Topic 6: magnetism, electromagnetism

Content:

- introduction
- basic quantities in magnetism
- magnetic intensity and induction
- magnetic permeabilty,
- diamagnetic, paramagnetic and ferromagnetic material
- magnetic flux
- laws of electromagnetic interaction:
- a. Lorentz force (law)
- b. Biot-Savart law
- c. Amper's law
- d. Faraday's law of induction
- comments to units

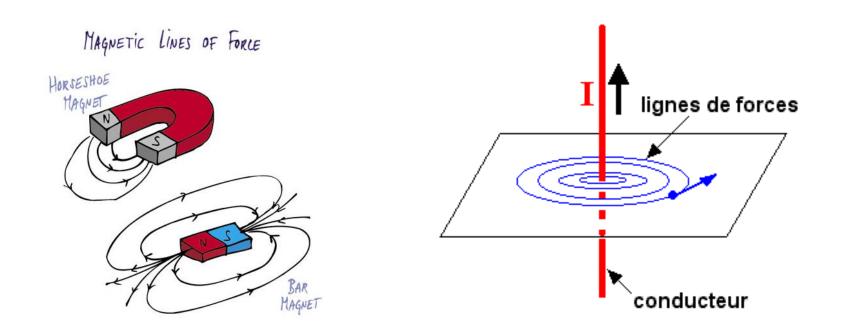
magnetism

Magnetism is a class of physical phenomena that are mediated by magnetic fields.

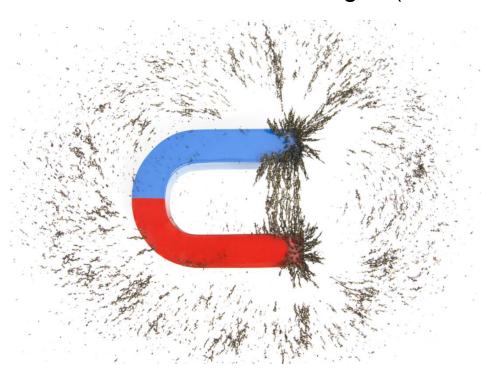
It refers to physical phenomena arising from the force caused by :

- a) permanent magnets (objects that produce fields that attract or repel other objects),
- b) electric current (fields in the vicinity of objects with el. current).

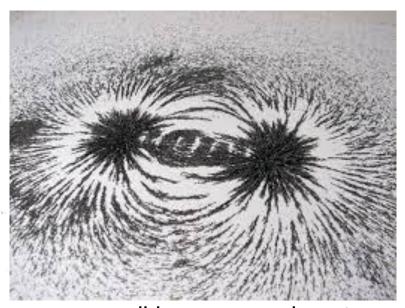
Magnetism is one aspect of the combined **electromagnetic force**.



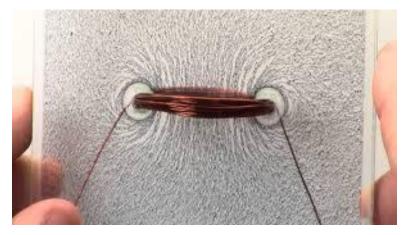
<u>Comment:</u> Also the magnetic field of a permanent magnet is caused by the movement of electric charges (inside of the atomic structure of the material).







very well known experiment with a paper and iron fillings



The magnetic field is often represented by magnetic field lines, which show the direction of the field at different points.





- a) experiment iron fillings around a bar magnet: https://www.youtube.com/watch?v=8llkHQtaOlg
- b) experiment iron fillings around a wire: https://www.youtube.com/watch?v=opJYLFvI-RE

Electrons circulating around atomic nuclei, electrons spinning on their axes, and rotating positively charged atomic nuclei all are magnetic dipoles. The sum of these effects may cancel so that a given type of atom may not be a magnetic dipole. If they do not fully cancel, the atom is a permanent magnetic dipole, as are iron atoms.

Permanent magnets are build from ferromagnetic materials such as iron (Fe), cobalt (Co), nickel (Ni) or gadolinium (Gd).

Bar magnets attract or repel other magnets.
These 4 elements have so called ferromagnetic properties, which we will touch later on.

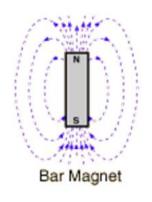


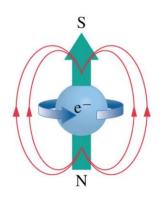
lodestone (magnetite)

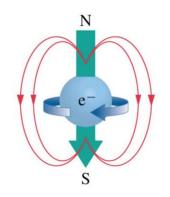
Magnetism, phenomenon associated with the motion of electric charges.

electric current → electromagnets

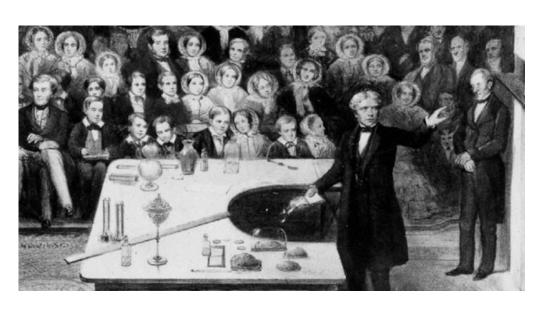
all effects of electrons → permanent (bar) magnets

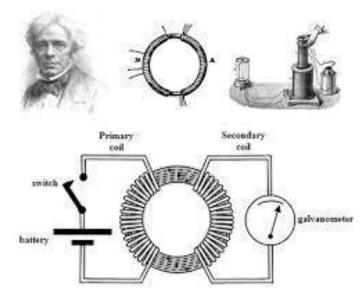






magnetic moment of an electron - caused by its spin





There were several physicists, who have studied properties of magnetism:



William Gilbert (1540-1603)



Carl F. Gauss (1777-1855)



Hans Ch. Oersted (1777-1851)



Andre M. Ampere (1775-1836)



Michael Faraday (1791-1869)

Magnetic field has exclusively a dipole character !!!

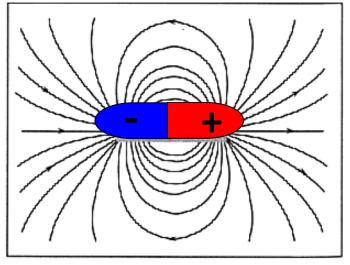
Recall that an electric dipole consists of two equal but opposite charges separated by some distance, such as in a polar molecule. Every <u>bar magnet</u> or an electormagnet is a <u>magnetic dipole</u>.

Note how the **E** field due an electric dipole is just like the magnetic field (**B** field) of a bar magnet. Field lines emanate from the + or N pole and re-enter the - or S pole.

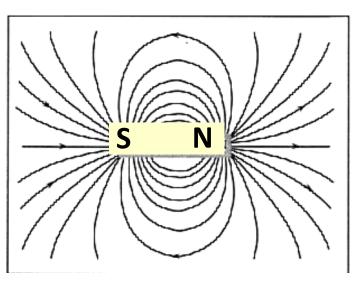
Although they look the same, they are different kinds of fields. **E** fields affect any charge in the vicinity, but a **B** field only affects moving charges. As with charges, opposite poles attract and like poles repel.

