ALPINE-CARPATHIAN GRAVIMETRY WORKSHOP

MAY 22-23 Bratislava

Qualitative analysis of higher derivatives ratios of Bouguer anomaly map from pan-Alpine region

Roman Pašteka[1], Miroslav Bielik[1,2], Pavol Zahorec[2], Juraj Papčo[3], Roland Karcol[1], Ema Nogová[1,2]

[1]Department of Engineering Geology, Hydrogeology and Applied Geophysics, Comenius University in Bratislava

[2]Earth Science Institute, Slovak Academy of Sciences, Bratislava

[3]Department of Global Geodesy and Geoinformatics, Slovak University of Technology, Bratislava

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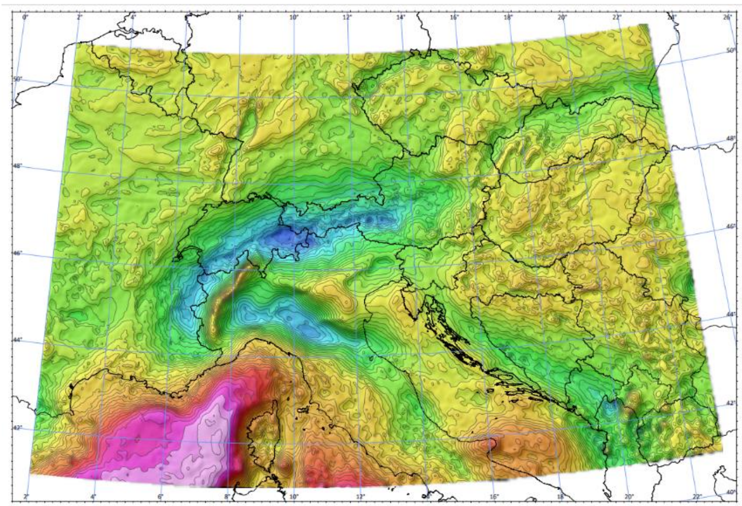
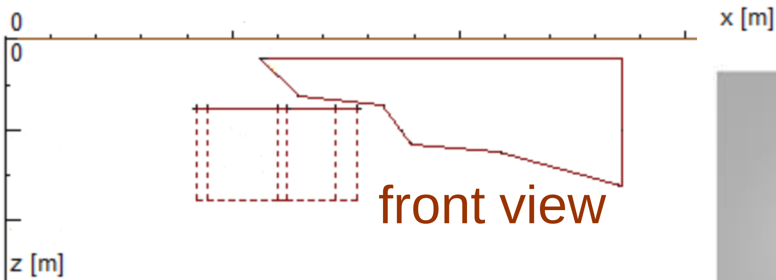
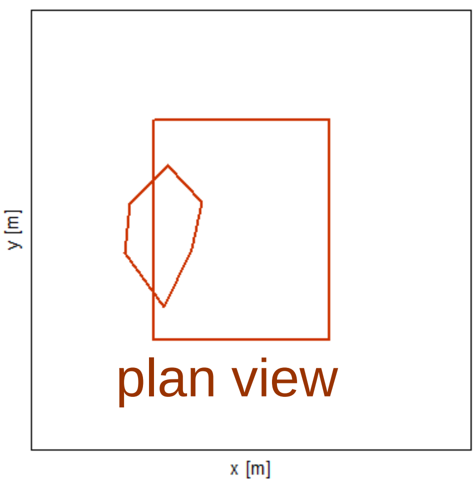
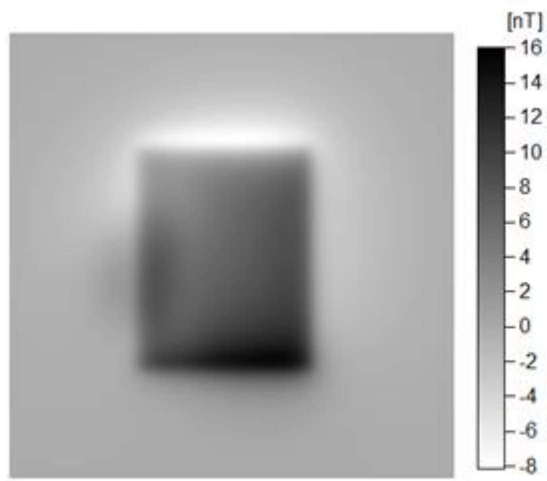


Motivation:

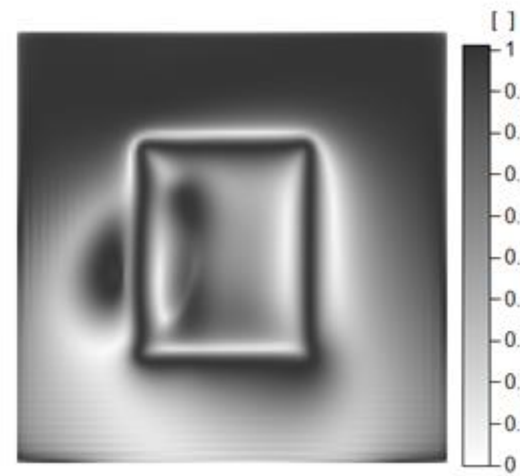
- use of **higher derivatives** (**and their ratios**) as edge mappers for better delineation of interpreted structures







original field (TMI)

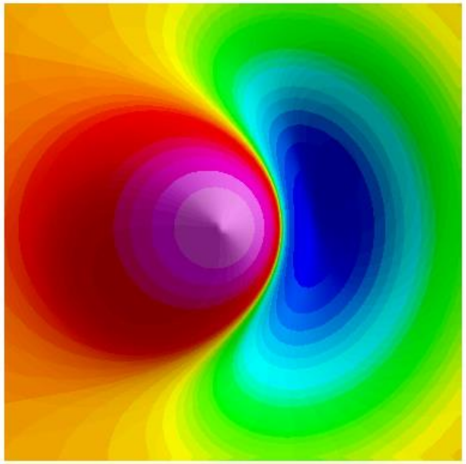


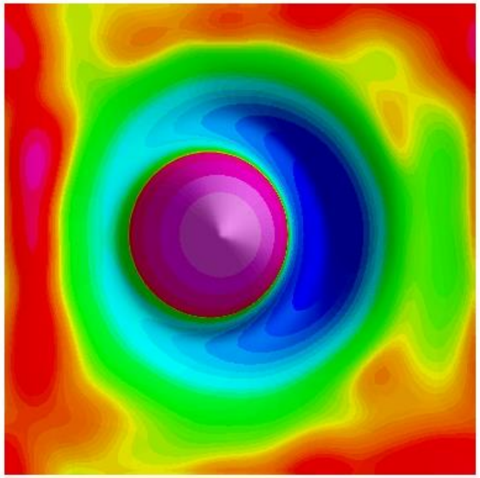
so called theta-derivative transformation
(exact description will be given later)

... but:

- what to do with the strong **noise/errors amplification** during higher derivatives evaluation?
- in other words – evaluation of numerical derivatives is **an instable operation** and we have to find ways for its stabilization

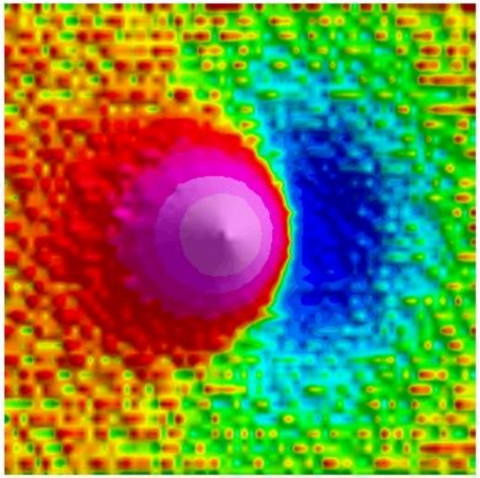
synthetic magnetic field of a sphere (without noise)

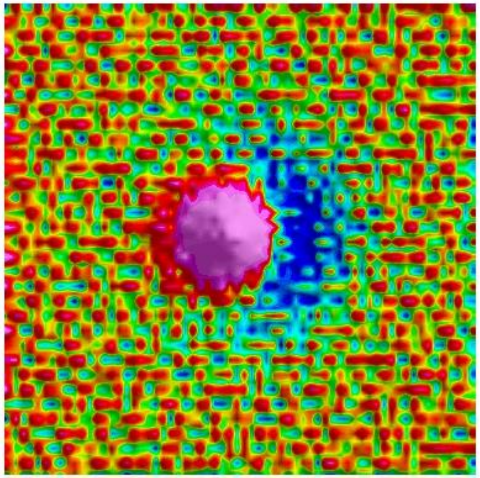




numerically evaluated vertical derivative

synthetic magnetic field of a sphere (with 5% Gaussian noise)





numerically evaluated vertical derivative (intensive noise amplification – up to 30-40%)

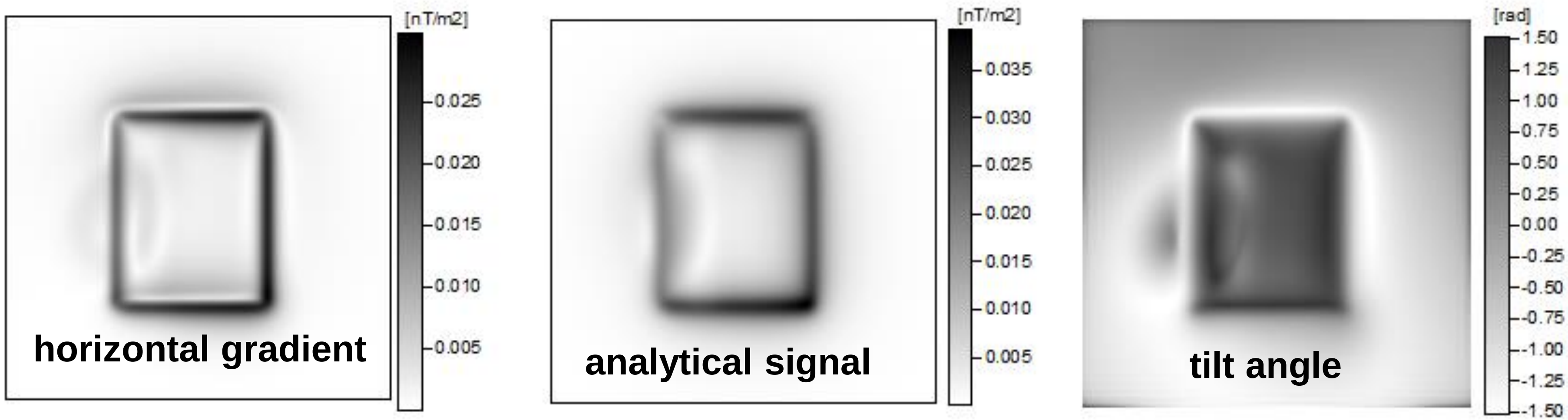
Qualitative analysis of higher derivatives ratios of Bouguer anomaly map from pan-Alpine region

