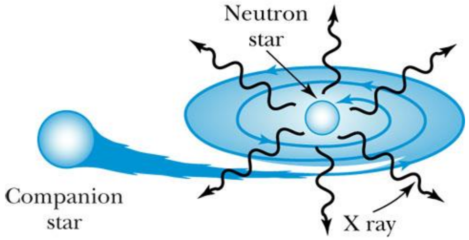
  This result is used to detect a black hole by its Hawking radiation.
- Mass falling into a black hole would create a rotating accretion disk. Internal friction would create heat and emit x rays.

Black Holes

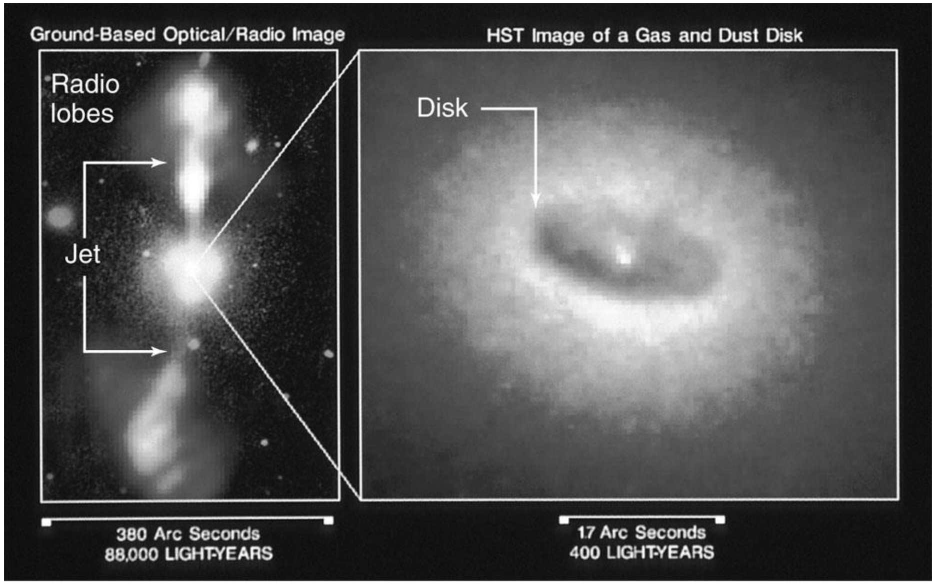
- There were several plausible candidates:
  - Cygnus X-1 is an x ray emitter and part of a binary system in the Cygnus constellation. It is roughly 7 solar masses.
  - The galactic center of M87 is 3 billion solar masses (found in 1981).
  - NGC 4261 is a billion solar masses.

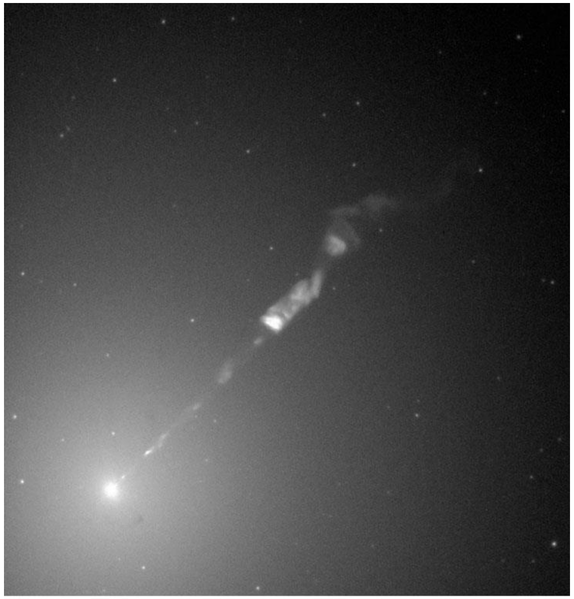


(a)