Magnetic dipole moment – important parameter (will be explained later on).

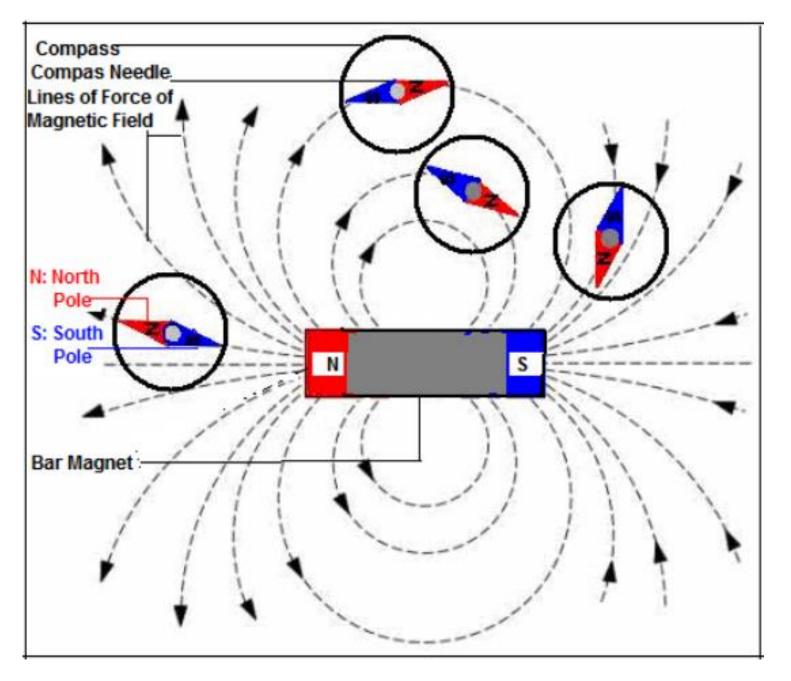
B is the field of magnetic induction (will be explained later on).



electric dipole and *E* field

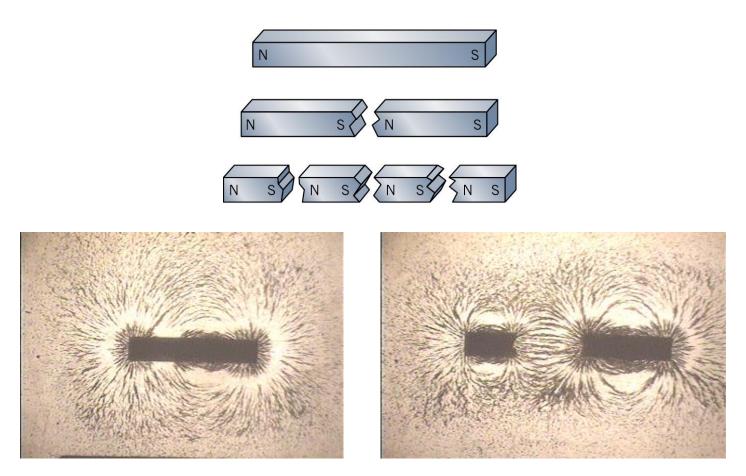


magnetic dipole and **B** field



Field lines emanate from the + or N pole and re-enter the - or S pole.

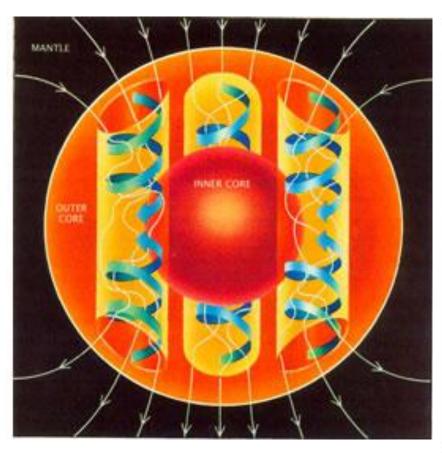
important comment:

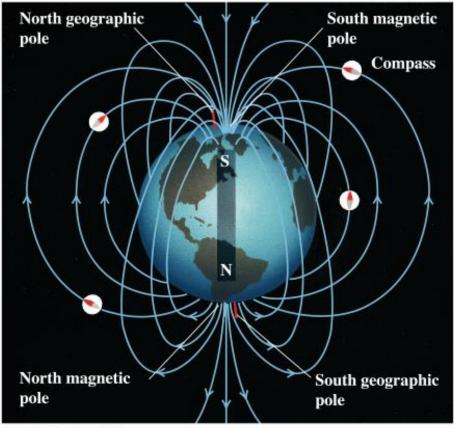


Magnetic monopoles do not exist (!), meaning it is impossible to isolate a N or S pole. When we try to separate the two poles by breaking the magnet, we only succeed in producing two distinct dipoles.

Some particle theories predict their existence, but there is no experimental evidence that they exist.

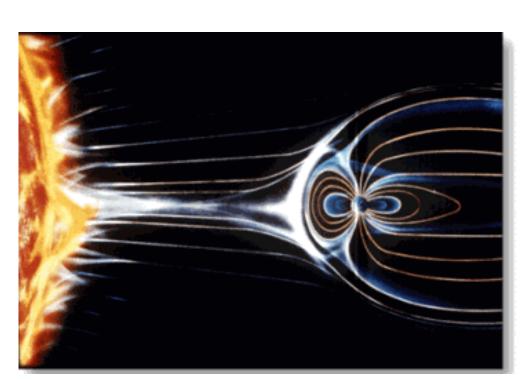
Earth magnetic field is generated by the effect of movement of conductive masses in the outer Earth core – so called hydromagnetodynamic effects.

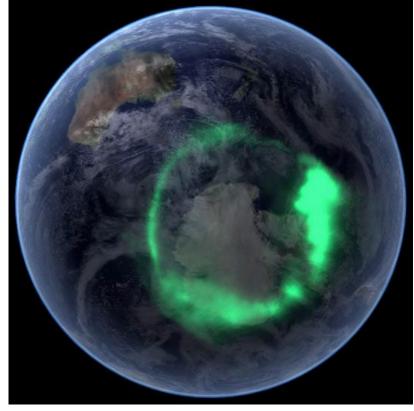




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Thanks to the Earth magnetic field the life conditions are acceptable on the Earth – magnetic field (called also magnetosphere) build "a shield", which protects Earth before so called solar wind (stream of high-energy particles emanated from the Sun).





magnetism - basic quantities (intensity vs. induction)

Magnetic field is described by means of two very close vector quantities:

magnetic intensity \mathbf{H} (unit, $[A \cdot m^{-1}]$) and magnetic induction \mathbf{B} (unit, [T] = tesla); Relation between them is given by this simple equation:

$$\mathbf{B} = \mu \mathbf{H}$$
,

where μ – magnetic permeability, (unit: [H·m⁻¹ = N·A⁻²]), it describes the ability of a material to be magnetized by the action of an outer magnetic field.

We use often so called relative magnetic permeability:

$$\mu_r = \mu/\mu_0 \Rightarrow \mu = \mu_r \cdot \mu_0$$

 μ_0 – permeability of vacuum (free space) or magnetic constant $(4\pi \cdot 10^{-7} \text{ H} \cdot \text{m}^{-1})$.

