Calculation of distant relief effect in gravimetry using ellipsoidal Earth approximation

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Alpine-Carpathian Gravimetry Workshop, Bratislava, 22-23 May, 2024

introduction 1

DRE – a look back

Bullard (1936)	just one calculation point, topography & bathymetry (no ice)
Pick et al. (1960)	limited territory, topography & bathymetry (no ice)
Liu and Liu (1984; 1986)	limited territory, topography & bathymetry ? (no ice)
Kotake and Hagiwara (1987)	limited territory, topography & bathymetry (no ice)
Mikuška et al. (2006)	global, 5' DEM, topography & bathymetry (no ice)
Mikuška et al. (2011, unpub**lished**)	global, 5' DEM, ice
Hirt et al. (2019)	global, up to 3'' DEM, topography (no bathymetry, no ice)

However, all of the above quoted estimates were based on spherical Earth approximation.

Ellipsoidal approach emerged

Sjöberg (2004) introduced ellipsoidal corrections to the topographic geoid effects.

Novák and Grafarend (2005) calculated the gravitational potential, that was generated by the topographical masses above GRS80, as well as its vertical gradient (i.e., an effect on measured gravity), over a test area in the Canadian Rocky Mountains. They did not consider bathymetry.

Hinze et al. (2005) called for the use of ellipsoidal heights and for ellipsoidal Earth approximation in general.

Vajda et al. (2006), among others, discussed in detail the geophysical indirect effect (GIE), which consists of two terms, namely GIE1 or normal-gravity term, and GIE2 or mass term.

Vajda & Pánisová (2007) continue in the discussion of GIE. This discussion is important, because, when confronting our current ellipsoidal DRE calculations with the older spherical ones, we will be dealing with the distant component of the mentioned second GIE term, namely DGIE2.

Vajda et al. (2008) explained that the allowing for topography and bathymetry should be global and based on the reference ellipsoid.

Claessens and Hirt (2013) stated among others, that in the spectral domain gravity modeling, the traditional spherical approximation has become insufficient.

Grombein et al. (2016) applied the earlier introduced rock-water-ice (RWI) approach on an ellipsoidal reference surface.

introduction 2

The motivation for our recent ellipsoidal DRE calculations was primarily disconnecting from dependence on the spherical Earth approximation and accepting the fact that "the Earth is to a much higher degree of accuracy modeled by an oblate ellipsoid of revolution" (Claessens and Hirt, 2013, p. 5992, ☺).

Using polyhedral bodies of global extent with triangular sides based on the maximally regular triangular net on ellipsoid we were capable of calculating the gravitational effects of all the three basic types of environments, namely R, W and I, considering their respective (constant) densities. Our method could be briefly characterized as a direct and a space-domain one. All this was based on the achievements of V. Pohánka from the period 1988- 2008, more detailed quotations will be given later.

One of our recent goals was to calculate quantities which would in principle be commeasurable with those calculated by our 2006 approach and to find out to what extent the two kinds of outputs differ.

The sources of primary information about the basic topographical environments and their surfaces were the data of Schaffer et al. (2016) and Tozer et al. (2019).

Under the Earth relief we will further understand the solid rock masses whose upper boundary differs from the reference Earth ellipsoid (i.e. the rock masses located above the ellipsoid together with the missing rock masses below the ellipsoid), plus the masses of ocean waters and ice bodies.

method 1

Our calculation of the Earth Relief Effect (ERE) was based on the method presented in Pohánka (1988). Two aspects of this method are of special importance:

* The gravitational effect of the body with constant density can be calculated as a surface integral over the boundary of the body;
* This surface integral can be calculated with reasonable approximation by replacing its actual surface by a surface of certain polyhedral body – the simplest case being that all polygons were triangles as then the sides of the body are automatically planar and uniquely defined by the vertices of those triangles.

According to its adopted definition, the ERE will consist of the following three contributions: that of the masses of solid rock above the ellipsoid (and lack of them below), and the masses of ocean waters and ice bodies.

In accordance with Schaffer et al. (2016) we have considered three boundary surfaces, extending over all the globe, namely: * upper boundary of the solid rock (bedrock topography, BT), * upper boundary of the ocean waters (ice base topography, IB), and * upper boundary of the ice masses (surface elevation, SE). Note that the order of boundaries corresponds to the decreasing density below each boundary.

Calculating the contribution of the solid rock. Let us consider some reference Earth density model consisting of layers with constant density. The upper boundary of the uppermost layer has to be identical with reference Earth ellipsoid RE, while the lower boundary LB of this layer should lie everywhere below the surface of the solid rock BT. Then the contribution of the uppermost layer of that reference Earth will be equal to the difference of the surface integrals over RE and LB, respectively. The same time, the contribution of the (uppermost layer) of the real solid Earth, with the same density, will be equal to the difference of the surface integrals over BT and LB, respectively.

method 2

Thus the resulting contribution to the ERE will be equal to the difference of the surface integrals over BT and RE, and the location and shape of the surface LB will have no effect.

Our notation is slightly different from that of Schaffer et al. (2016) which is due to computational reasons. We require that the lower boundary of the ocean water is identical with BT and the lower boundary of the ice masses is identical with IB, irrespective of the existence or nonexistence of the water and ice masses at the actual place. Of course, if there is no water, then BT = IB and if there is no ice then IB = SE.

There are also differences in the actual data, namely that in Schaffer et al (2016) the height IB is under continents as well as in the absence of ice equal to zero, whereas we have in this case BT = IB = SE.

The next step was to approximate all real boundaries (including the surface of the reference ellipsoid) by polyhedra with triangular sides. In the case of the Earth ellipsoid this was done according to the method described in articles Pohánka (2006; 2007; 2008). The maximally regular triangular net (on the spherical and ellipsoidal surface) is uniquely defined for each degree of the net, up to a rotation with respect to the polar axis. While the starting net of degree 0 has 12 vertices and 20 sides, the net of degree n ($n \geq 1$) has $10*4^n+2$ vertices and $20*4^n$ sides.