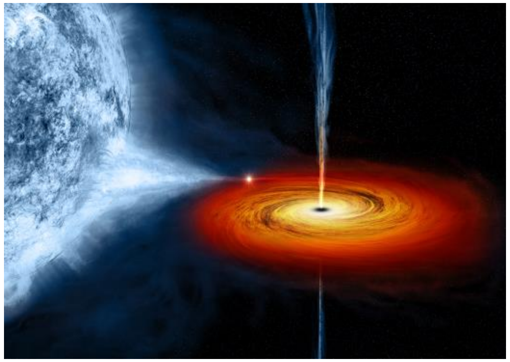


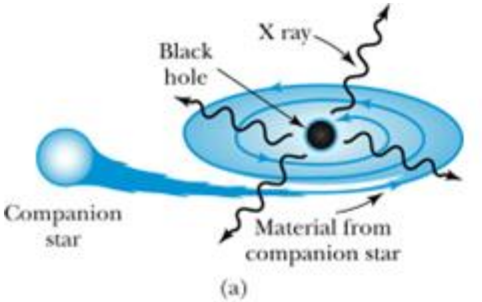
(b)

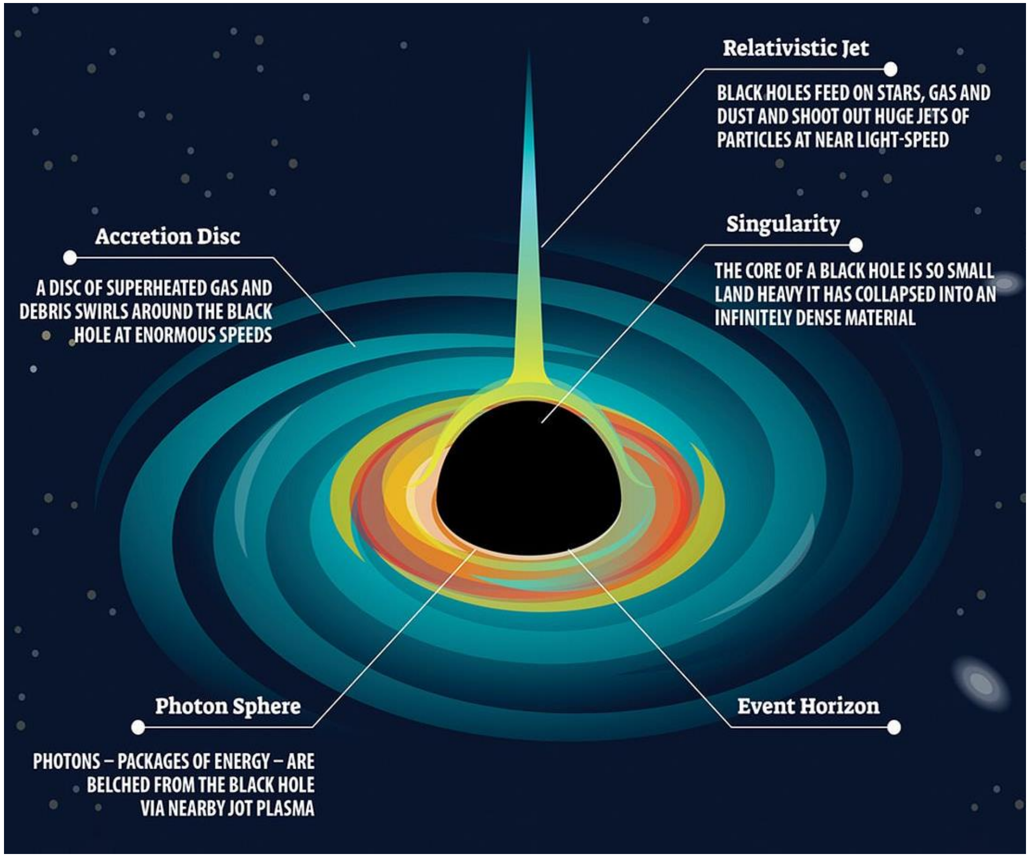




Black Holes




X ray
Black hole
Companion star
Material from companion star
(a)