Content of the presentation:

- motivation
- edge detectors (based on derivatives ratios)
- Tikhonov regularization in numerical derivatives evaluation
- derivative edge detectors: Bouguer anomaly map from the pan-Alpine region
- conclusions

edge mappers (used in potential field geophysics):

- majority of them is based on derivatives ratios



- derivatives of the input field: $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$,
- horizontal gradient: $HG = \sqrt{(\partial f/\partial x)^2 + (\partial f/\partial y)^2}$,
- analytical signal: $AS = \sqrt{(\partial f/\partial x)^2 + (\partial f/\partial y)^2 + (\partial f/\partial z)^2}$,
- tilt derivative: $tilt = \mathrm{arctg}\dfrac{\partial f/\partial z}{HG}$, (Miller and Singh, 1994; Verduzco et al., 2004)
- theta derivative: $\cos(\theta) = \dfrac{HG}{AS}$, (Wijns et al., 2005)
- TDX derivative: $TDX = \mathrm{arctg}\dfrac{HG}{\partial f/\partial z}$, (Cooper and Cowan, 2006)

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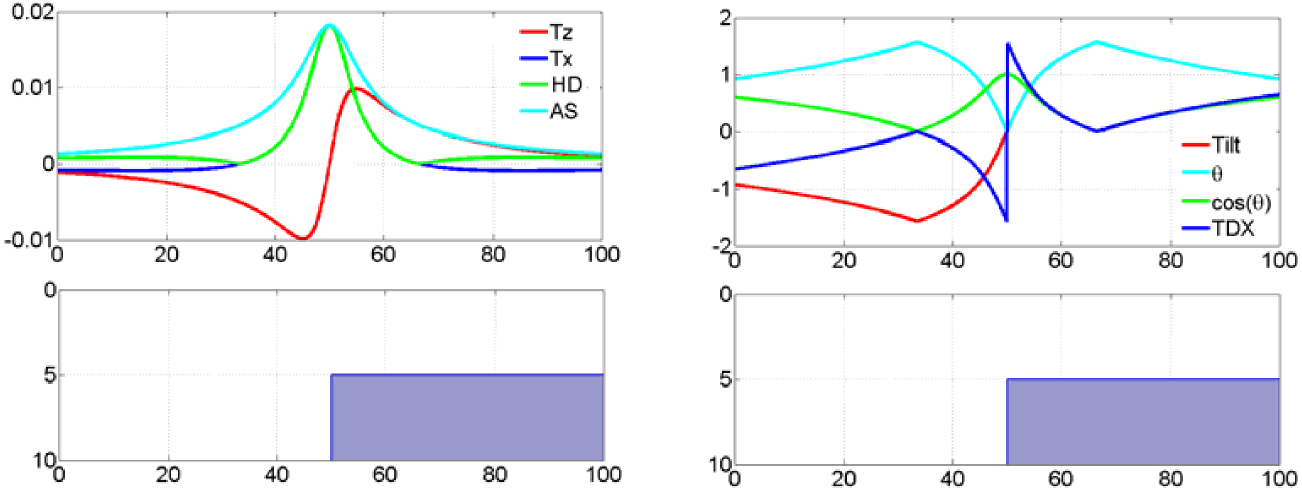

(from Fairhead and Williams, 2006)

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edge mappers (used in potential field geophysics):

- majority of them is based on derivatives ratios (newer modifications)

Ferreira et al. (2013)
(ratio of HG vert. deriv. to horiz. deriv. of HG)

$$\mathrm{TAHG} = \tan^{-1}\left(\frac{\frac{\partial \mathrm{HG}}{\partial z}}{\sqrt{\left(\frac{\partial \mathrm{HG}}{\partial x}\right)^2 + \left(\frac{\partial \mathrm{HG}}{\partial y}\right)^2}}\right).$$

Arisoy and Dikmen (2013)
(ratio of vert. deriv. to horiz. deriv. of AS)

$$\mathrm{ETilt} = \tan^{-1}\left(K\frac{\frac{\partial f}{\partial z}}{\sqrt{\left(\frac{\partial AS_0}{\partial x}\right)^2 + \left(\frac{\partial AS_0}{\partial y}\right)^2}}\right)$$

Nasuti and Nasuti (2018)
(ratio of 2. vert. deriv. to horiz. deriv. of AS_n)

$$N_n\mathrm{Tilt} = \tan^{-1}\left(k^n\frac{\frac{\partial^2 f}{\partial z^2}}{\sqrt{\left(\frac{\partial \mathrm{AS}_n}{\partial x}\right)^2 + \left(\frac{\partial \mathrm{AS}_n}{\partial y}\right)^2}}\right).$$

... and many others.

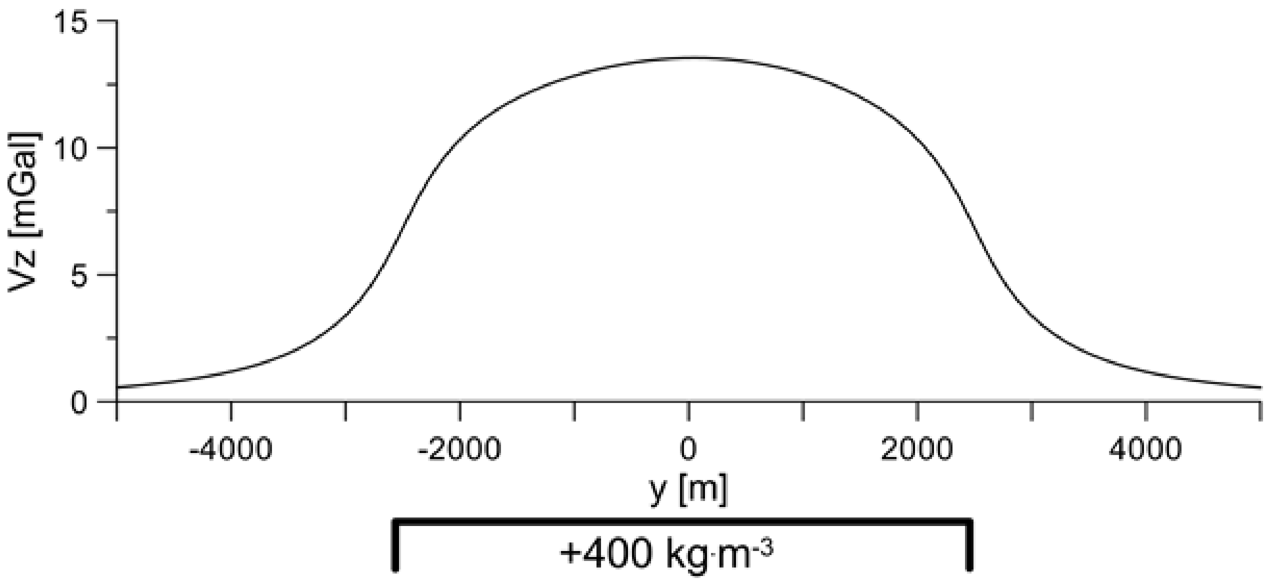
Excellent review paper from Núñez-Demarco et al. (2020, Surveys in Geophysics).

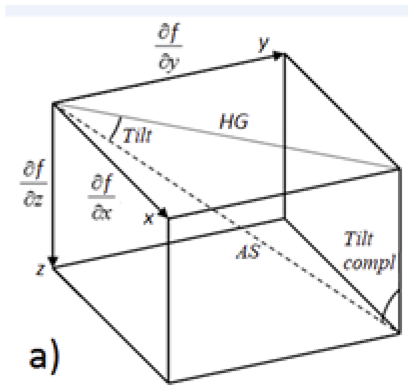
In this study - we have tried 2 new transformations (TPC and MHGA)

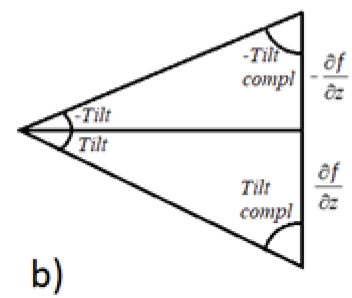
1) tilt angle plus and complementary angle = TPC

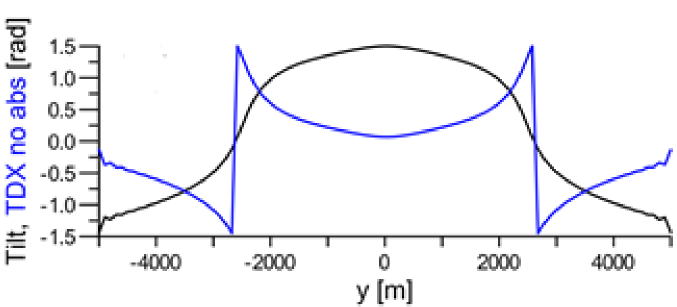
(reaches values +π/2 over sources and -π/2 outside them)

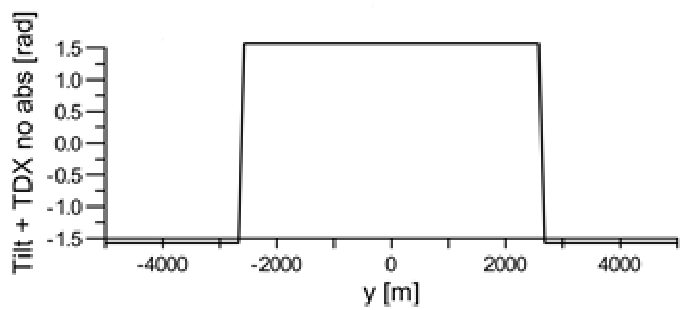
- tilt derivative: $\text{tilt} = \text{arctg}\dfrac{\partial f / \partial z}{HG}$,
- TDX derivative: $\text{TDX} = \text{arctg}\dfrac{HG}{\partial f / \partial z}$,