By choosing the value of the degree we define the (average) dimension of the triangle side on the surface of the Earth ellipsoid. In majority of our calculations we used n = 10, thus 10485762 vertices and 20971520 sides. Then the average area of a triangle is 24.3 km^2 and average length of the triangle sides is 7.5 km.

The problem of coastal triangles. We divide each coastal triangle into three smaller triangles and the calculation proceeds as before, but separately for each of these three smaller triangles. The reason is to preserve the (local) horizontality of the upper boundary of the ocean water surface.

method 3

Calculation the contributions of distant masses to the ERE. We divide all triangles into three categories: distant, boundary and near, with regard to the cutting distance. The criterion is the distance of the base vertices of a triangle (i.e. the vertices lying at the ellipsoid) from the location of the base point of the calculation point (again lying at the ellipsoid). The distance itself is calculated in 3-D space and not along the ellipsoid – if we talk about distances like 167 km, the differences are negligible.

There are two variants of calculation of the contribution of the distant masses.

In the first variant, there is simply considered only the contribution of the distant triangles. This simplifies the calculation, but it has a significant drawback, namely the boundary of the layer is not closed – there are missing the vertical parts of such a boundary.

This drawback was removed in the second variant of calculation – there are considered also the boundary triangles, but these are modified as follows: the vertices of the upper boundary nearest to the calculation point are changed to be identical with the corresponding vertices of the lower boundary. In this way the upper and lower boundary represent closed boundary of a body (layer).

We used constant densities 2.670, 1.023 and 0.917 g/cm^3 (R, W and I, respectively; Schaffer et al., 2016).

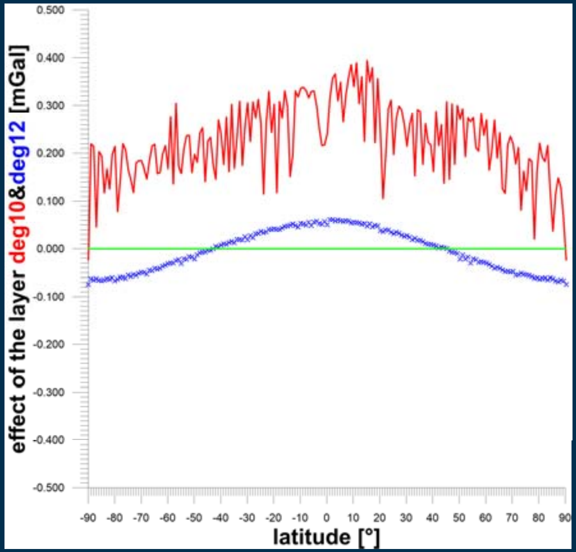
The input data used:
RTopo-2.0.1_30sec_bedrock_topography.nc (BT)
RTopo-2.0.1_30sec_ice_base_topography.nc (IB)
RTopo-2.0.1_30sec_surface_elevation.nc (SE)
RTopo-2.0.1_30sec_aux.nc mask (0 - sea, 1 - ice on the continent, 2 - ice on the sea, 3 - continent)
geoid EGM96 WW15MGH.GRD

Interpolation was performed using the method presented in Pohánka (2005).

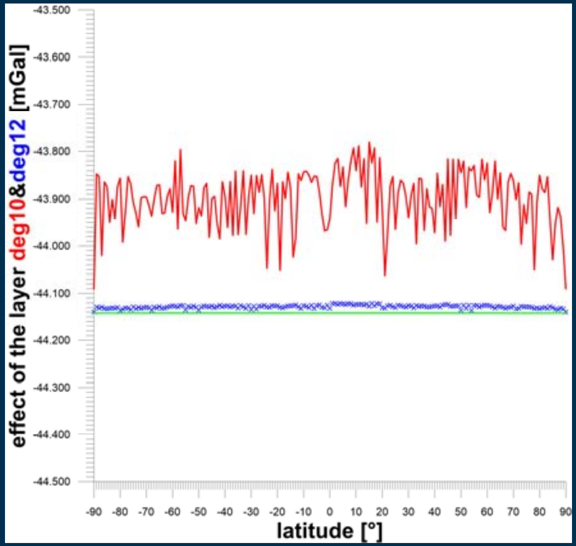
ellipsoidal layer *1 calculation points at the layer bottom*

This is the first among the achieved results that we would like to present today. We calculated the gravitational effects of a synthetic model in the form of an ellipsoidal layer with a constant thickness of 400 m and a constant density of 2.67 g/cm^3, residing on the surface of the WGS84 ellipsoid.

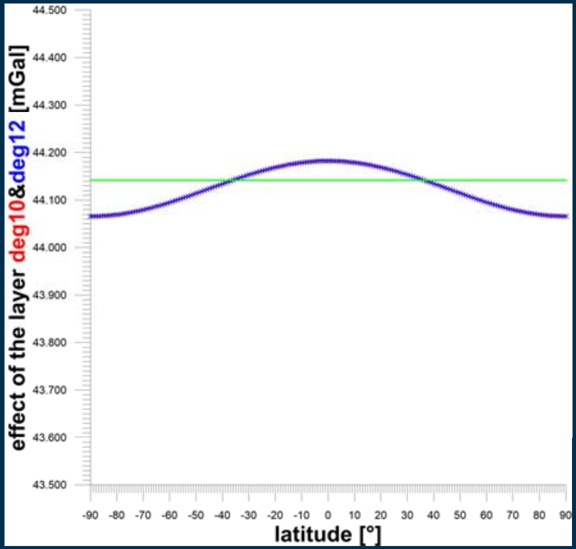
The calculation points were arranged along a meridian and first they were located at the level of the layer's bottom.