Comment: Units tesla and henry will be explained later on.

magnetism – basic quantities (intensity vs. induction)

Magnetic intensity \mathbf{H} ([A·m⁻¹]).

Describes the measure of force effects (magnetisation effects) of the magentic field. In comparison with magnetic induction, it does not include the reaction of the material to the outer magnetic field.

Magnetic induction **B** ([T]).

Characterizes force effect of the magnetic field on a moving charge or a conductor with electric current, it does include the reaction of the material to the outer magnetic field (its magnetisation or "resistance" to it).

The best way to understand **H** vs. **B** is the so called hysteresis (we will come to it in a moment).

Alternative names for B and H

В			
name	used by		
magnetic flux density	electrical engineers		
magnetic induction	electrical engineers		
magnetic field	physicists		
Н			
name	used by		
magnetic field intensity	electrical engineers		
magnetic field strength	electrical engineers		
auxiliary magnetic field	physicists		
magnetizing field	physicists		

magnetism – basic quantities (permeability)

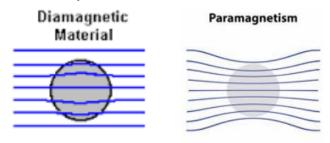
Based on the value μ_r ($\mu_r = \mu/\mu_0$) we can divide materials to:

diamagnetic (μ_r < 1), they weaken the magnetic field, they are repelled by the outer magnetic field (form an opposite induced field) (e.g.: water, organic substance, but also some metals: Cu, Ag, Au, Hg, Bi,)

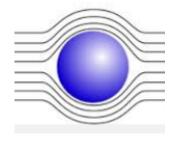
paramagnetic (μ_r > 1), slightly amplify the magnetic field, they are attracted by the outer magnetic field (form a field in the direction of the outer field), but do not remember this magnetisation (most of chemical elements, e.g.: Al, Mn, Cr, Pt)

ferromagnetic ($\mu_r >> 1$) strongly amplify the magnetic field, they are attracted by the outer magnetic field (form a field in the direction of the outer field), but do remember this magnetisation (4 metals: Fe, Ni, Co, Gd).

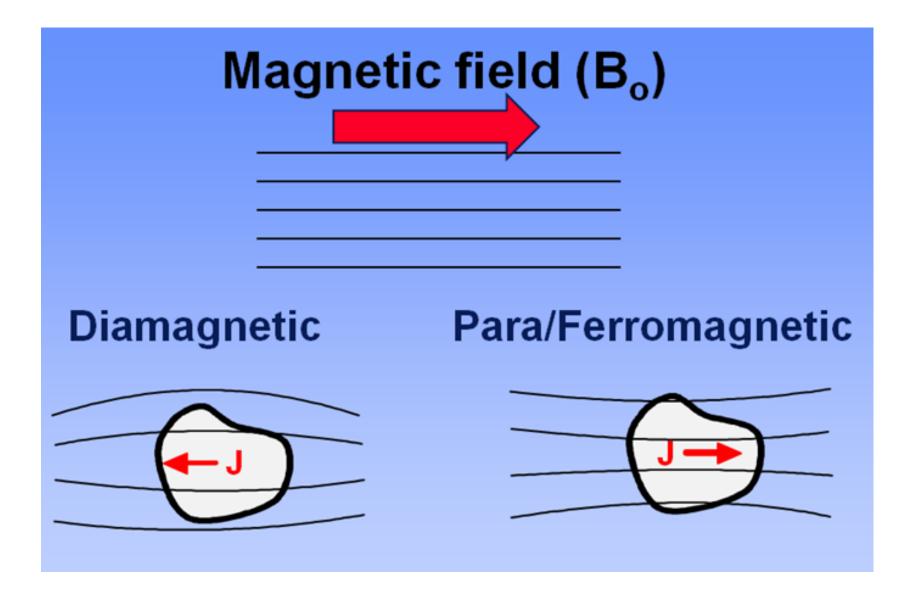
Comment: Strongest diamagnetic behaviour show superconductors, which can almost fully prohibit the entering of the outer magnetic field into their volume (beside a thin surface layer), so for their centre is valid $\mu_r = 0$.



superconductor:



magnetism - basic quantities (permeability)



J is sometimes named as magnetisation vector

diamagnetic (μ_r < 1), paramagnetic (μ_r > 1) , ferromagnetic (μ_r >> 1)

Material	Туре	Relative permeability
Bismuth	Diamagnetic	0.99983
Silver	Diamagnetic	0.99998
Copper	Diamagnetic	0.999991
Lead	Diamagnetic	0.999983
Water	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1
Air	Paramagnetic	1.0000004
Aluminum	Paramagnetic	1.00002
Palladium	Ferromagnetic	1.0008
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Mild Steel (0.2 C)	Ferromagnetic	2,000
lron (0.2 impurity)	Ferromagnetic	5,000
Silicon Iron	Ferromagnetic	7,000
Mumetal	Ferromagnetic	100,000
Purified iron (0.05 impurity)	Ferromagnetic	200,000
Supermalloy	Ferromagnetic	1,000,000

comment – Nd permanent magnets

Neodymium - Nd

In its pure form is paramagnetic (!) (below temperature 20 K partly ferromagnetic).

But in composition Nd₂Fe₁₄B it is strongly ferromagnetic.

This is used for the production of so called neodymium magnets. (used in many technical applications – in microphones, physics, etc.).

video:

http://www.dailymotion.com/video/x15ryzj



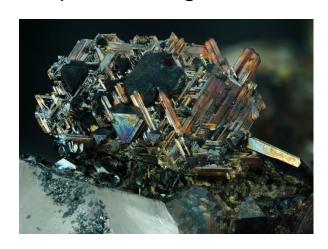
magnetism - basic quantities (susceptibility)

Some scientific branches work more with magnetic susceptibility $kappa \chi$. Its connection with permeability:

$$\mu_r = 1 + \chi$$

diamagnetic (χ < 0), paramagnetic (χ > 0), ferromagnetic (χ >>0)

Examples of magnetic minerals (in the nature):



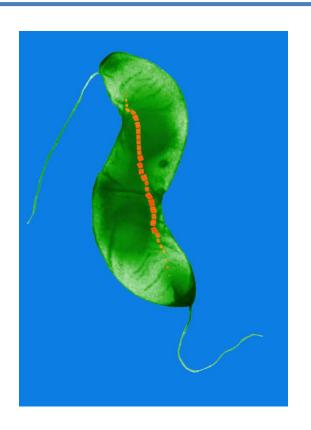
Ilmenite: FeTiO₃ Magnetit: Fe₃O₄

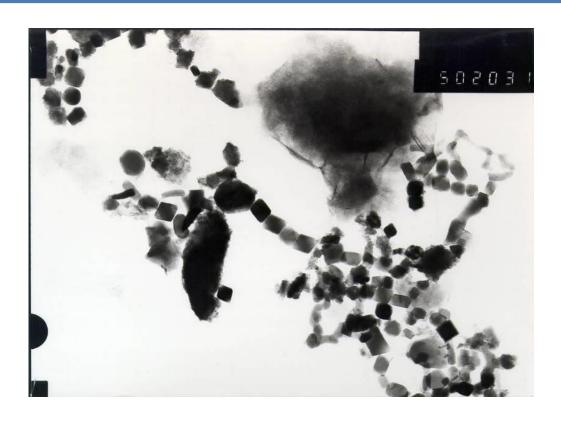


magnetic minerals (in biology)

Many organisms, from bacteria to pigeons to humans harbor tiny crystals of magnetite or other magnetic minerals. Experiments have show that some can also sense the Earth's magnetic field for the purposes of navigation.