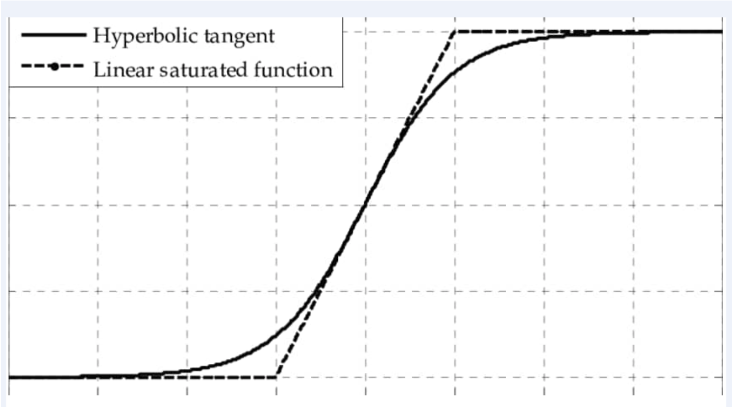




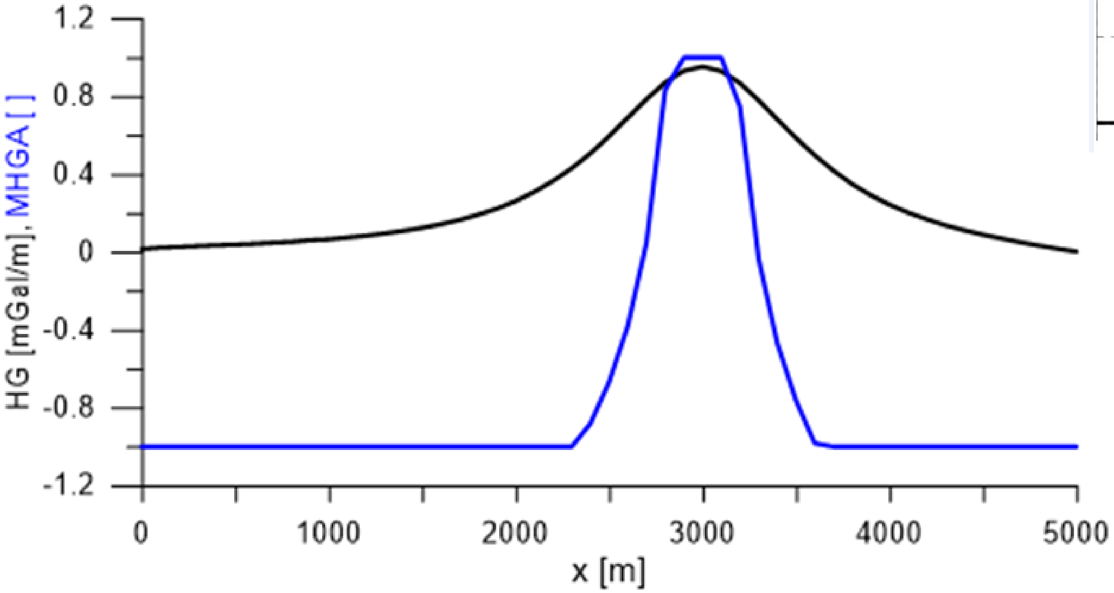
2) next (new) transformation – MHGA (Hanbing et al., submited):

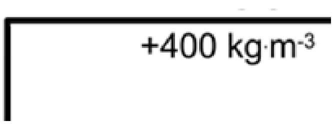
(reaches values +1 over edges and -1 outside them)

intermediate function R is calculated, and afterwards function MHGA (using the concept of linear saturated function):

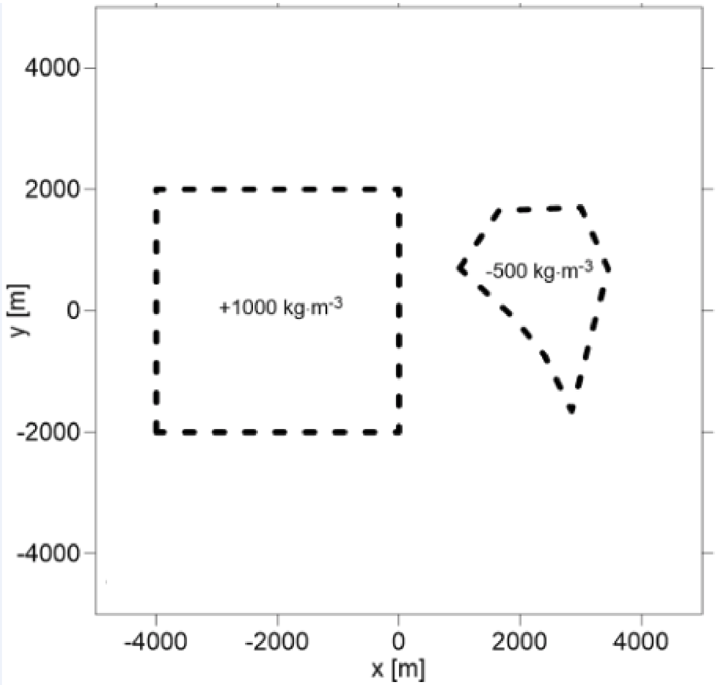


$$R = \frac{HG_z}{\sqrt{HG_x^{\ 2} + HG_y^{\ 2}}}, \quad MHGA = \frac{|R| - |R-2|}{2}$$

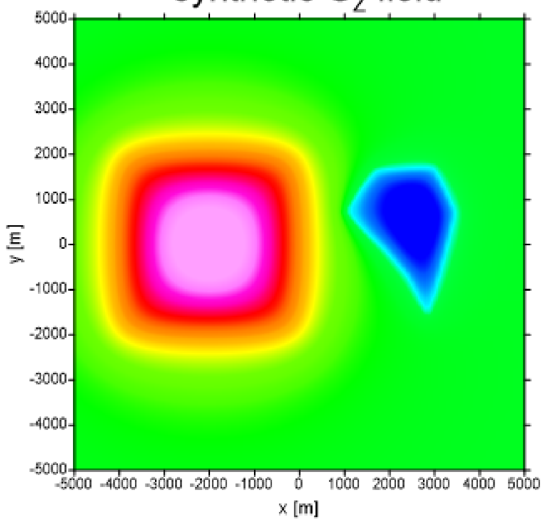




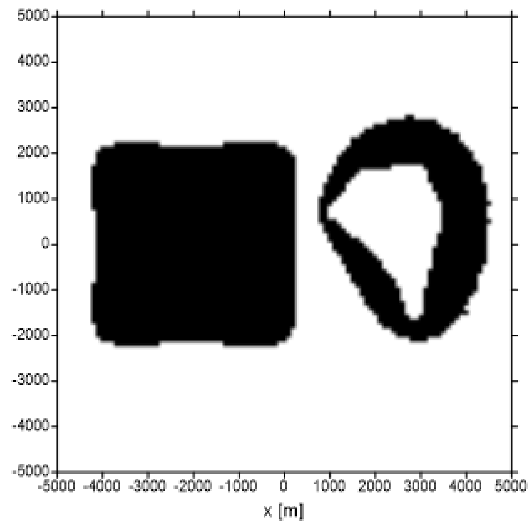
synthetic model (2 objects):



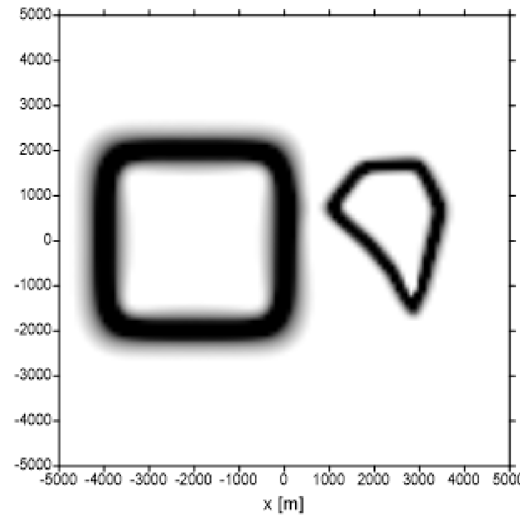
synthetic G_Z field



TPC transformation

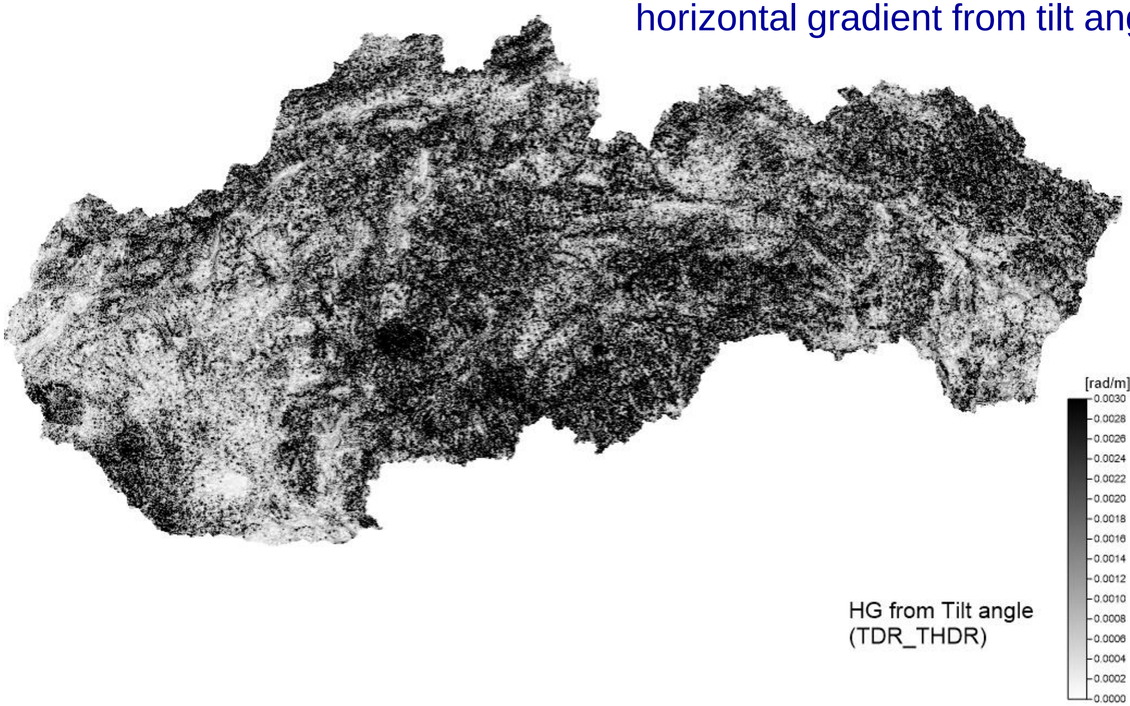


MHGA (MS Hainbing et al.)