$h_P = 0$ 0-180° (complete)



$h_P = 0$ 0-1.499444° (near)



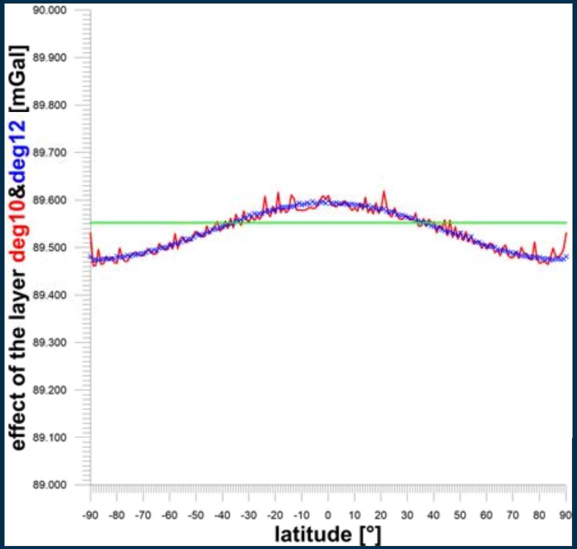
$h_P = 0$ 1.499444-180° (distant)

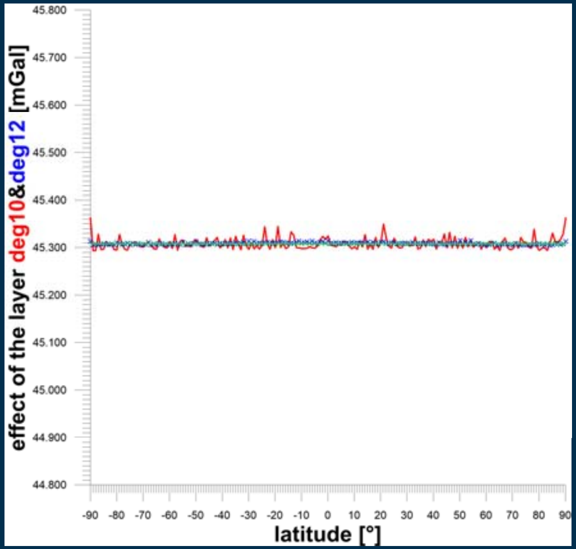
Green lines represent gravitational effects of the equivalent (truncated) spherical layers with values 0.000, -44.142 and +44.142 mGal, respectively.
Spherical values were calculated according to our 2006 formula.

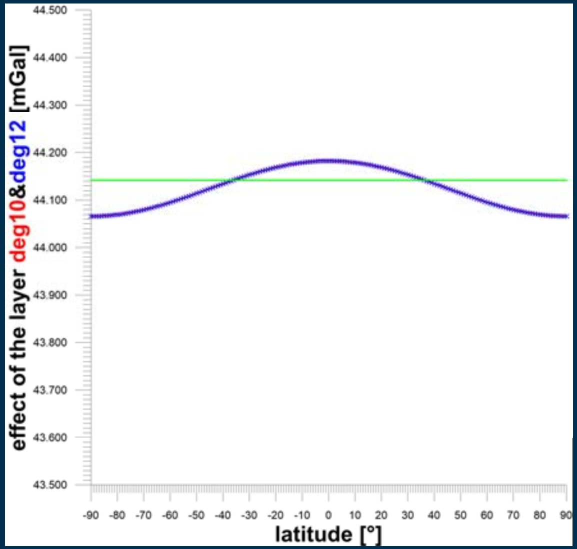
ellipsoidal layer *2 calculation points at the layer surface*

The arrangement of the calculation points as well as the model properties were identical with the previous example. The step was 1° along the meridian. Now the points were located at the layer surface.

Oscillations of the effects of the complete and near layers for the degree 10 of the triangular net now have significantly lower amplitudes.







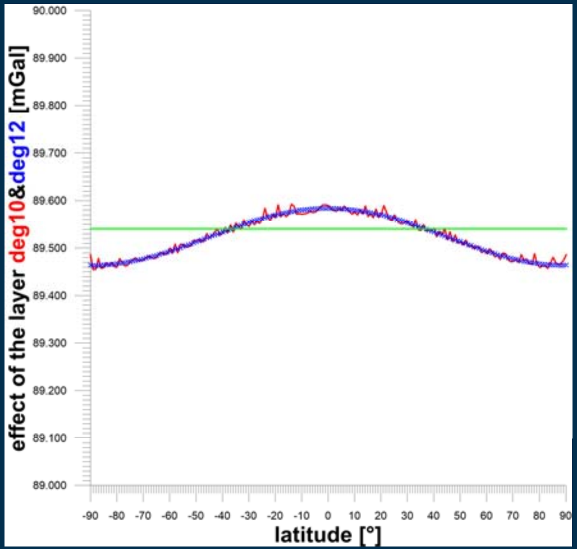
h_P = 400 0-180° (complete) h_P = 400 0-1.499444° (near) h_P = 400 1.499444-180° (distant)

Green lines represent gravitational effects of the equivalent (truncated) spherical layers 89.552, 45.308 and 44.244 mGal, respectively.
Spherical values were calculated according to our 2006 formula.

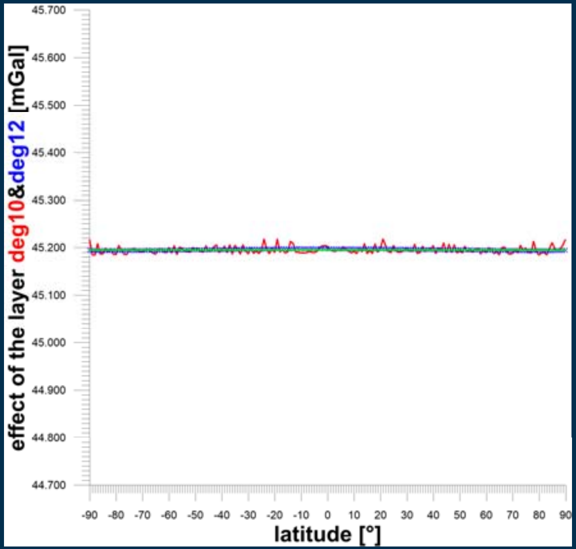
ellipsoidal layer *3 calculation points 400 m above the layer surface*

The arrangement of the calculation points as well as the model properties were identical with the previous examples. Now the calculation points were located 400 meters above the layer surface.