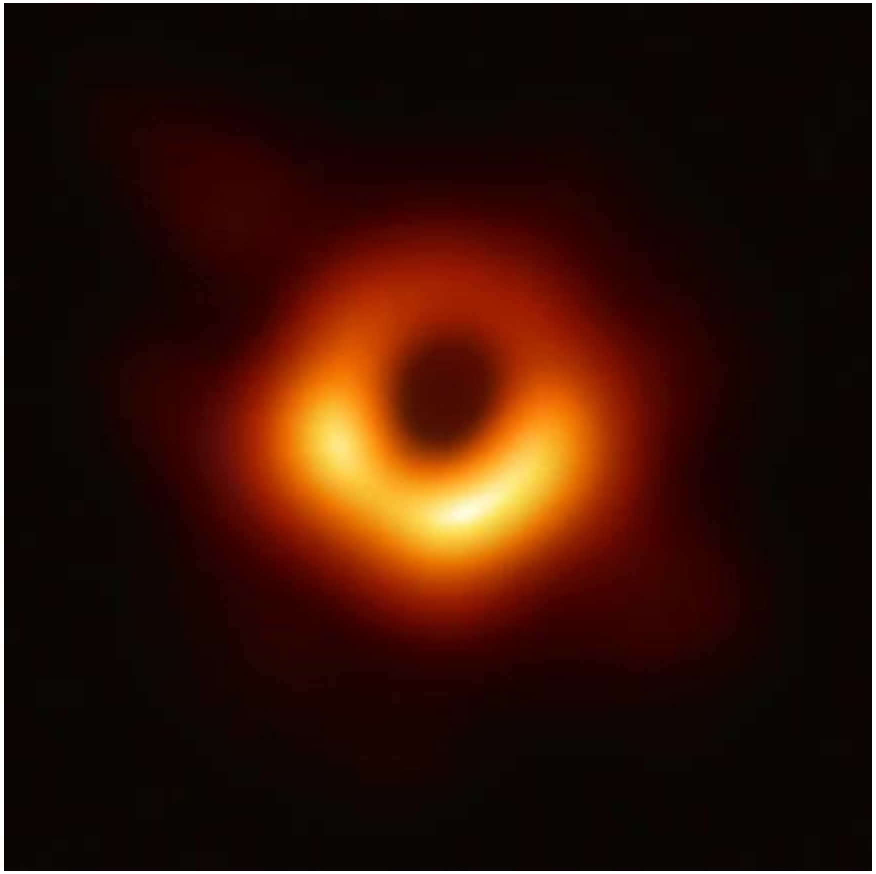

Relativistic Jet
BLACK HOLES FEED ON STARS, GAS AND DUST AND SHOOT OUT HUGE JETS OF PARTICLES AT NEAR LIGHT-SPEED
Singularity
THE CORE OF A BLACK HOLE IS SO SMALL LAND HEAVY IT HAS COLLAPSED INTO AN INFINITELY DENSE MATERIAL
Accretion Disc
A DISC OF SUPERHEATED GAS AND DEBRIS SWIRLS AROUND THE BLACK HOLE AT ENORMOUS SPEEDS
Photon Sphere
PHOTONS – PACKAGES OF ENERGY – ARE BELCHED FROM THE BLACK HOLE VIA NEARBY JOT PLASMA
Event Horizon

Black Holes

- First photo of a black-hole:

In 2019 the EHT released the first ever image of black hole at the centre of the supergiant elliptical galaxy M87, which is in the constellation Virgo.

It revealed a bright ring-like structure with a dark central region — the black hole's shadow.

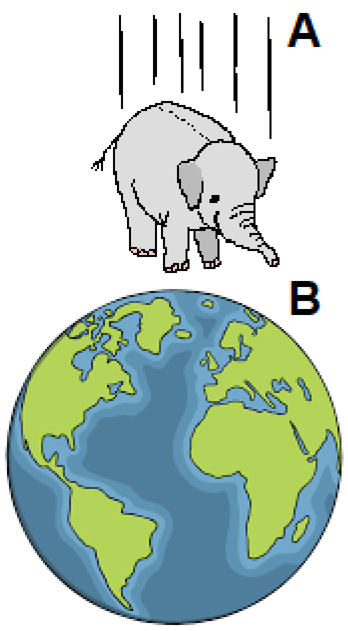


Gravitational time dilation

It is a form of time dilation, an actual difference of elapsed time between two events as measured by observers situated at varying distances from a gravitating mass.