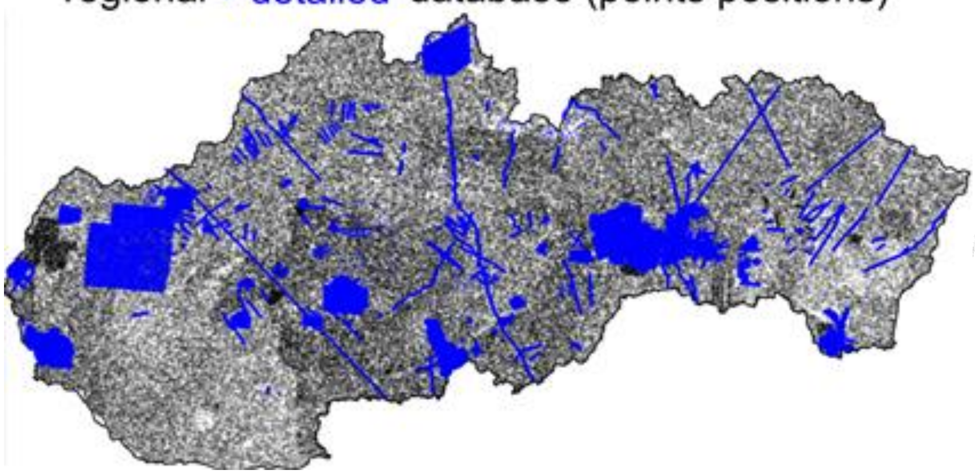


too much derivatives… (?)

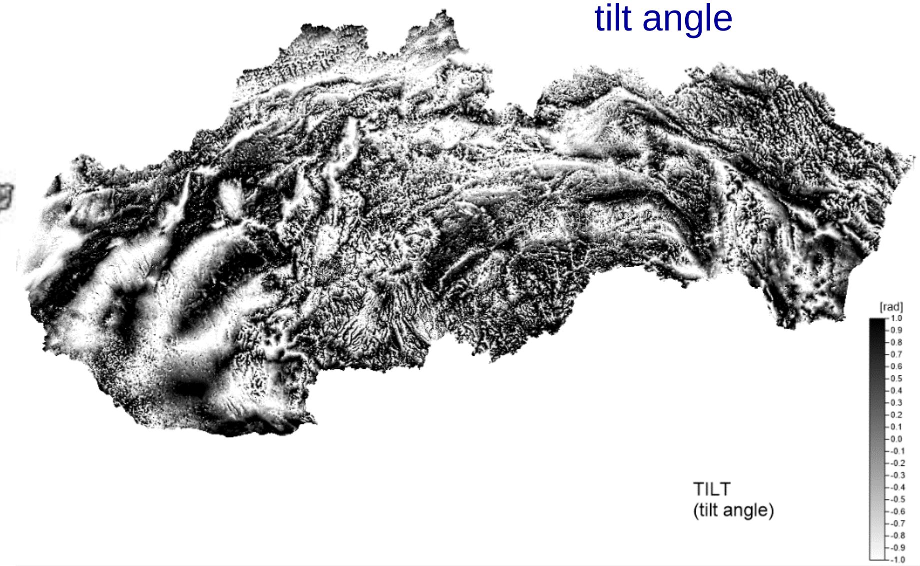
horizontal gradient from tilt angle



regional + detailed database (points positions)



tilt angle



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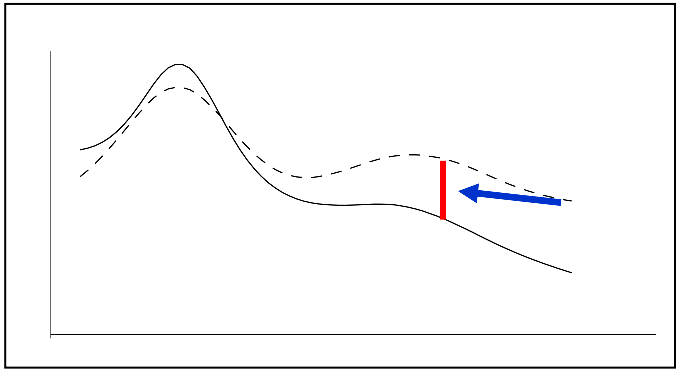
Stabilization of derivatives - Tikhonov regularization:

- classical low-pass filtering in the space domain (e.g. Hamming window) or in the Fourier domain (e.g. Butterworth filter),
- more sophisticated methods utilize the Wiener optimum filtering approach (e.g., Pawlowski and Hansen 1990) or enhanced derivatives (Fedi and Florio 2001) – the so called ISVD method; upward continuation
- we use in this contribution the **Tikhonov's regularization approach** (Tikhonov et al., 1968; Pašteka et al., 2009), which derives the shape of low-pass filter as a result of an optimisation problem (minimisation of two main functionals):

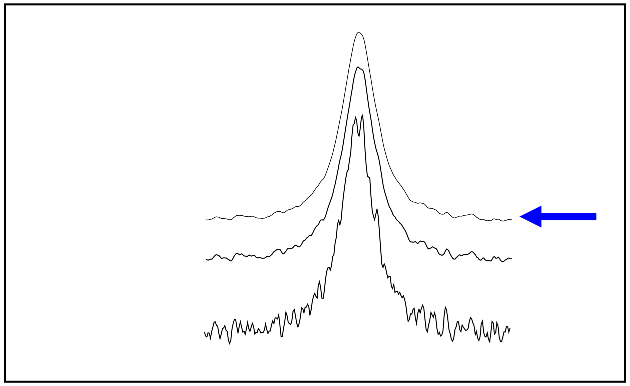
minimisation of two functionals (error functions)

the first describes the closeness to the classical solution for the derivative

the second describes its stability (smoothnes)



α



Stabilization of derivatives - Tikhonov regularization:

solution for the regularized derivative (in horizontal direction) in Fourier domain, 1D case (Pašteka et al., 2009):

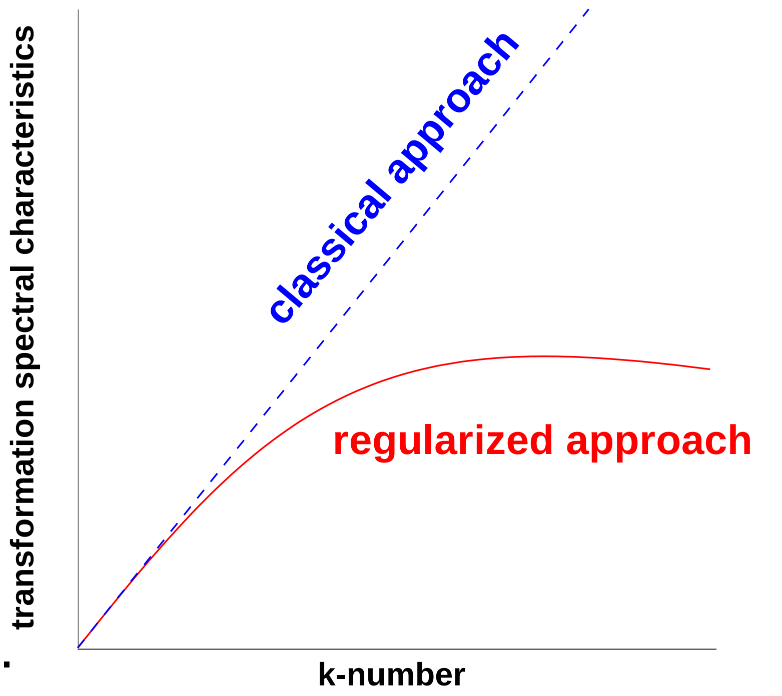
$$\boxed{\tilde{y}(k) = \frac{1}{1+\alpha k^2} ik\tilde{U}_\delta(k)}$$

where: k – wave number,

α **– regularization parameter**,

$\tilde{U}_\delta(k)$ – spectrum of original function,

$\tilde{y}(k)$ – spectrum of regularized derivative.



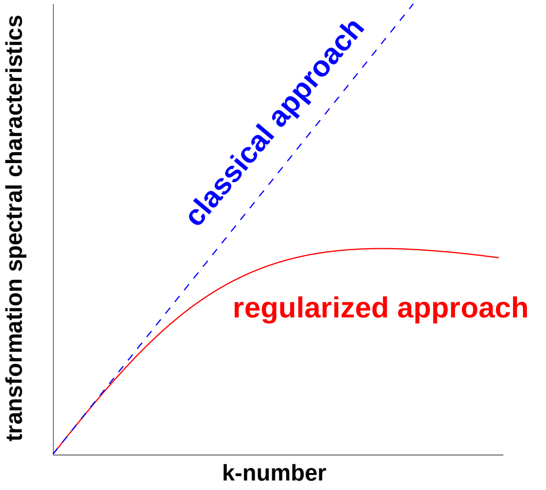
In 2023 colleague Assoc. Prof. Roland Karcol has changed the basic optimalization problem formulation – result of this is a new (better) solution, which we call as „general form“ of the regularized derivative operator.

(Karcol, Pašteka, 2024, submitted to the journal Geophysics):

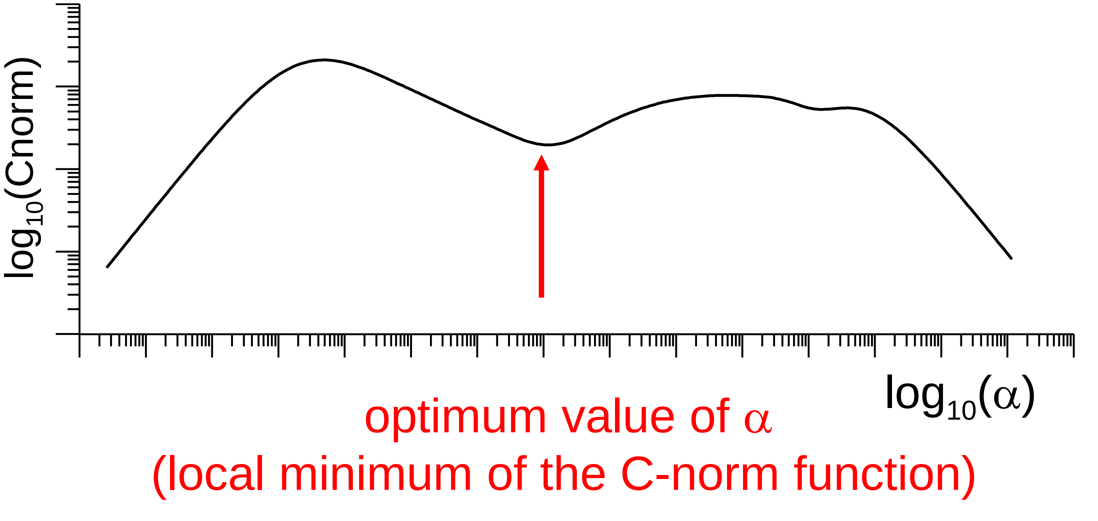
$$\boxed{\tilde{y}(k) = \frac{1}{1+\alpha(ik)k^2} ik\tilde{U}_\delta(k)}$$