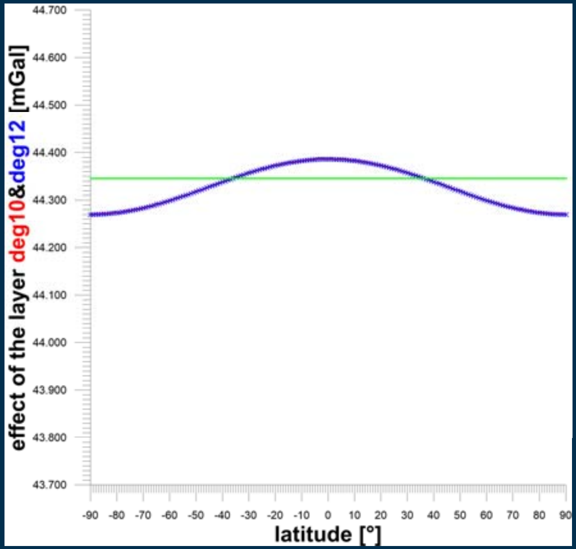
Oscillations of the effects of the complete and near layers for the degree 10 of the triangular net now have the lowest amplitudes. Red curves are hidden behind the blue symbols in all graphs of the effects of the distant models.



hP = 800   0-180° (complete)



hP = 800   0-1.499444° (near)



hP = 800   1.499444-180° (distant)

Green lines represent gravitational effects of the equivalent (truncated) spherical layers 89.541, 45.195 and 44.346 mGal, respectively.
Spherical values were calculated according to our 2006 formula.

ellipsoidal layer *4 brief summary*

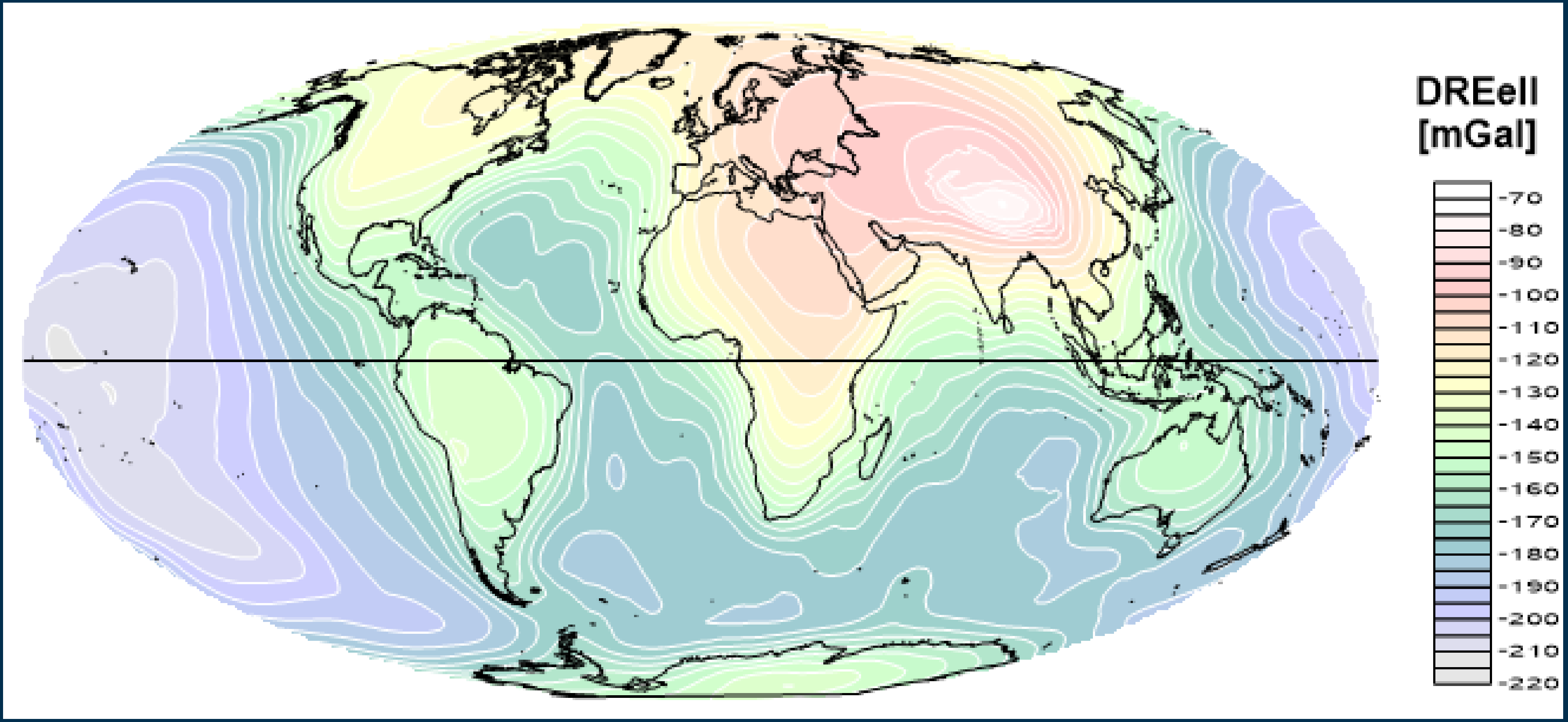
The effect of the Earth ellipticity is visible for the distant zones and for the complete layers. However, numerically, the changes due to ellipticity are well within the first hundreds of microgals. On the other hand, the effect of ellipticity is practically indiscernible within the near zones.