The higher the gravity acceleration (the closer the clock is to the source of gravitation), the slower time passes,
speeding up as the gravity acceleration decreases (the clock getting away from the source of gravitation).

If the force of gravity increases as we move downward, a free-falling object falls faster at a point on the surface, say B, than it does at a higher altitude, say A. For the free-falling object, according to Special Relativity, time at B must pass relatively slower than it would pass at A, because the object's velocity is faster at B."



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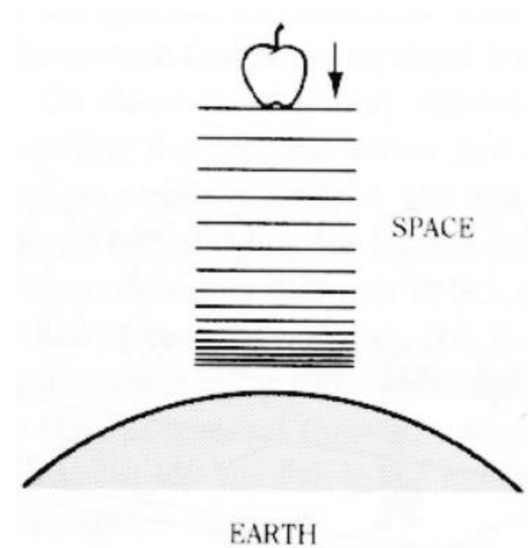
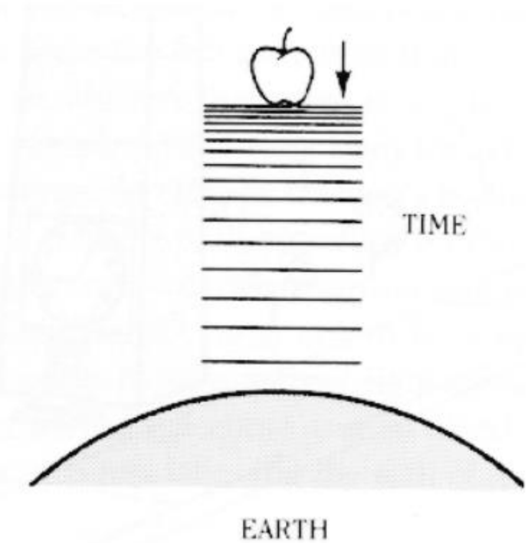


Space contracts near mass and dilates away from it.



Time dilates near mass and contracts away from it.

Gravitational time dilation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - R_S / r}}$$

Δt – dilated time interval,
Δt_0 – original time interval,
R_S – Schwarzschild radius
R – distance from the blackhole centre (singularity)

Schwarzschild radius = radius below which the gravitational attraction between the particles of a body must cause it to undergo irreversible gravitational collapse. This phenomenon is thought to be the final fate of the more massive stars (so called neutron stars).

---

only to remember: this was the time dilation equation from special theory of relativity:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}}$$

Gravitational time dilation

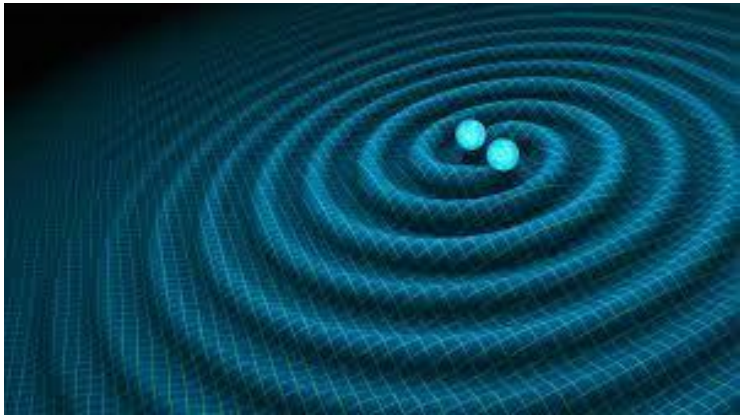
The higher the gravity acceleration, the slower time passes.