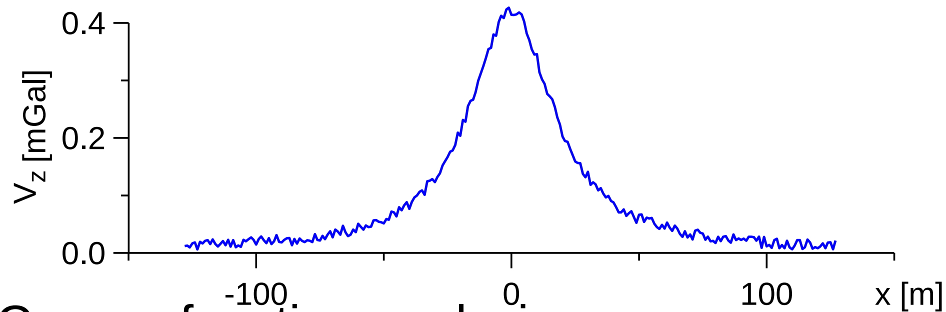
so called general form

Stabilisation of derivatives - Tikhonov regularization:



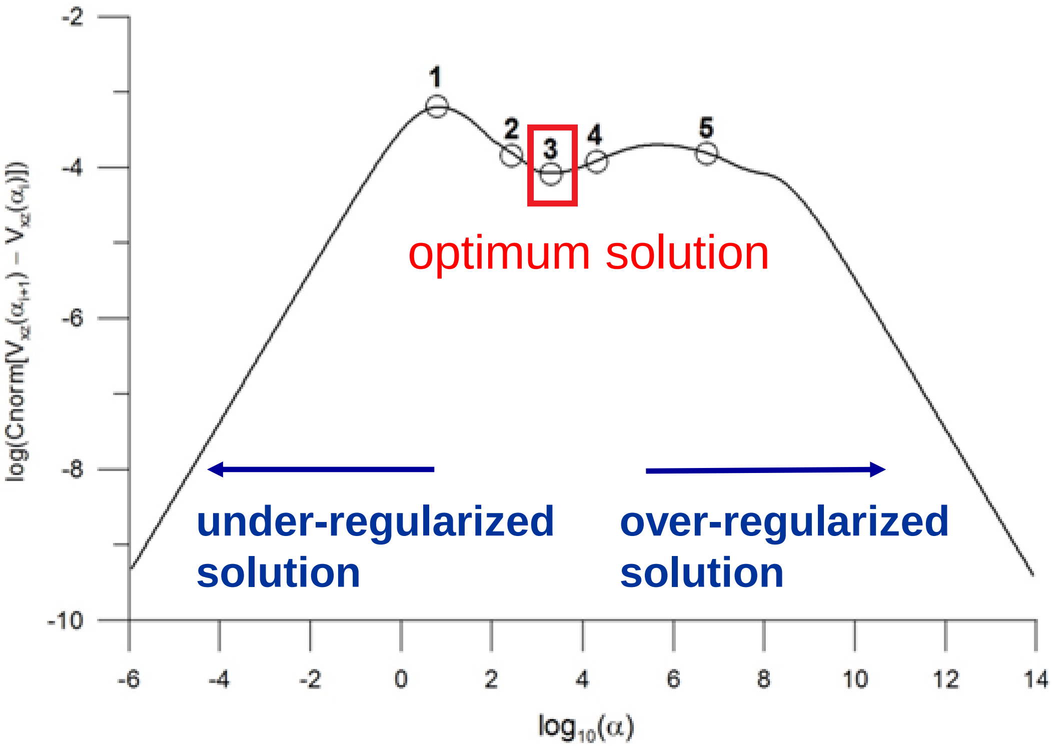
The most important task in regularization techniques – **selection of the optimum value of regularization parameter**, there exist a variety of methods (e.g. the L-curve, GCV method), we use the **C-norm** approach (Tikhonov et al., 1968).

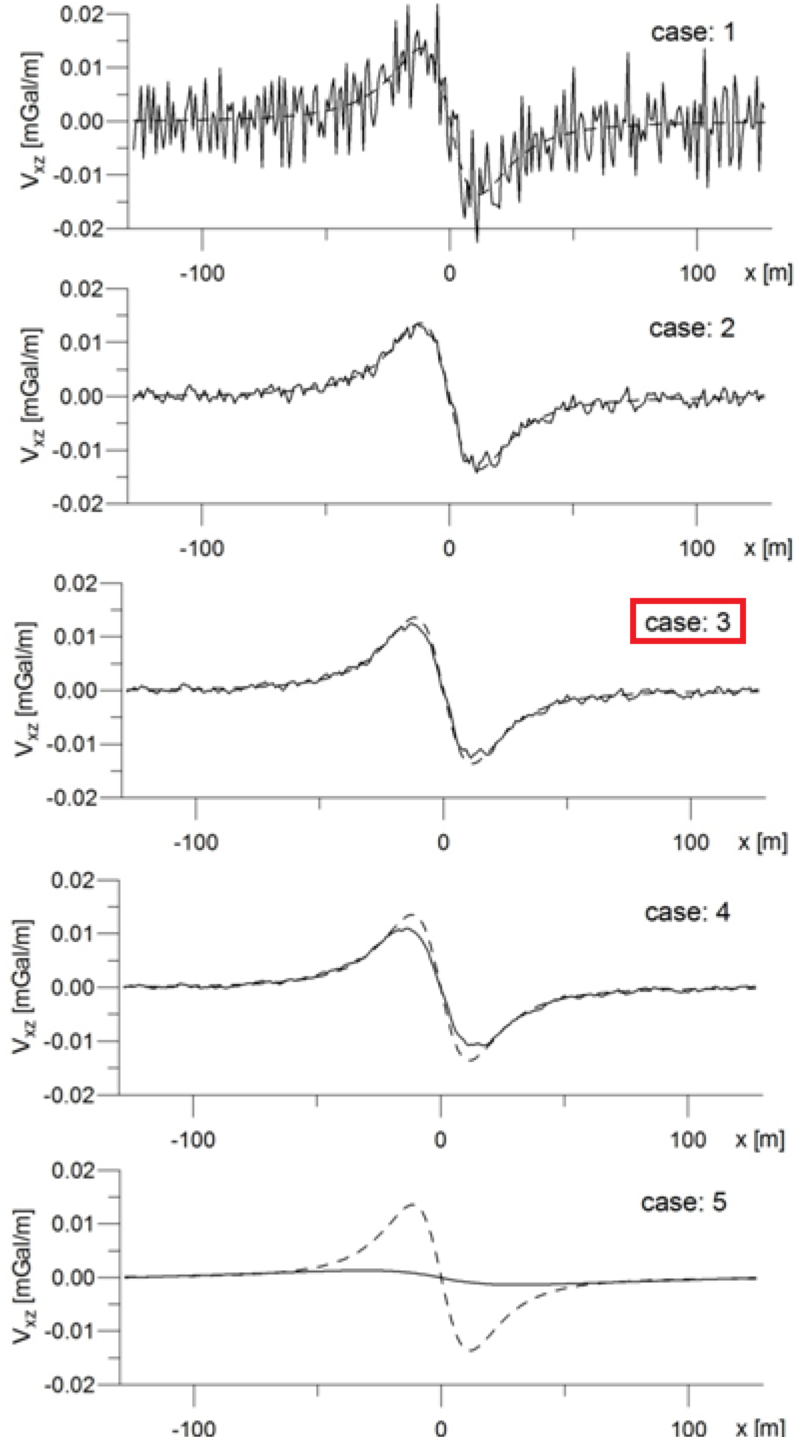




C-norm function analysis

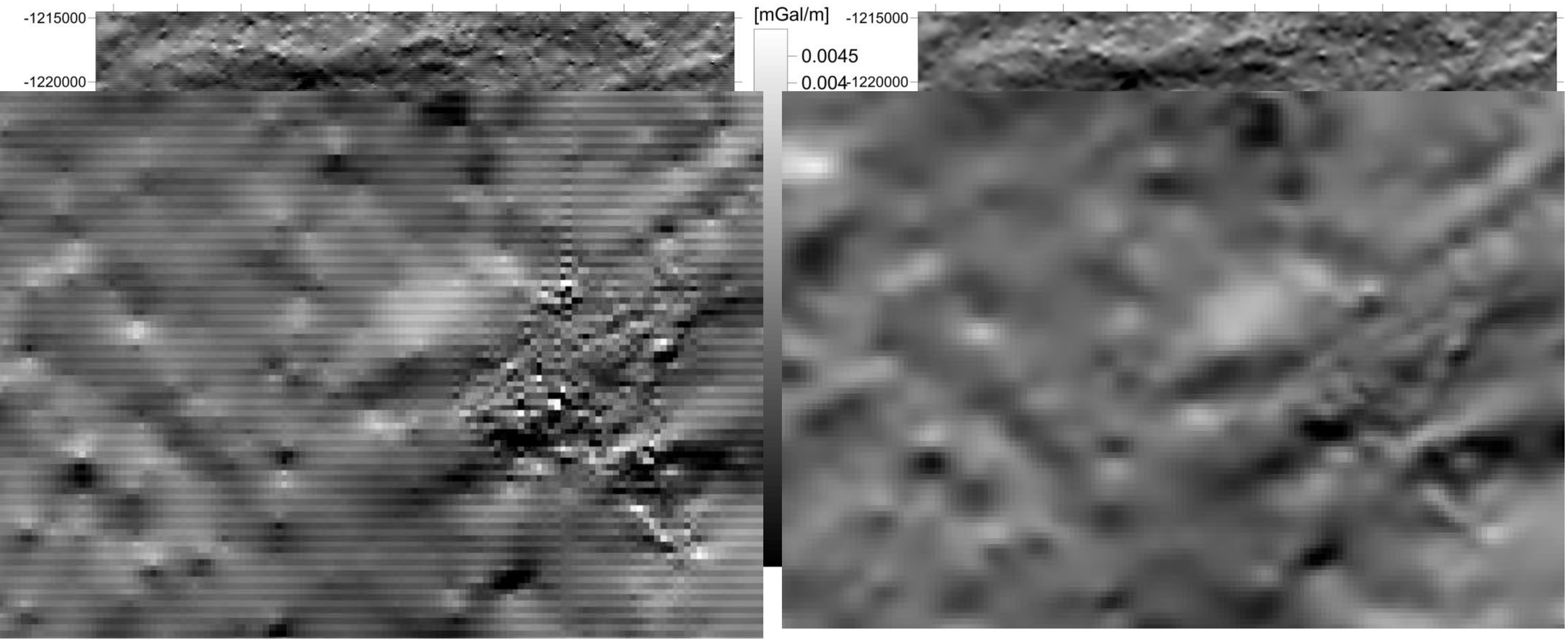
(example for x-derivative of 1D gravitational acceleration field – a horiz. cylinder effect)