The impact of the size of the triangles can be considered more serious for the complete layers and for the near zones, especially for the bottom calculation points, and for the degree 10 of the triangles (average area of a triangle is about 24.3 km^2 and average length of the triangle sides is approximately 7.5 km). For the degree 12 (average area of a triangle is about 1.5 km^2 and average length of the triangle sides is approximately 1.9 km), the triangle size impact is very moderate. Last but not least, for the distant zones, the triangle size impact is negligible even for the degree 10 of the triangles. The last mentioned finding encouraged us to calculate larger datasets using the faster 10-degree triangles and, the same time, while not having to expect serious damages to the outputs.

Incidentally, 400 m can be considered a common thickness of topographical masses. For example, the average (orthometric) height of all 318133 points in the Gravimetric Database of the Slovak Republic is about 446 m.

The findings which we present in this section certainly contribute to better understanding to the relationships between our new "ellipsoidal" outputs in the 1°x1° global grid (which will follow) and their older "spherical" counterparts.

1°x1° global net of calculating points 1 DREell

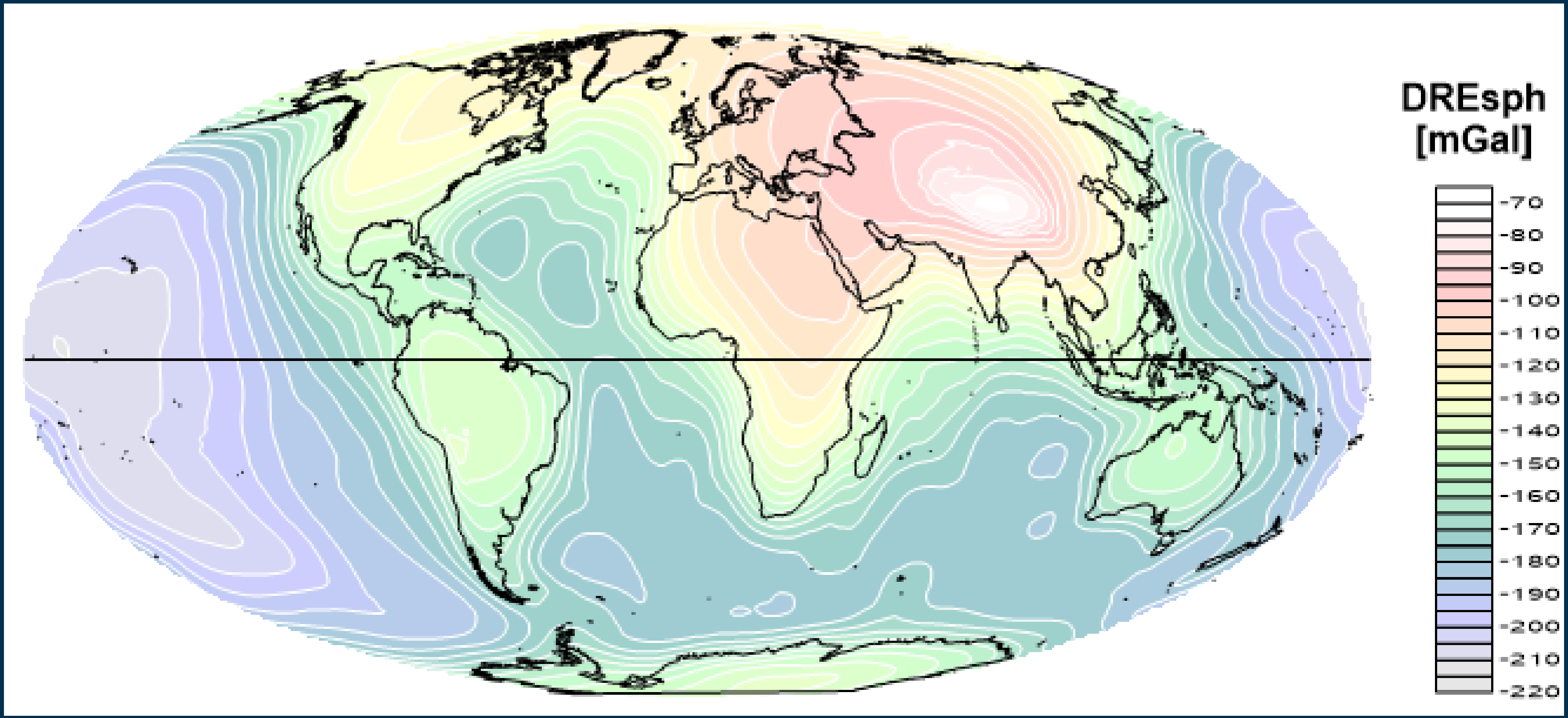


min -210.5 max -73.9 range 136.7

(DTE+DBE)ell	recently	degree 10	+2.67 & -2.67
DWEell	recently	degree 10	1.023
DIEell	recently	degree 10	0.917