But figuring out where the magnetoreceptors are hidden in an organism is no simple task.

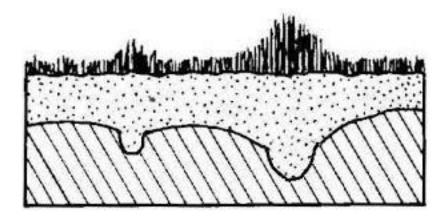




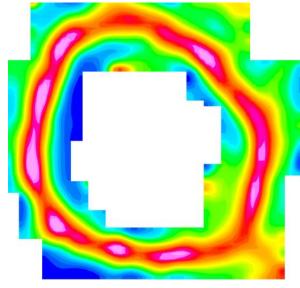
<u>Comment:</u> Some ferromagnetic minerals in a colloidal form (even in small nano-crystals) are products of the metabolism of some humus bacteria, when rests of plants are transformed into humus.

magnetic minerals (in biology)





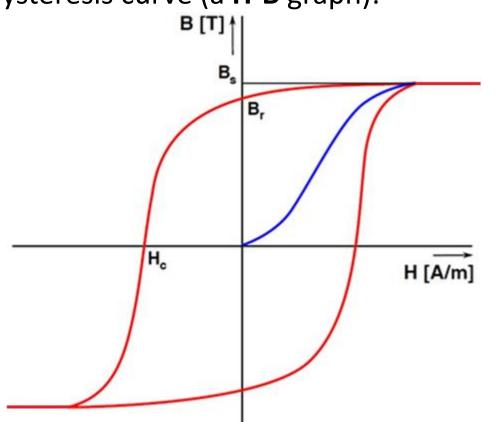






magnetism - basic properties (hysteresis)

Ferromagnetic materials "can hold" magnetisation also after "switching off" the outer magnetising field. This is connected with the so called magnetic hysteresis — often displayed in a form of a hysteresis curve (a **H-B** graph):



B_S – saturation stage
 (max. value of B),
 B_r – remanent
 magnetisation,
 H_C – coercitivity intensity,

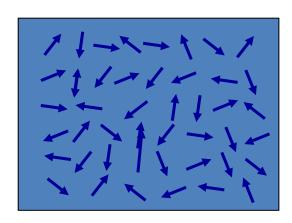
Based on the H_C value we dived ferromagnetic materials to soft (small H_C) and hard (large H_C).

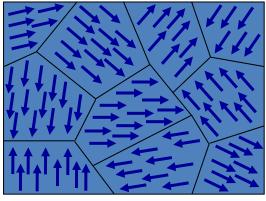
To demagnetize the remanent magnetisation requires heat or acting of a magnetic field in the opposite direction.

This is the effect that provides the element of memory in a hard disk drive.

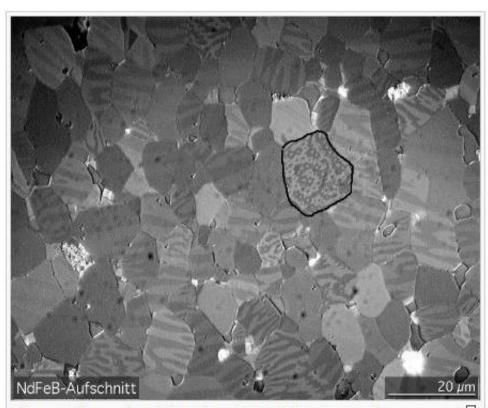
magnetism - basic properties (magnetic domains)

Electrones build with their movement a local (elementary) magnetic dipole moment of an atom. These magnetic moments can be cancelled (paramagnetic) or amplified (ferromagnetic), when they are self-organized in special zones, so called magnetic domains.





schematic visualisation of magnetic domains

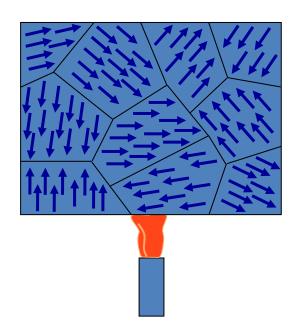


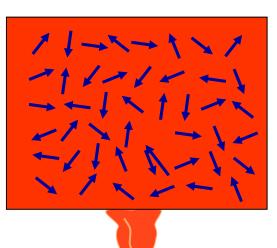
Microcrystalline grains within a piece of NdFeB (the alloy used in neodymium magnets) with magnetic domains made visible with a Kerr microscope. The domains are the light and dark stripes visible within each grain.

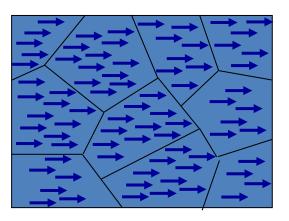
magnetism – basic properties (Curie's temperature)

Influence of high temperature (so called Curie's temperature) on remanent magnetisation

When some ferromagnetic material is heated to some temperature (so called **Curie's temperature**), the internal structure of the matter is changed to that of a paramagnetic one. After cooling down, the matter became again ferromagnetic and the structure with magnetic domain is re-established.

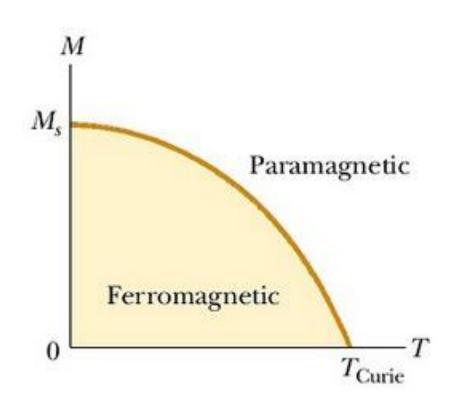






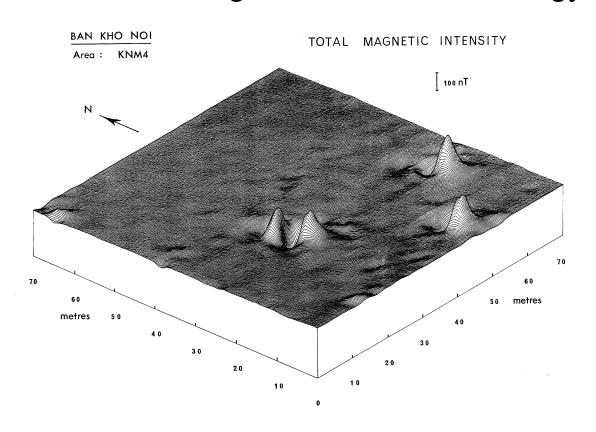
magnetism – basic quantities (Curie's temperature)

Material \$	Curie temperature (K)
Iron (Fe)	1043
Cobalt (Co)	1400
Nickel (Ni)	627
Gadolinium (Gd)	292
Dysprosium (Dy)	88
Mn Bi	630
MnSb	587
CrO ₂	386
MnAs	318
EuO	69
Iron(III) oxide (Fe ₂ O ₃)	948
Iron(II,III) oxide (FeOFe ₂ O ₃)	858



magnetism – basic quantities (Curie's temperature)

This can be again used in archaeology...