example of regularized derivative calculation: y-derivative

(part of CBA from central Slovakia)



standard y-derivative
(without regularization)

regularized y-derivative

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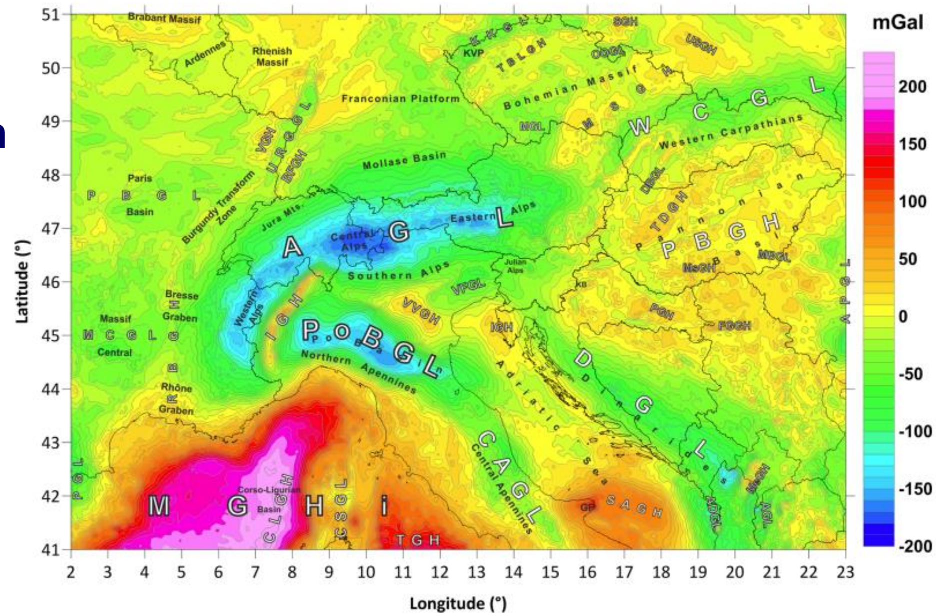
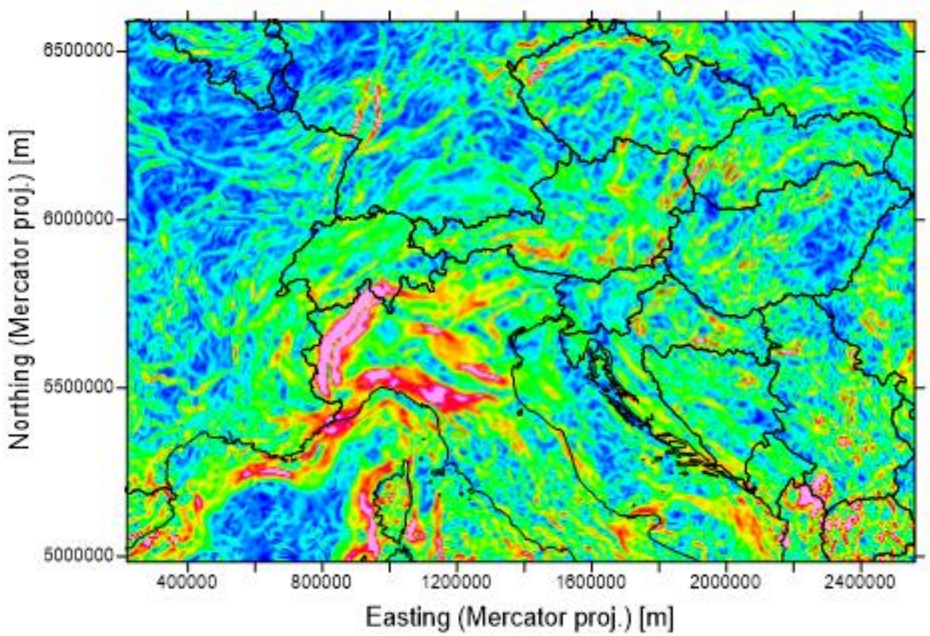
transformations: Bouguer anomaly map from pan-Alpine region (with regularized derivatives)

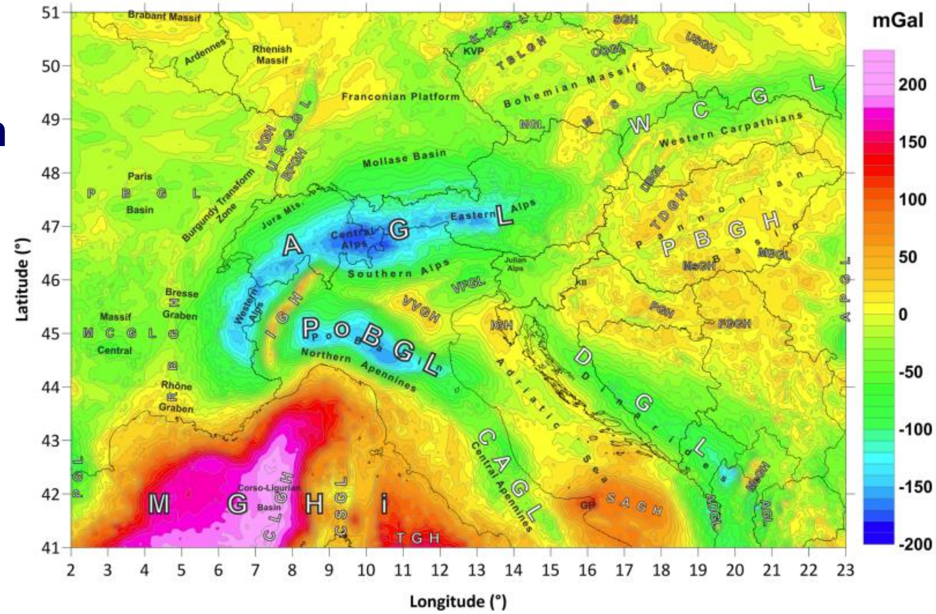


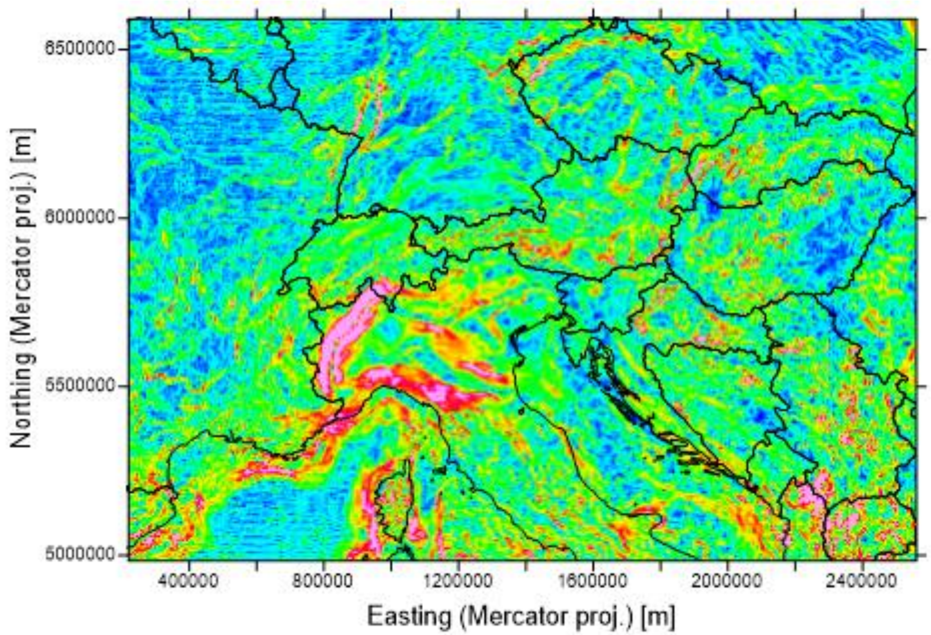


local maxima define boundaries (tectonics)

HG – horizontal gradient

transformations: Bouguer anomaly map from pan-Alpine region (WITHOUT regularized derivatives)