DREell = (DTE+DBE)ell + DWEell + DIEell

1°x1° global net of calculating points *2 DREsph*

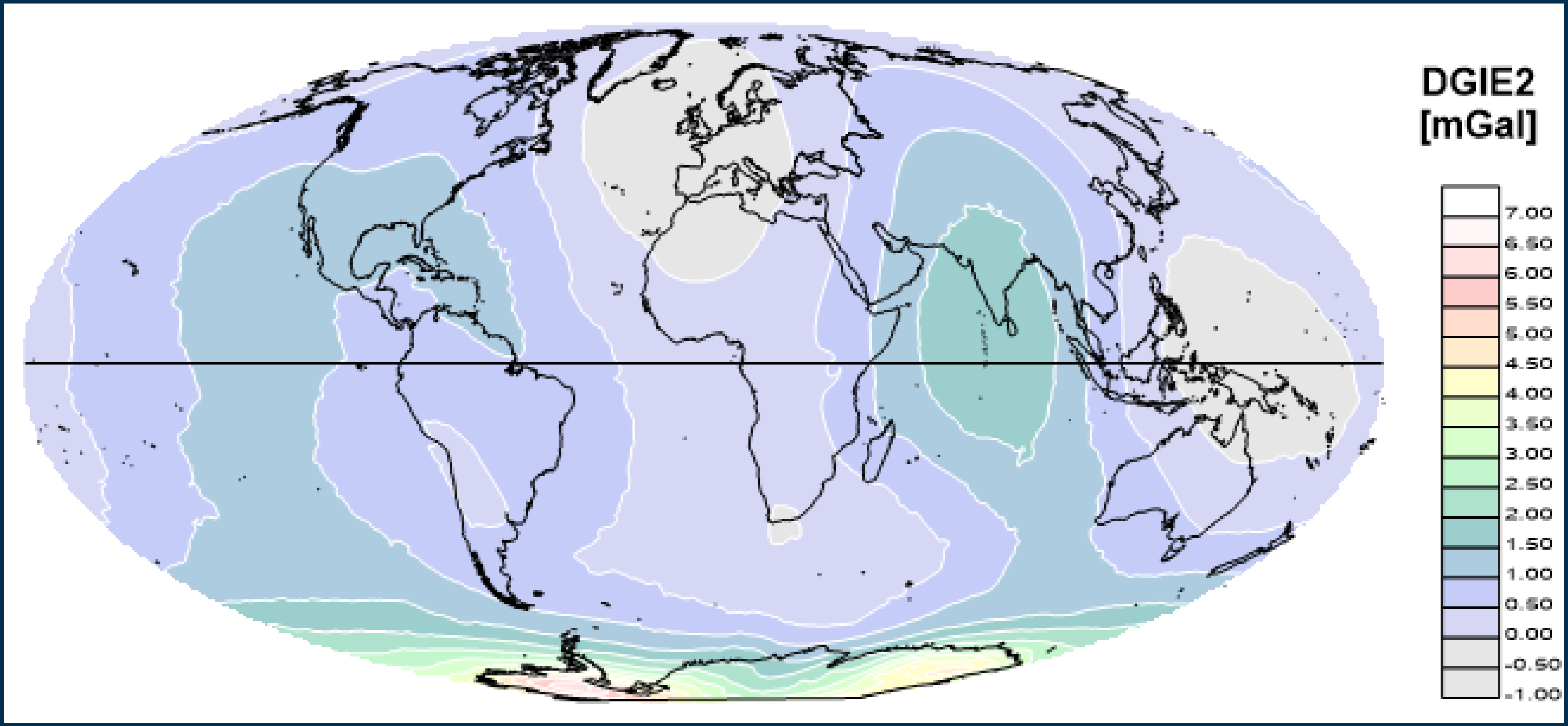


min -210.2 max -72.4 range 137.8

DTEsph	2006	etopo5bed	2.670
DWEsph	2006	etopo5ice	1.023
DIEsph	2011	etopo5bed&ice	0.917

DREsph = (DTE+DBE)sph + DWEsph + DIEsph

1°x1° global net of calculating points *3 DGIE2*



min -0.5 max +6.7 range 7.2

DGIE2 = DREsph - DREell

1°x1° global net of calculating points *4 concluding remarks*

A brief look into the recent past would tell us that e.g. Hackney & Featherstone (2003) did not mention the GIE distant component at all. On the other hand, e.g. Vajda & Pánisová (2007) called up that „integration must be carried out over the whole globe".

To our knowledge our contribution represents the first estimate of the Distant Relief Effect in an ellipsoidal Earth approximation, allowing for geoid, and thus considering the masses of topography, ocean waters and ice, as well as the calculating points, in their real positions. Calculating points in the recent calculations were „formally identical" with the 2006 ones.

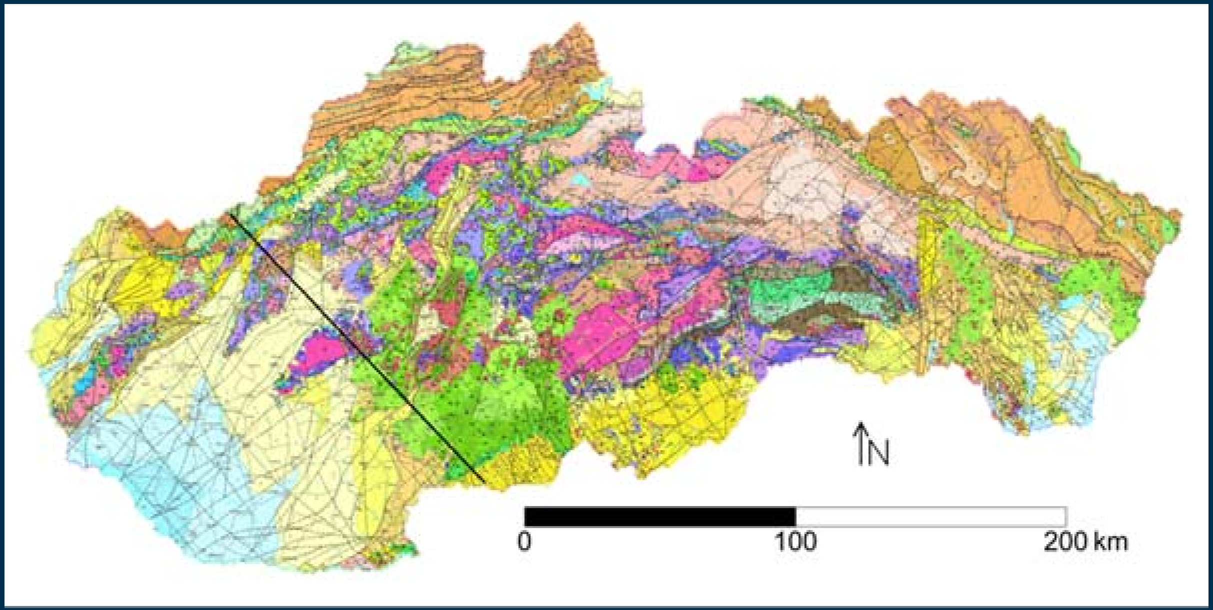
However, the estimated DREell and DREsph do not differ from each other as much as might have been expected. In other words, the range of the DGIE2 is relatively small. Also the impact of the Earth ellipticity has been different from what we originally anticipated.