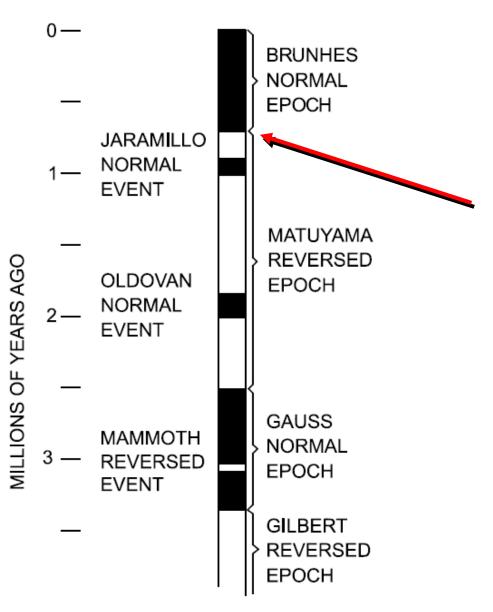


site Ban Kho Noi, Thailand



ceramics kilns have been excavated

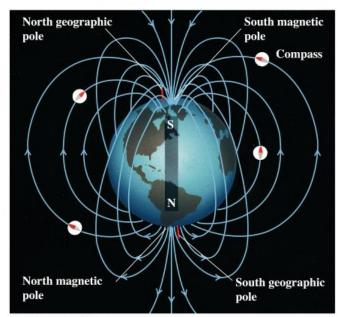
Earth's magnetic field - changes



 periode of changing the poles is close to thousands up to millions of years –

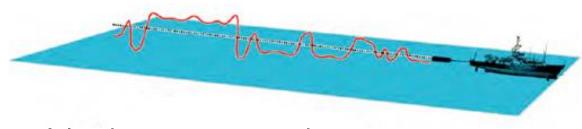
in average 250.000 years,

- we live in the so called Brunhes normal epoch, which started approx. 780.000 years ago,

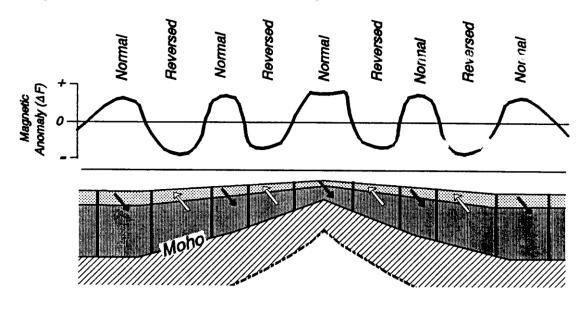


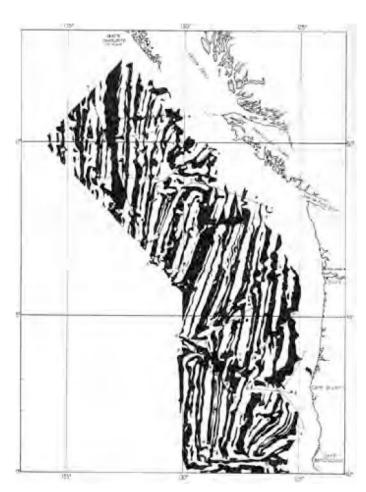
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Earth's magnetic field - changes



Atlantic ocean magnetic maps (Rald and Mason, 1961)

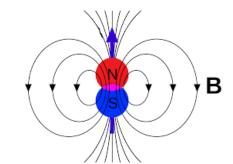




magnetism - basic quantities (scalar magnetic potential)

Field of the magnetic can be expressed with the magnetic potential (scalar and also vector):

$$\psi(P) = \frac{1}{4\pi} \frac{\vec{m} \cdot \vec{r}}{r^3} = \frac{1}{4\pi} \frac{m r \cos \theta}{r^3} = \frac{1}{4\pi} \frac{m \cos \theta}{r^2}$$



where:

 \vec{m} is the magnetic dipole moment (unit: [A·m²]),

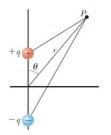
 \vec{r} the distance vector between the centre of the dipole and point P,

 θ the angle between \vec{m} and \vec{r} .

Unit: [T·m].

For comparison: potential of electric dipole (previous lecture)

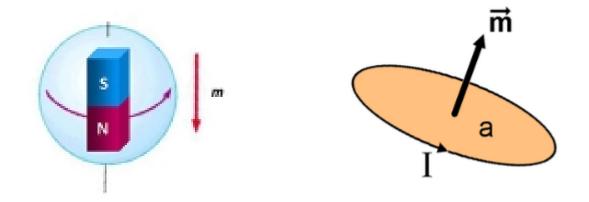
$$\phi(P) = \frac{1}{4\pi\epsilon_0} \frac{\vec{m} \cdot \vec{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{m\cos\theta}{r^2}$$



magnetism - basic quantities (magnetic moment)

Magnetic (dipole) moment $\hat{\mathbf{m}}$ is a vector variable, describing the strength and orientation of a magnetic dipole.

Example: Magnetic moment of a current I, enclosing an area a.



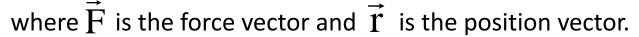
Unit of magnetic moment: $[A \cdot m^2 = J/T]$.

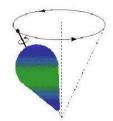
In atomic physics we define the electron magnetic moment: $-9.284764 \cdot 10^{-24}$ J/T and proton magnetic moment: $-1.41066 \cdot 10^{-26}$ J/T .

magnetism - basic quantities (magnetic moment)

From mechanics we know that moment (torque) can be expressed:

$$\vec{M} = \vec{F} \times \vec{r} \quad [N \cdot m]$$





Moment of force (torque) causes a <u>precessional movement</u> around the rotational axis.

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Torque is causing that a spinning particle to precess (rotate about the axis of the applied field).

magnetic flux

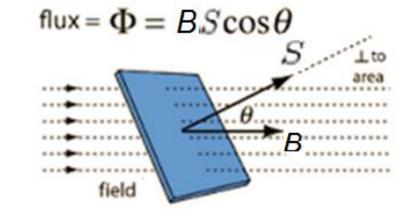
Magnetic flux Φ_B is the measure of flow of the magnetic field ${f B}$

through a given area A.