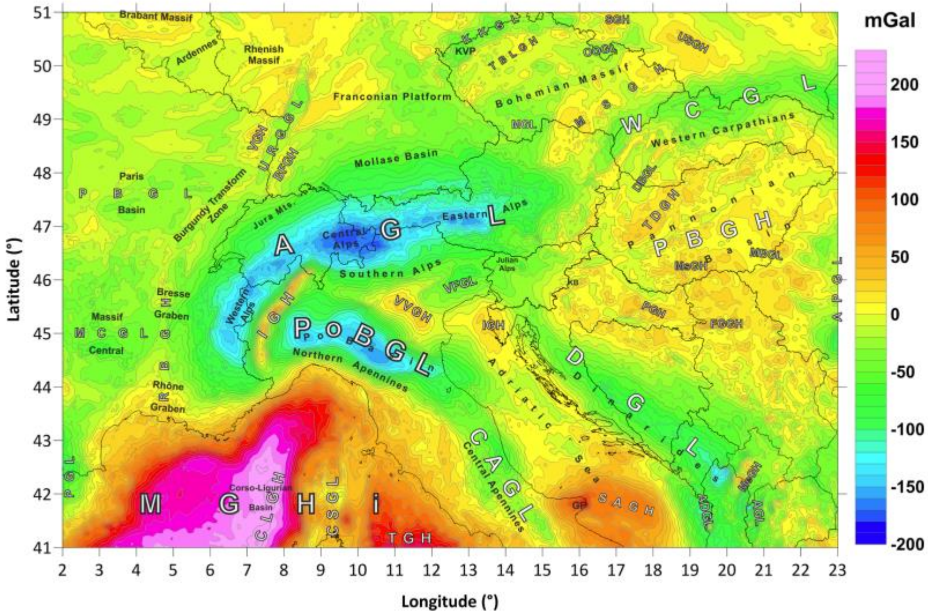


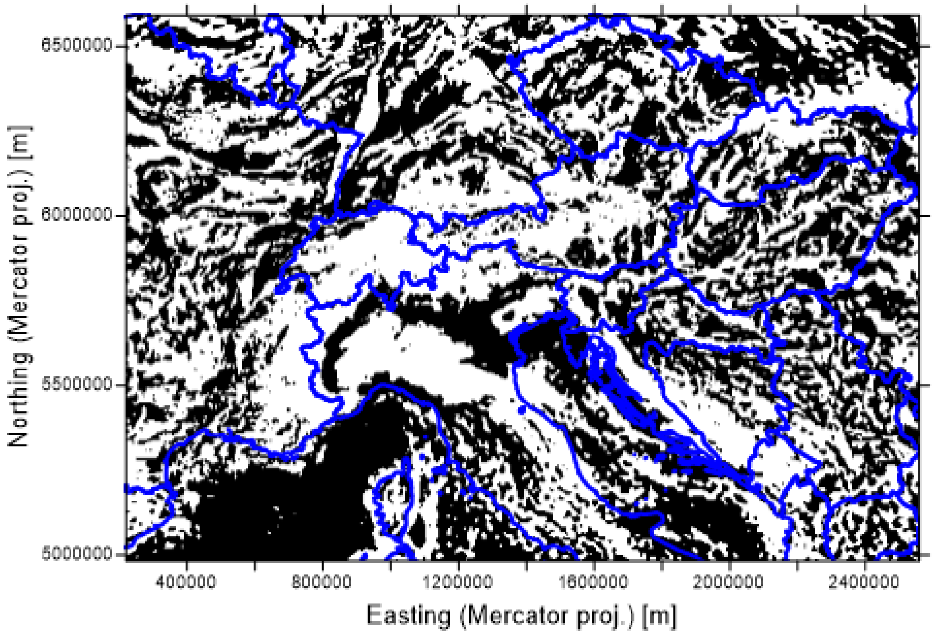


local maxima define boundaries (tectonics)

HG – horizontal gradient

**transformations:
Bouguer anomaly map
from pan-Alpine region
(with regularized
derivatives)**

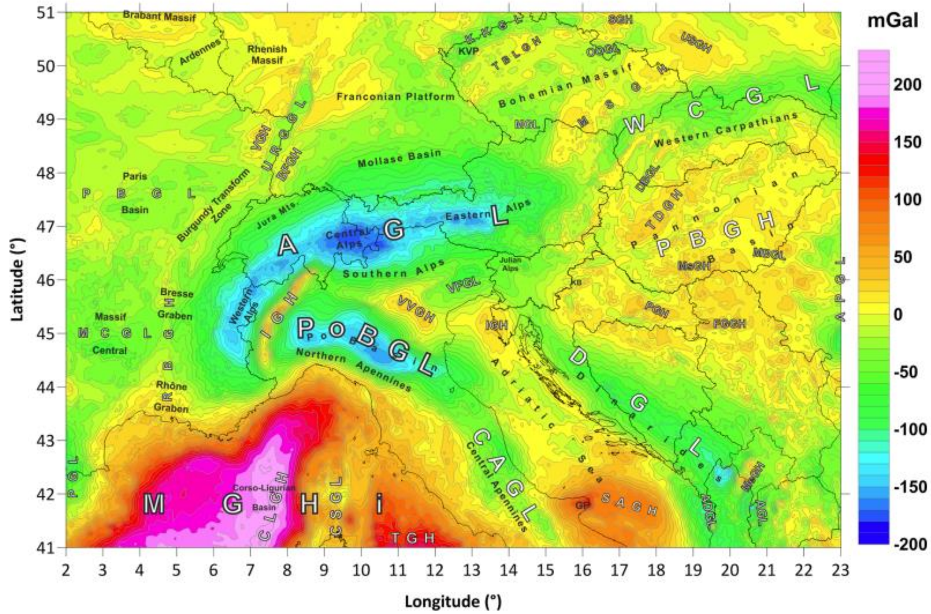


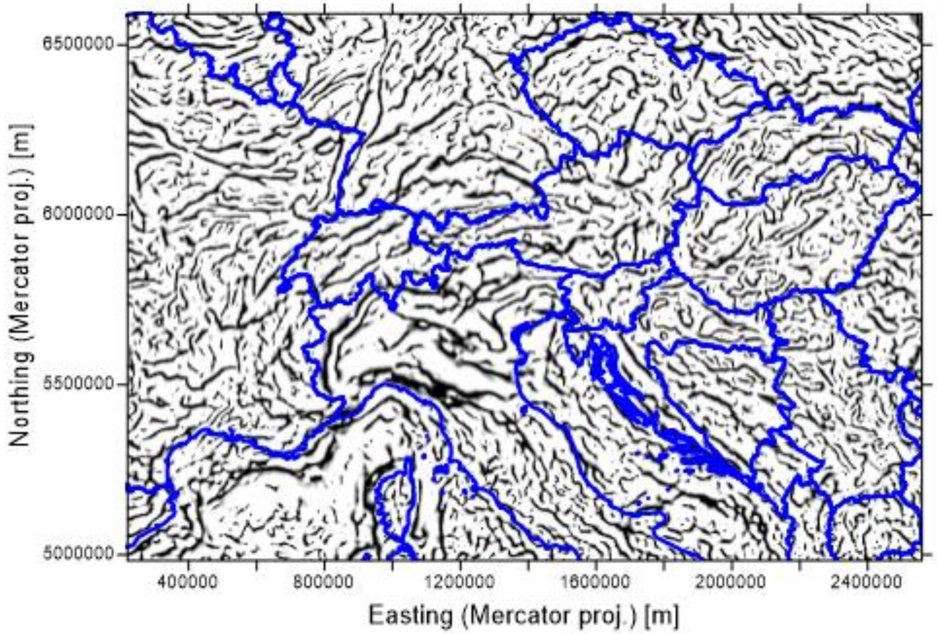


**binary map:
+π/2 over sources
(positive anomalies)
-π/2 outside them**

TPC - tilt angle plus and compl. angle

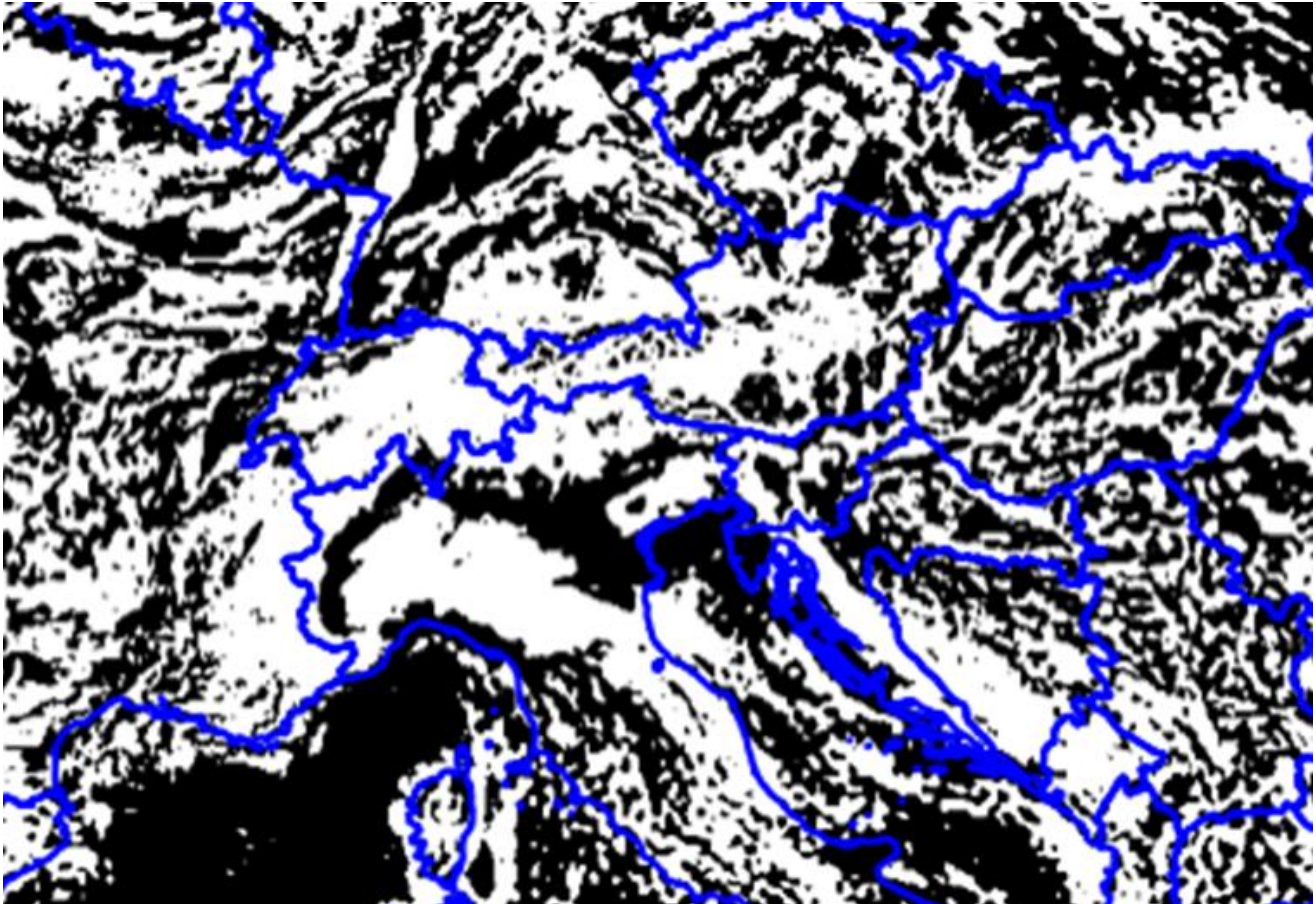
transformations: Bouguer anomaly map from pan-Alpine region (with regularized derivatives)





local maxima define boundaries (tectonics)

MHGA



Conclusions (methodical):

- **stabilized derivatives** (e.g. by means of the proposed Tikhonov approach) can **improve the information content** of the edge mappers and remove noise amplification

- during the **C-norm function analysis**, there is still need for a skill from the side of the user, but its involvement can help to select the **proper interval of low-pass filter parameter**