movie Interstellar

During one hour spent on Miller's planet (close to the black-hole Gargantua), seven years pass on the Earth.
Captain Cooper and few other crew members make a several hours trip to Miller's planet. During this time their colleague (Romilli) is waiting for them on the Endurance space ship - and he gets older by 23 years…

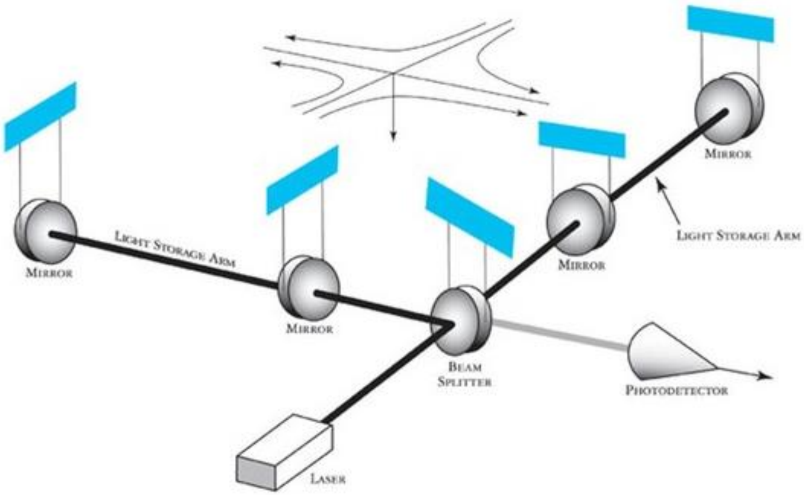
Gravitational Waves

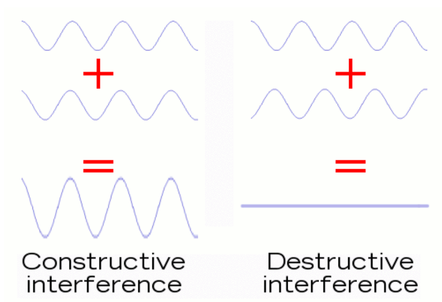
- When a charge accelerates, the electric field surrounding the charge redistributes itself. This change in the electric field produces an electromagnetic wave, which is easily detected. In much the same way, an accelerated mass should also produce gravitational waves.
- Gravitational waves carry energy and momentum, travel at the speed of light, and are characterized by frequency and wavelength.
- As gravitational waves pass through spacetime, they cause small ripples. The stretching and shrinking of the spacetime is on the order of 1 part in 10^{21} even due to a strong gravitational wave source.
- Detection of gravitational waves was one of the greatest challenges in the physics of 20th century.