Unit: weber, $[Wb] = [V \cdot s]$

For a planar area A we can write:

$$\Phi_B = \mathbf{B} \cdot \mathbf{S} = BS \cos \theta,$$



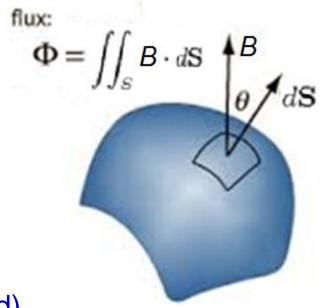
For an irregular area A we have to write an integral:

$$\Phi_B = \iint_S \mathbf{B} \cdot d\mathbf{S}.$$

and for closed irregular area A:

$$\Phi_B = \iint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

This is so called Gauss's law for magnetism (due to the dipole character of magnetic field).



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Lecture 6: magnetism, electromagnetism

laws of electromagnetic interaction

Lecture 6: magnetism, electromagnetism

laws of electromagnetic interaction

These laws describe the relation between electric and magnetic field in their common interaction (so called electromagnetic forse).

We know following basic EM laws:

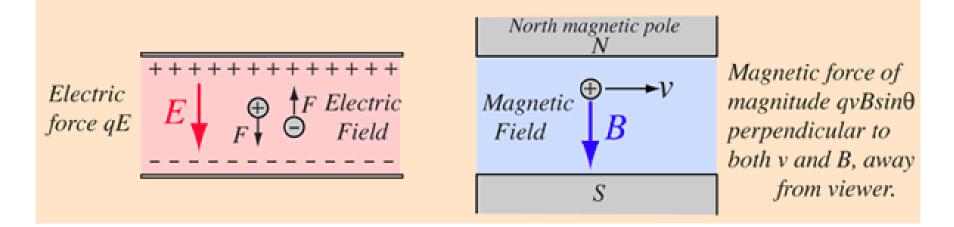
Lorentz force (law), Biot-Savart' law, Ampere's law, Faraday's law and Lenz' law.

Lorentz force (law)

If a particle of charge *q* moves with velocity *v* in the presence of an electric field *E* and a magnetic field *B*, then it will experience a force:

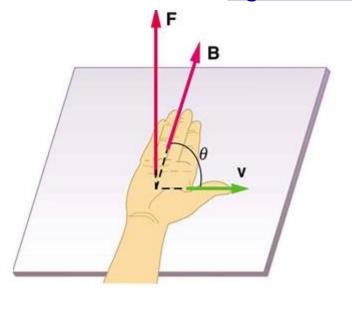
 $\vec{F} = q\vec{E} + q\vec{v}x\vec{B}$ Electric Magnetic force force

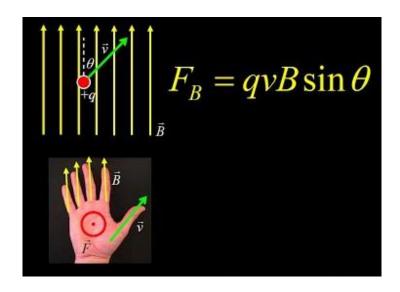
The electric force is straightforward, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right hand rule.

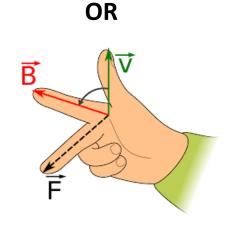


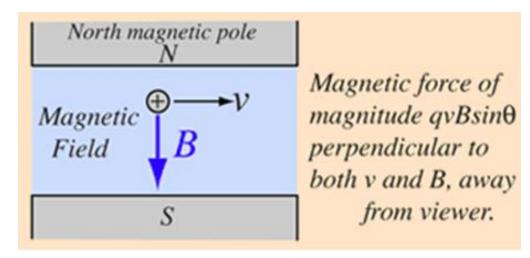
Important: so called <u>right hand rule for the magnetic force</u>.

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Used symbols: • vector points out of the screen

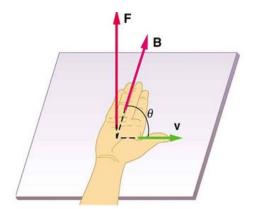
- vector points inside the screen

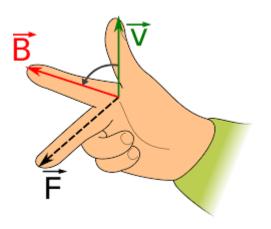
Lorentz force (law)

$$\vec{F} = q\vec{E} + q\vec{v}\vec{x}\vec{B}$$
Electric force force force



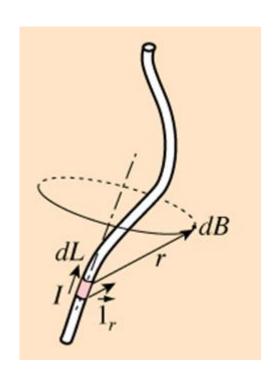
Beam of electrons moving in a circle, due to the presence of a magnetic field. Purple light is emitted along the electron path, due to the electrons colliding with gas molecules in the bulb.





Biot-Savart law

The Biot-Savart Law relates magnetic fields to the currents, which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources (finding the magnetic field resulting from a current distribution involves the vector product).



From empirism we know: $\left| dB \right| = K \frac{1}{r^2}$

Resulting field **B** we get as a result of integration along the wire.

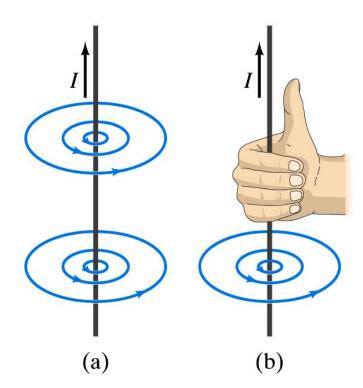
$$\left| \vec{B} \right| = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Biot-Savart law

Result after integration (for a direct wire):

$$\left| \vec{B} \right| = \frac{\mu_0}{2\pi} \frac{I}{r}$$

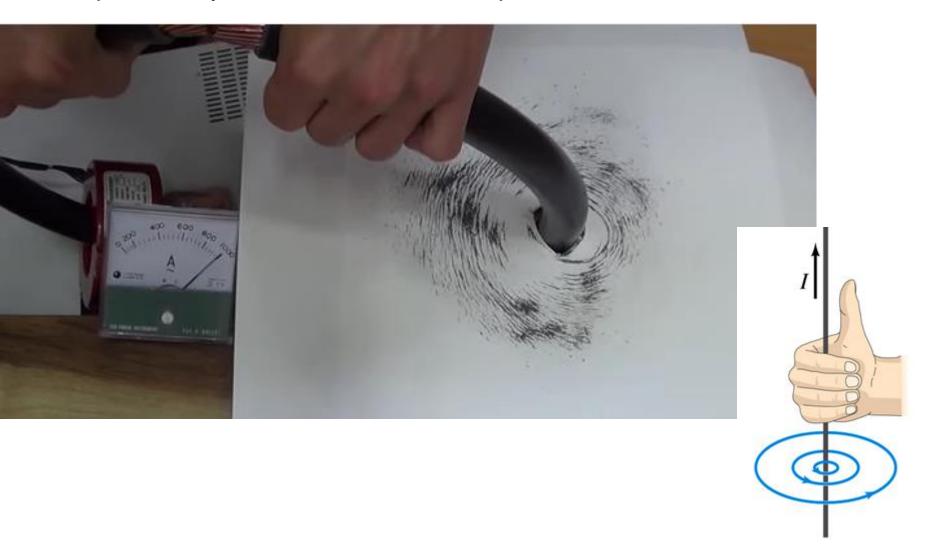
here *r* is the perpendicular distance from a direct wire (with direct current *l*)



Here we have a next right hand rule – showing the direction of **B**.