We think we cannot say that we have calculated some definitive values of the DRE. Rather, all the figures should be considered estimates.

In the future calculations, there can be expected some impact of the size of the triangles (i.e. the degree of the triangular net) on the calculated outputs. Another issue will be the detailedness and accuracy of the input DEMs.

Of course there can be other distant effects except of those caused by the distant Earth relief.

gravity profile across the state territory *1 introduction*



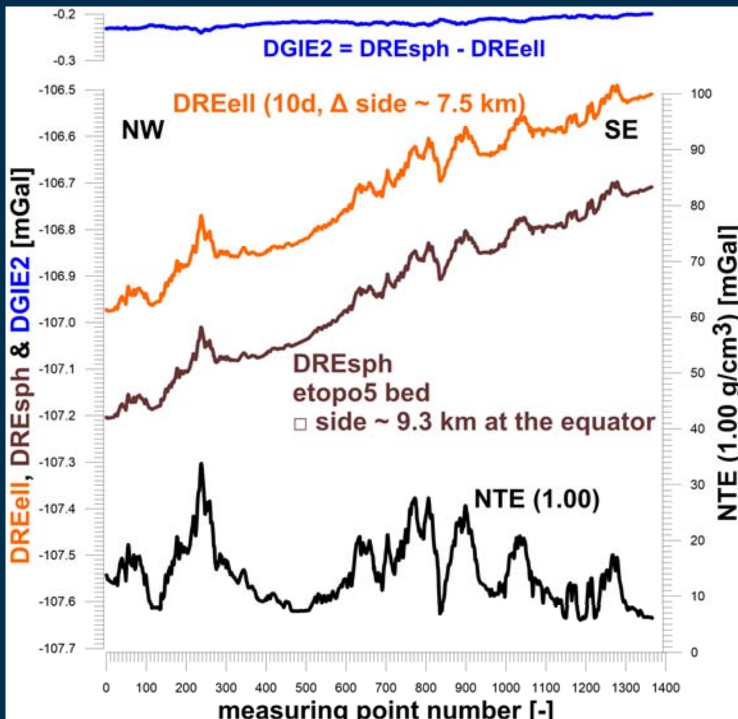
total length	136.5 km
number of points	1366
average step	100 m
elevation range	730 m
geoid over ellipsoid ca.	43.5 m
DGIE2 range	0.041 mGal
crossing many geological units	

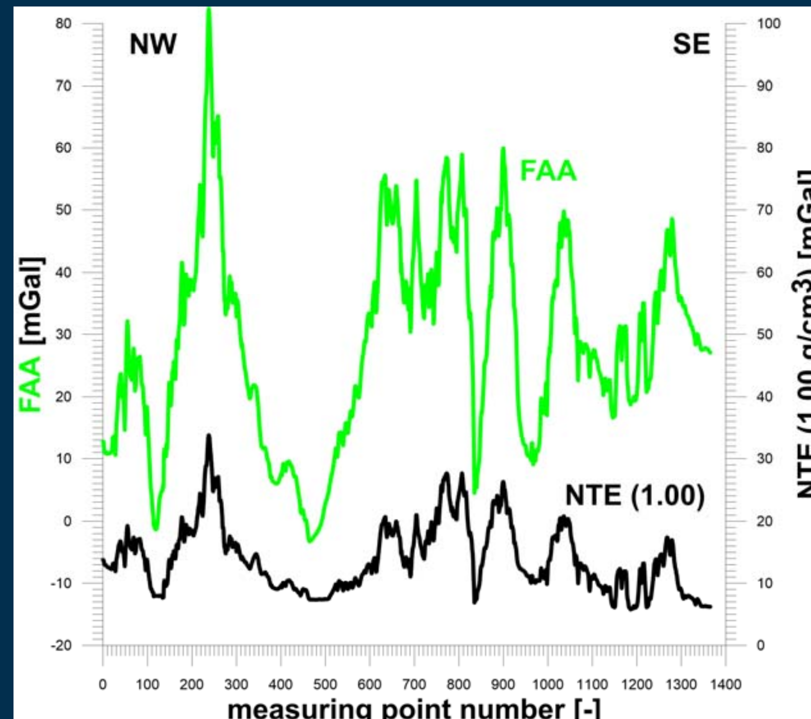
This is an example of DRE calculations along a relatively long and, the same time, relatively detailed gravity profile, i.e. detailed regional? ☺. It is evident that **this** is a significantly different application, when compared with the calculations in a 1°x1° global network which we saw in the previous example.

In the past we already presented various processing & interpretative techniques applied to the profile data in question which originally come from the unpublished report of Szalaiová et al. (2005), Geocomplex, a.s., Bratislava.