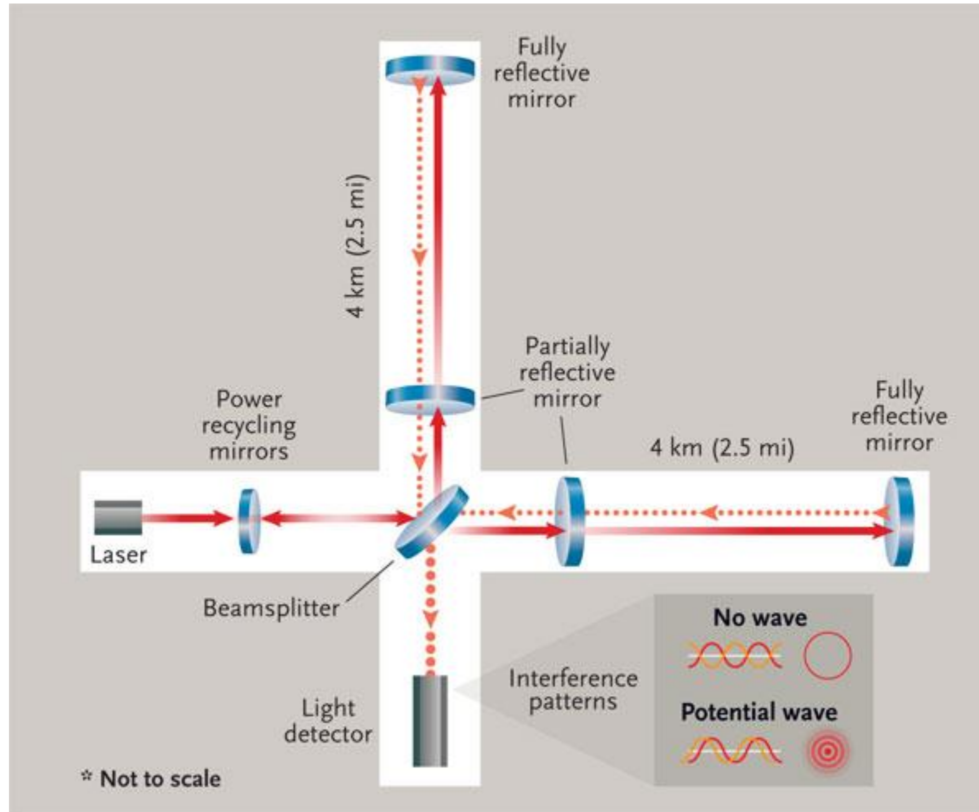


Gravitational Waves

project LIGO = Laser Interferometer Gravitational-Wave Observatory







Gravitational waves cause space itself to stretch in one direction and simultaneously compress in a perpendicular direction. This causes <u>one arm</u> of the interferometer to <u>get longer</u> while <u>the other gets shorter</u>, then vice versa, back and forth as long as the wave is passing. A beam in a shorter arm will return to the beam splitter before the beam in a longer arm – and this causes special effects in the interferometer picture.

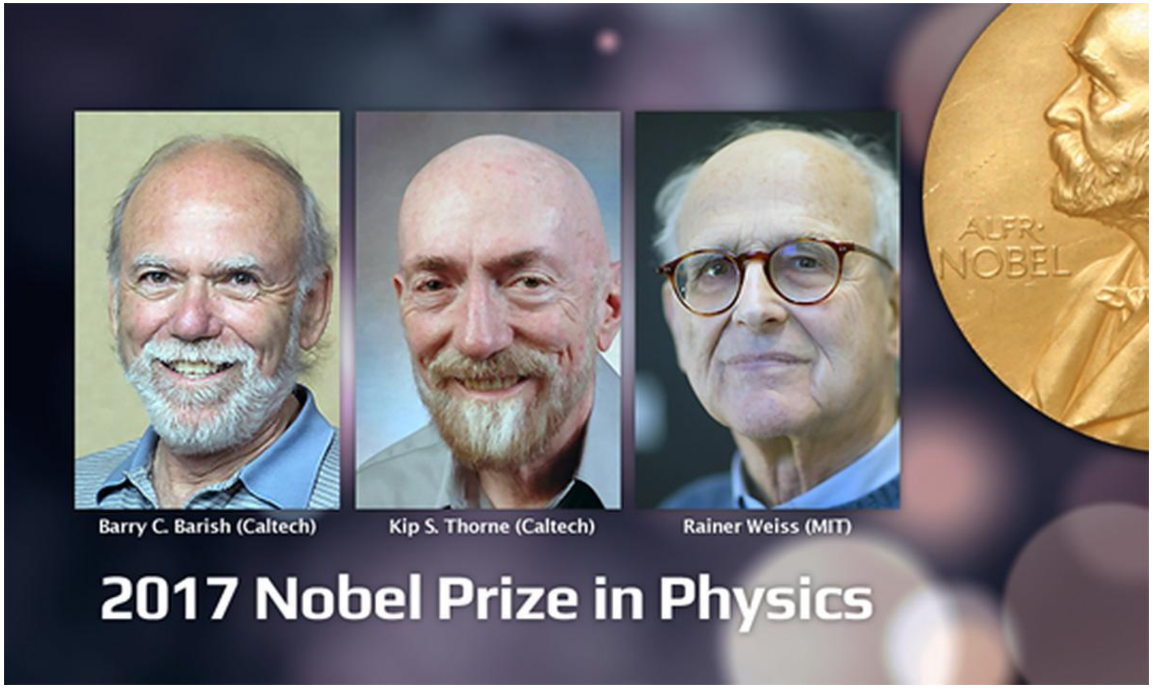
video, explanation: https://ligo.caltech.edu/system/video_items/files/21/Einsteins_messengers_hi_res_Nov_17_MPEG720p.mp4?1447873693