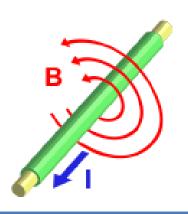
Biot-Savart law

The video from the begin of this lecture: https://www.youtube.com/watch?v=opJYLFvI-RE



Amper's law (as a consequence of Biot-Savart law)

Also called as Amper's circuital law.



Derivation:

C dl

r - radius of the circle (around the wire with *l*)

Along a circle the angle between these two vectors (**B** and d**l**) is zero, so it follows:

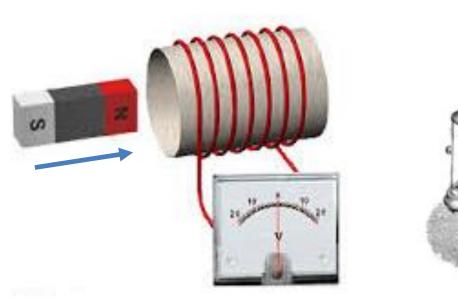
$$\oint_{I(S)} \vec{B} \cdot d\vec{l} = \oint_{I(S)} |\vec{B}| dl = |\vec{B}| 2\pi r = \mu_0 I$$

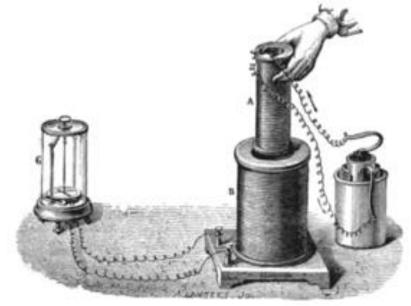
$$|\vec{B}| = \frac{\mu_0}{2\pi} \frac{I}{r}$$

When we now enter for the size of vector **B** the Biot-Savart law expression, we get the final result.

Faraday's Law of Induction describes how an electric current produces a magnetic field and, conversely, how a changing magnetic field generates an electric current in a conductor.

English physicist Michael Faraday gets the credit for discovering magnetic induction in 1830; however, an American physicist, Joseph Henry, independently made the same discovery about the same time.





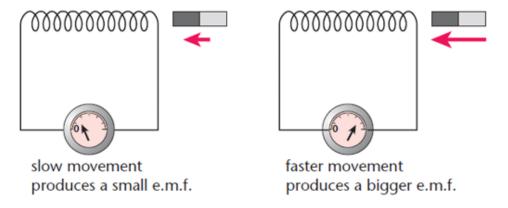
Faraday's law of induction (experiment): https://www.youtube.com/watch?v=vwldZjjd8fo

Quantitative aspects of Faraday's law of induction describe the change of the magnetic flux with time – it is equal to the electromotive force \mathcal{E} (EMF), measured in volts (!):

$$\varepsilon = Emf = -\frac{d\Phi_{B}}{dt}$$

unit:
$$[W/s = V \cdot s/s = V]$$

The direction of the electromotive force is given by Lenz's law.



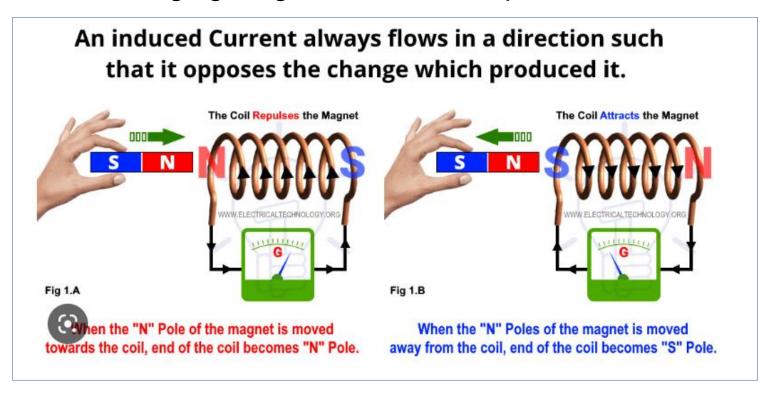
Comment: The word "force" is somewhat misleading, because EMF is not a force, but rather a "potential" to provide energy.

An electromotive force can be originated also in a situation, when a conductor is moving in a magnetic field – principle of a dynamo.

Faraday's law of induction (Lenz's law)

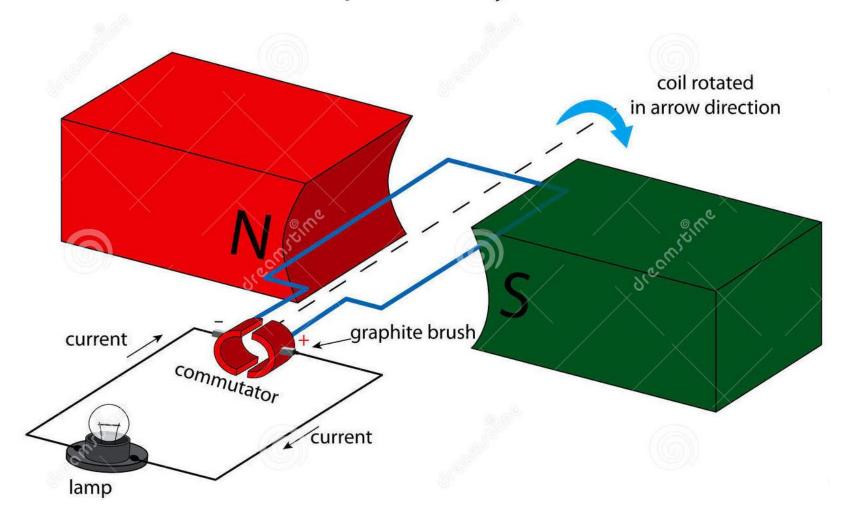
The direction of the electromotive force is given by Lenz's law:

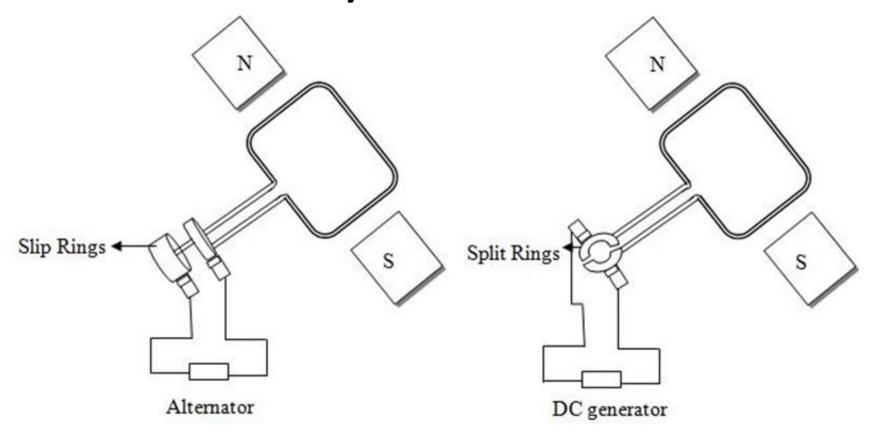
Lenz's law of electromagnetic induction states that the direction of the current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes the initial changing magnetic field which produced it.