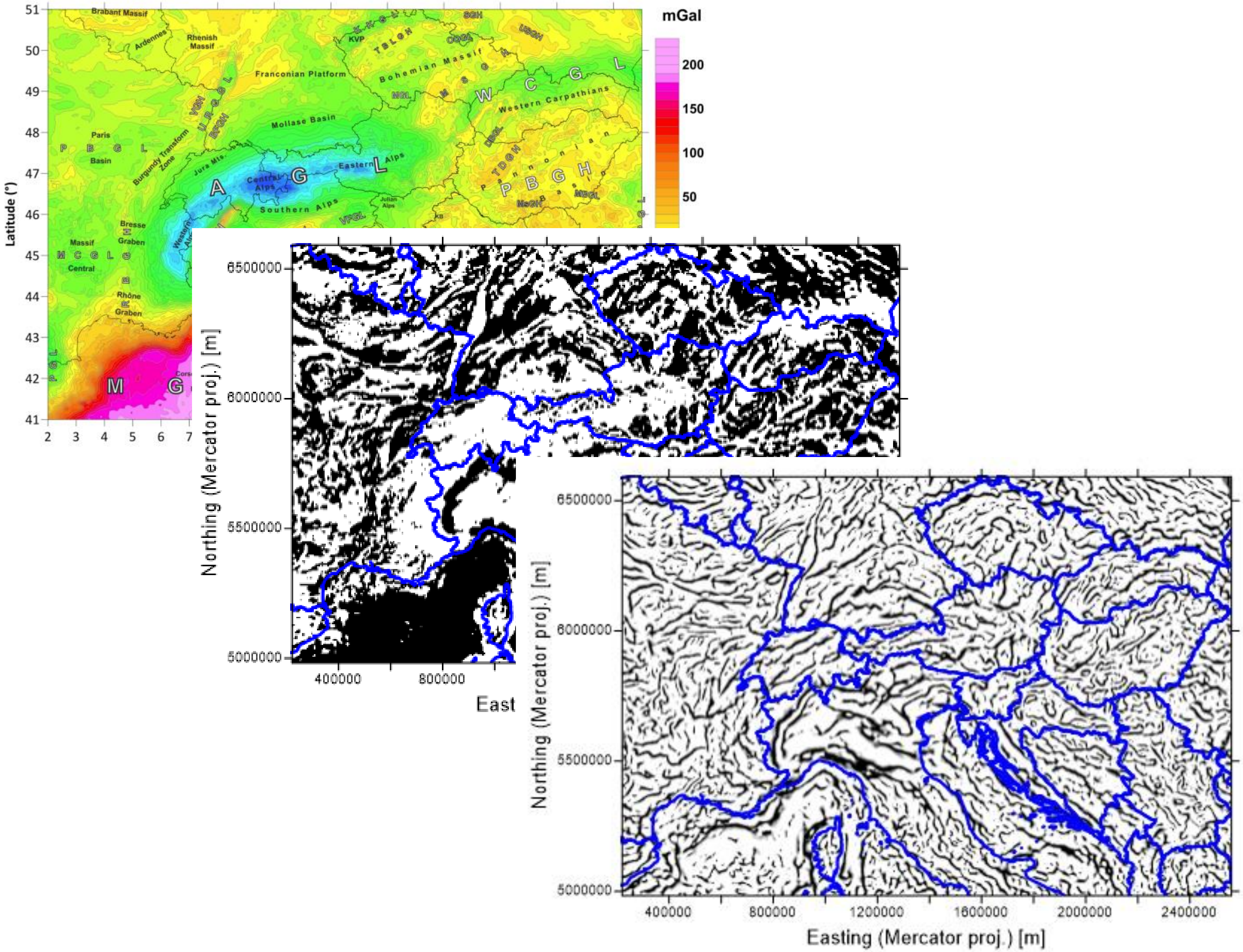
- in general, we are not **able now to recommend “the best” edge mapper** (we usually try several of them and then try to select)

Conclusions (interpretational):

- general experience: „**edge mappers can not recover on principle new features** (compared with the original field), but can help **better to understand** the most important features

- **manifestation of tectonics can be deformed** – due to the resolution of used maps (grids) – here it is the case of 4x4m, or 2x2km

- we will be happy, if other experts will use these results (we can send them in any graphical form and coordinate system)



Thanks for your attention.

algorithm of Tikhonov's regularization of derivatives evaluation

input: measured field

direct FFT

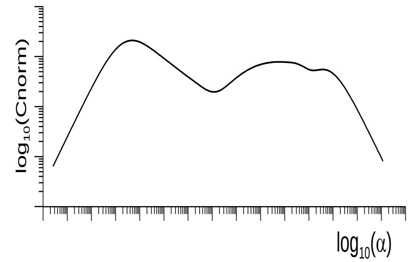
spectrum multiplied by $ik \dfrac{1}{1+\alpha k^2}$

inverse FFT

calculation of the „difference“ (a norm function) between the actual and previous solution y

repeated for a geometrical sequence of α (usually from 1E-10 to 1E+20)

analysis of the norm function – search of the optimum α



reconstruction of the solution y (inv. FFT) for the optimum α

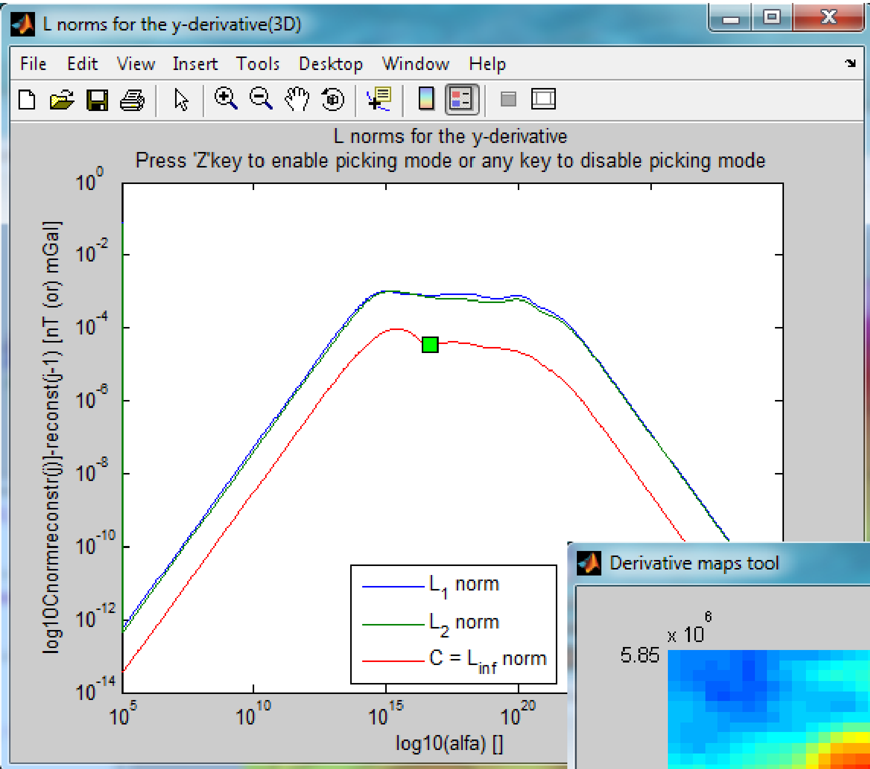
output: regularized (stable) derivative

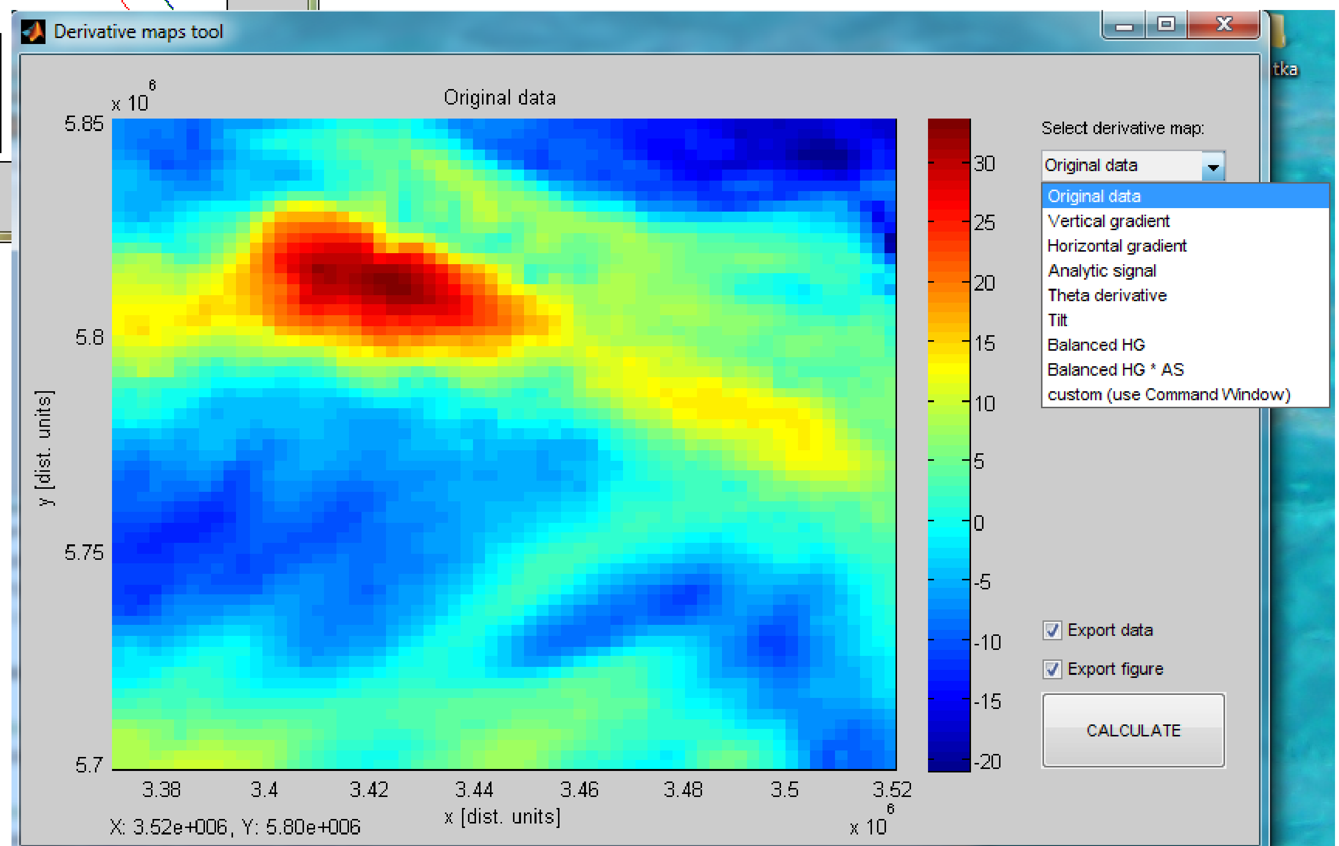
Neviem, či dať

Matlab script REGTILT

(free available upon request)

- C-norm analysis
- standard edge mappers selection
- „custom mode“ possibility





Neviem, či dať