Gravitational Waves

project LIGO = Laser Interferometer Gravitational-Wave Observatory

Laser interferometer of the LIGO project can detect a change in distance between its mirrors 1/10,000th the width of a proton!
Project is composed of two such hight-tech interferometer devices - in Louisiana and Washington (3000 km away).
The first search for gravitational waves began in 2002. Over 1000 researchers from 83 organizations and universities participated in the project.
And finally, on 14th Sept 2015 the first gravitational wave ever has been detected – signal (called GW150914) came from merging of two black holes (each about 30 times more massive than our Sun) approx. 1.3 milliards years ago.
These appearances have been then spotted 50 times.

Gravitational Waves



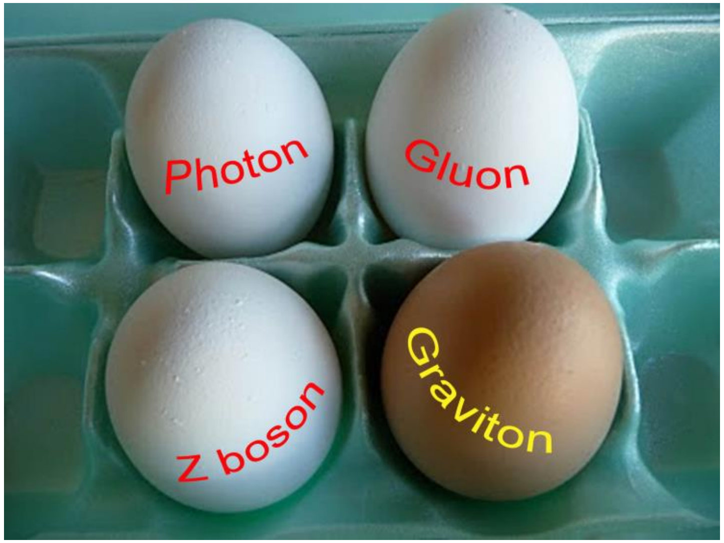
Kip S. Thorne received a golden medal from the Comenius University on 13th October 2021.

But one experiment is still missing
(in the theory of gravity) –
which one?

Graviton

In theories of quantum gravity, the graviton is the hypothetical quantum of gravity, an elementary particle that mediates the force of gravitational interaction.
There is no complete quantum field theory of gravitons due to an outstanding mathematical problem in the general relativity theory.

The graviton remains hypothetical, however, because at the present time, it's impossible to detect.



Summary

- two postulates in special relativity theory – "the principle of relativity" and "the constancy of the speed of light"
- time dilation and length contraction $\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$ $L = L_0 \sqrt{1 - v^2/c^2}$
- spacetime – light cones, curvature, black-holes, etc.
- Lorentz tranformations – were derived with respect to relativity theory
- two postulates of general relativity theory: "principle of equivalence" and "curvature of spacetime due to gravitation"
- gravitational waves pass through spacetime (by speed of light) - and they cause small ripples (stretching and shrinking is on the order of 1 part in 10^{21})
- first detection of gravitational waves – 14^{th} September 2014, project LIGO (Nobel prize in 2017)