An electromotive force can be originated also in a situation, when a conductor is moving in a magnetic field – principle of a dynamo.

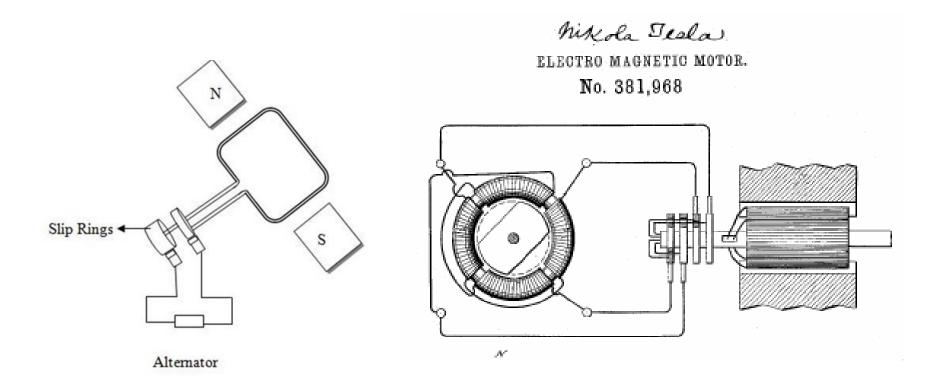
Simple d.c. Dynamo





Difference between a DC dynamo (DC generator) and AC dynamo (alternator) is the shape of the commutator.

In the case of DC dynamo it has split rings – to obtain the current of only one direction. An AC dynamo has slip rings.



Next very important contribution of Nicola Tesla: He suggested the use of so called rotating magnetic field in the case of the commutator (there was no need for the use of brushes). Result was the invention of so called induction motor.

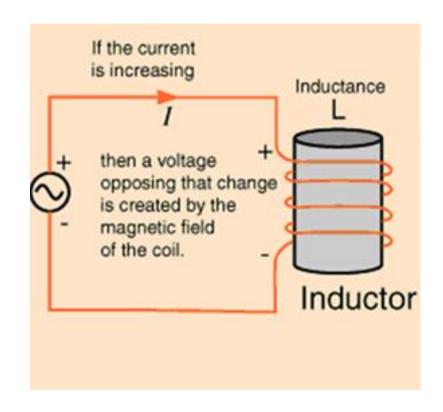
Faraday's law of induction - inductance

Inductance (L)

Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it (conductor is often in the shape of a coil). It is connected with the Electromotive force (Emf):

$$Emf = -L \frac{\Delta I}{\Delta t}$$

unit for L: [V-s/A] = [Henry]



Lecture 6: magnetism, electromagnetism

- comments to units

unit Weber

Unit of magnetic flux Φ_B .

A change in flux of one weber per second will induce an electromotive force of one volt (produce an electric potential difference of one volt across two open-circuited terminals).

The unit was named after Wilhelm Eduard Weber, a German physicist.

As an SI derived unit, the weber can also be expressed as:

$$Wb = \frac{kg \cdot m^2}{s^2 \cdot A} \neq V \cdot s \neq T \cdot m^2 = \frac{J}{A}$$

unit Farad

Unit of electrical capacitance.

It describes the ability of a body to store an electrical charge, it is equivalent to 1 coulomb per volt (C/V).

Named after the English physicist Michael Faraday (1791–1867).

$$F = \frac{s^4 \cdot A^2}{m^2 \cdot kg} = \frac{s^2 \cdot C^2}{m^2 \cdot kg} = \frac{C}{V} = \frac{A \cdot s}{V} = \frac{W \cdot s}{V^2} = \frac{J}{V^2} = \frac{N \cdot m}{V^2} = \frac{C^2}{J} = \frac{C^2}{N \cdot m} = \frac{s}{\Omega} = \frac{1}{\Omega \cdot Hz} = \frac{S}{Hz} = \frac{s^2}{Hz} = \frac{S}{Hz} = \frac{S}{Hz}$$

unit Tesla



Unit of magnetic induction **B**.

A particle, carrying a charge of one coulomb, and passing through a magnetic field of one tesla, at a speed of one metre per second, perpendicular to said field, experiences a force with magnitude one newton, according to the Lorentz force law.

One tesla is equal to one weber per square metre. The unit was announced during the General Conference on Weights and Measures in 1960 and is named in honour of Nikola Tesla.

As an SI derived unit, the tesla can also be expressed as:

$$T = \frac{V \cdot s}{m^2} = \frac{N}{A \cdot m} = \frac{Wb}{m^2} = \frac{kg}{C \cdot s} = \frac{N \cdot s}{C \cdot m}$$

In the older system CGS: The cgs unit is a Gauss (G)

$$\bullet$$
 1 T = 10⁴ G

unit Henry

Unit of electrical inductance L.

The inductance of an electric circuit is one henry when an electric current that is changing at one ampere per second results in an electromotive force of one volt across the inductor.

The unit was named in honour of Joseph Henry, an American physicist.

As an SI derived unit, the henry can also be expressed as:

$$H = \frac{kg \cdot m^2}{s^2 \cdot A^2} = \frac{kg \cdot m^2}{C^2} = \frac{J}{A^2} = \frac{T \cdot m^2}{A} = \frac{Wb}{A} = \frac{V \cdot s}{A} \neq \frac{s^2}{F} = \frac{1}{F \cdot Hz^2} = \Omega \cdot s$$

electromagnetism – some remarks to Maxwell's equations

Maxwell's equations are a set of 4 partial differential equations that, together with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits.

In the framework of this lecture, few laws build a part of the Maxwell's equation system (Gauss's law for magnetism, Ampere's circuital law and Faraday's law of induction),

But the formalism of Maxwell's equations will be the topic of next term – you can look forward to it! ;-)



Maxwell's equations

Name	Equation	
	Integral form	Differential form
Faraday's law of induction	$\oint_{c} \overrightarrow{E} \cdot d\overrightarrow{I} = -\iint_{S} \frac{\partial \overrightarrow{B}}{\partial t} \cdot d\overrightarrow{S}$	$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$
Ampère-Maxwell law	$\oint_{c} \overrightarrow{H} \cdot d\overrightarrow{I} = \iint_{S} \overrightarrow{J} \cdot d\overrightarrow{S} + \iint_{S} \frac{\partial \overrightarrow{D}}{\partial t} \cdot d\overrightarrow{S}$	$\nabla \times \overrightarrow{H} = \overrightarrow{J} + \frac{\partial \overrightarrow{D}}{\partial t}$
Gauss' electric law	$\iint_{S} \overrightarrow{D} \cdot d\overrightarrow{S} = \iiint_{V} \rho dV$	$ abla \cdot \overrightarrow{D} = oldsymbol{ ho}$
Gauss' magnetic law	$\oiint_{S} \vec{B} \cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$