Today we are going to present and analyze the values of Distant Relief Effects, with the stress put on the quantity DREell, calculated in the ellipsoidal approximation.

gravity profile across the state territory *2 proportionality*





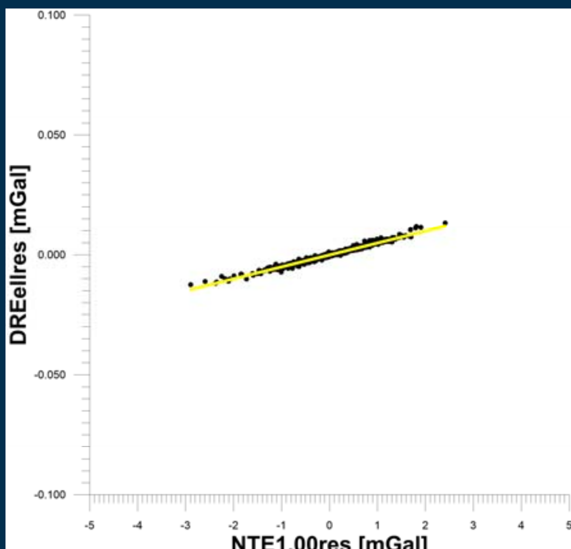
There are remarkable general trends in both DREell and DREsph, which would only be seen if we had just a global 1°x1° calculations (about two calculating points for the whole profile?!). But there are also evident manifestations of local proportionality of both quantities DREell & DREsph vs NTE(1.00), similar to the case of the FAA vs NTE(1.00). Is this an evidence of a local expression of DRE in mountainous areas?

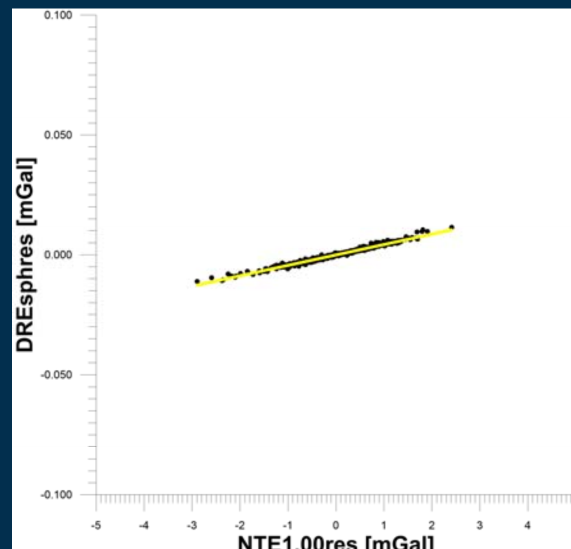
As if the influence of DRE caused that the local topographic masses were „denser“, or, in other words, as if the gravitational effect of the near topographic masses (NTE) were higher. In a loose analogy to magnetometry (the term "magnetization"), in 2014 we proposed the working term "gravitization" to describe such a kind of (a local) influence of gravitational effects of distant masses (here the DRE).

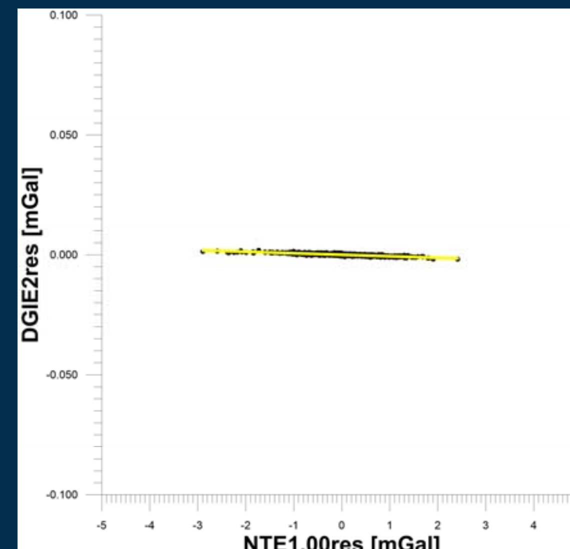
gravity profile across the state territory *3 residuals*

Introducing residuals. Residuals, as well as the method of the rock density estimates based on them were recently described in Zahorec et al. (2023). The parameters which are directly used for the purpose in question are the slopes of the approximation lines in scatter plots FAAres vs NTE(1.00)res.

Instead of FAAres we now calculated residuals DREellres, DREsphres and DGIE2res, and compared them with NTE(1.00)res. The results are interesting.







*DREellres ≈ 0.0050*NTE(1.00)res - 0.0* *DREsphres ≈ 0.0044*NTE(1.00)res - 0.0* *DGIE2res ≈ -0.0006*NTE(1.00)res - 0.0*

Slopes of the approximation lines (i.e. the apparent density indicators) in the graphs differ by -0.0006. Slightly higher value of the DREellres approx. line slope may indicate that the new ellipsoidal calculations were more detailed. The slope of the DGIE2 vs NTE(1.00) approx. line is identical with the difference of the first two slopes (this is of course a trivial check - just to illustrate that it works as it has to).

brief conclusions *& thanks*

In recapitulation, we would like to mention that
* In our contribution we present the first use of the maximally regular triangular net on ellipsoid for the global and regional/detailed DRE calculations, in other words – in an ellipsoidal approximation – which is advantageous in some aspects;
* There were estimated gravitational effects of an ellipsoidal layer, complete and truncated, i.e. near and distant, which proved to be quite instructive;
* We thoroughly compared the newly calculated „ellipsoidal“ values with the older „spherical“ ones;
* As an originally unintended „by-product“ of the mentioned comparisons we obtained first estimates of the DGIE2, according to its definition similar to the one of Vajda a Pánisová (2007);
* Although the ellipsoidal approximation was not easy to realize, its implementation definitely makes things more clear and correct. We believe that it will gradually replace the spherical approximation.

Finally we would like to express our gratitude to
* Xiong Li for informing us about relevant Chinese authors and providing us with copies of their contributions to calculations of spherical quantities;
* Ivan Marušiak for programming and contributions to the estimations both of „spherical“ quantities which were necessary for the comparisons and for the means used here for the analysis of the profile data, within approximately the past two decades;
* To the Slovak Research and Development Agency, namely to its finished project APVV-0827-12 as well as to the ongoing one, APVV-19-0150, for financial support.